Econ 301
Intermediate Microeconomics
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Final

You have 2h to complete the exam and the final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = p_2 = 10$ and Ace's income is $m = 300$. His utility function is

$$U(x_1, x_2) = (x_1)^{20} (x_2)^{20}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (80, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Show geometrically the optimum bundle of Ace – do not calculate it.

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = \min(x_1, x_2)$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2}\ln(x_1) + \frac{1}{2}\ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^A(x_1, x_2) = x_1 + x_2$ and $U^G(x_1, x_2) = x_1 + 2x_2$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$20 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 4 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = x^2$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, increasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Suppose the cost function is such that $ATC^{MES}= 2$ and $y^{MES}= 1$ and the demand is $D(p) = 4 - p$. Determine a number of firms in the industry given the free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 0$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces ten pounds of honey which sells at the price of \$2 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 2h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$5 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 5(t + h) - \frac{1}{2}t^2$$

a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)