Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final

You have 2h to complete the exam and the final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = p_2 = 10$ and Ace's income is $m = 300$. His utility function is

$$U(x_1, x_2) = (x_1)^{20} (x_2)^{20}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (80, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Show geometrically the optimum bundle of Ace – do not calculate it.

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = \min(x_1, x_2)$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2}\ln(x_1) + \frac{1}{2}\ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^A(x_1, x_2) = x_1 + x_2$ and $U^G(x_1, x_2) = x_1 + 2x_2$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$20 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 4 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = x^2$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, increasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Suppose the cost function is such that $ATC^{MES}= 2$ and $y^{MES}= 1$ and the demand is $D(p) = 4 - p$. Determine a number of firms in the industry given the free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 0$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces ten pounds of honey which sells at the price of \$2 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 2h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$5 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 5(t + h) - \frac{1}{2}t^2$$

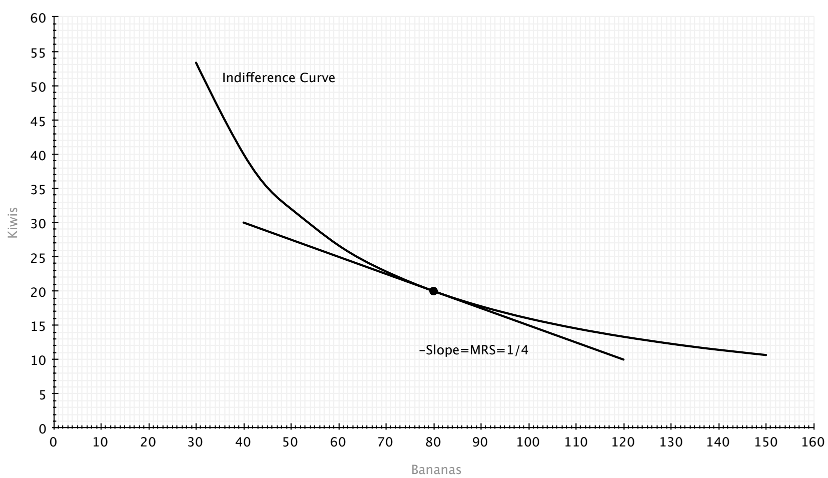
a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)

Final Solutions
ECON 301
May 13, 2012

Problem 1

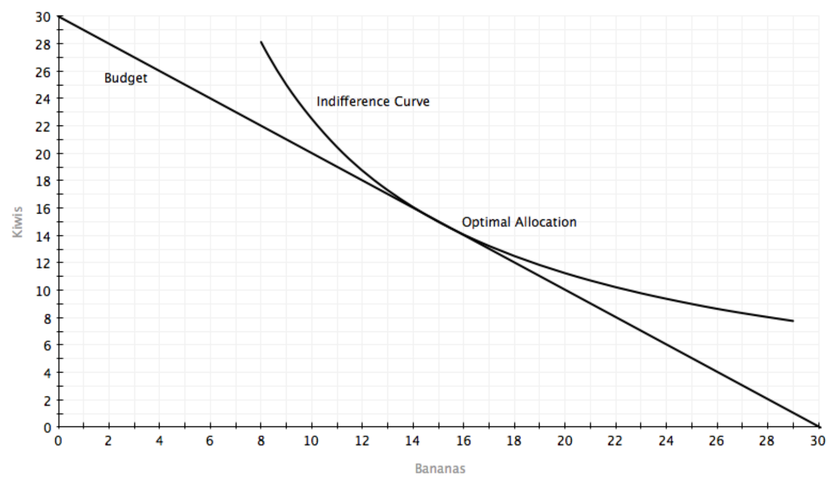
a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{1}{x_2}} = \frac{x_2}{x_1}$. At $(x_1, x_2) = (80, 20)$, $MRS = \frac{20}{80} = \frac{1}{4}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)

- Budget: $10x_1 + 10x_2 = 300$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

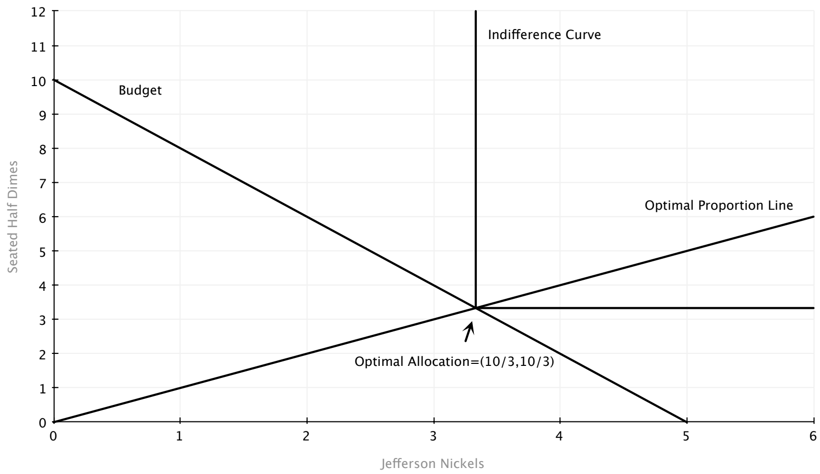
c) The optimal allocation is shown in the graph below



Problem 2

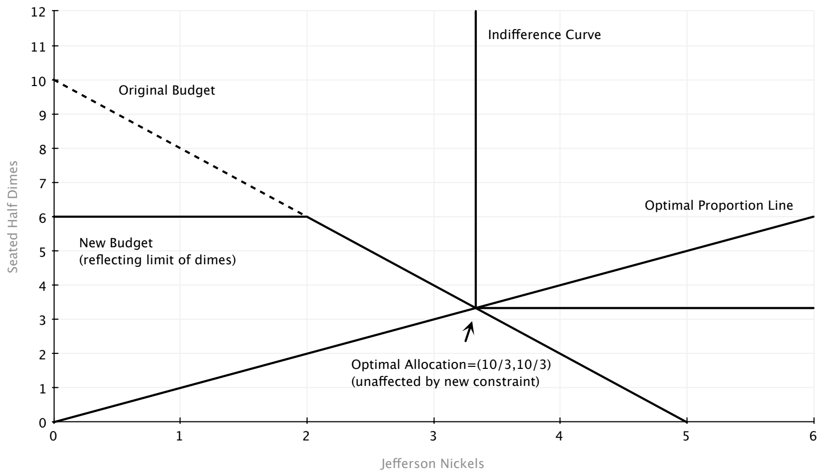
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * min(x_1, x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) The optimal bundle occurs where the optimal proportion line, $x_1 = x_2$, crosses the budget line, $4x_1 + 2x_2 = 20$. This happens when $(x_1, x_2) = (\frac{10}{3}, \frac{10}{3})$.



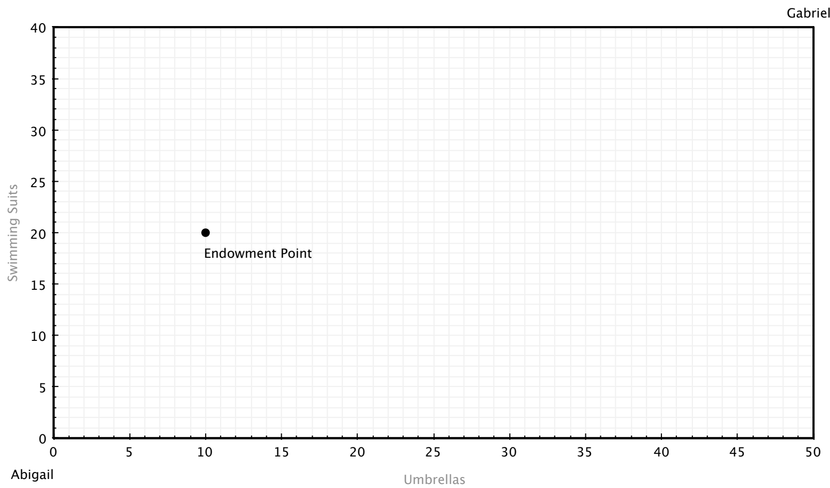
c) Giffen goods are goods that you consume more when their own price increases. Here we have $x_1 = x_2 = \frac{m}{p_1+p_2}$, so x_1 and x_2 are decreaseing in their own price: not Giffen goods.

d) The additional constraint is shown in the graph below, but it is not binding.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

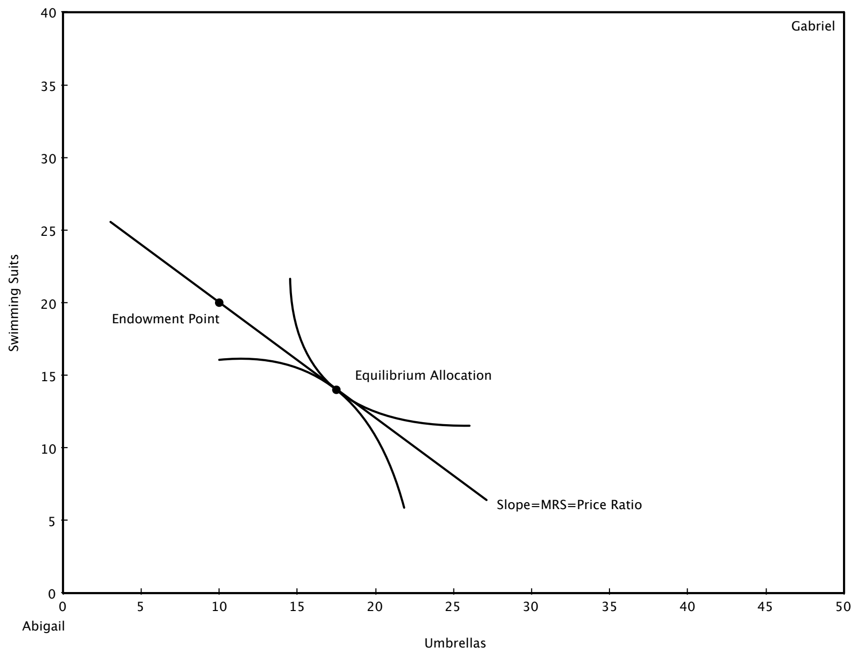
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

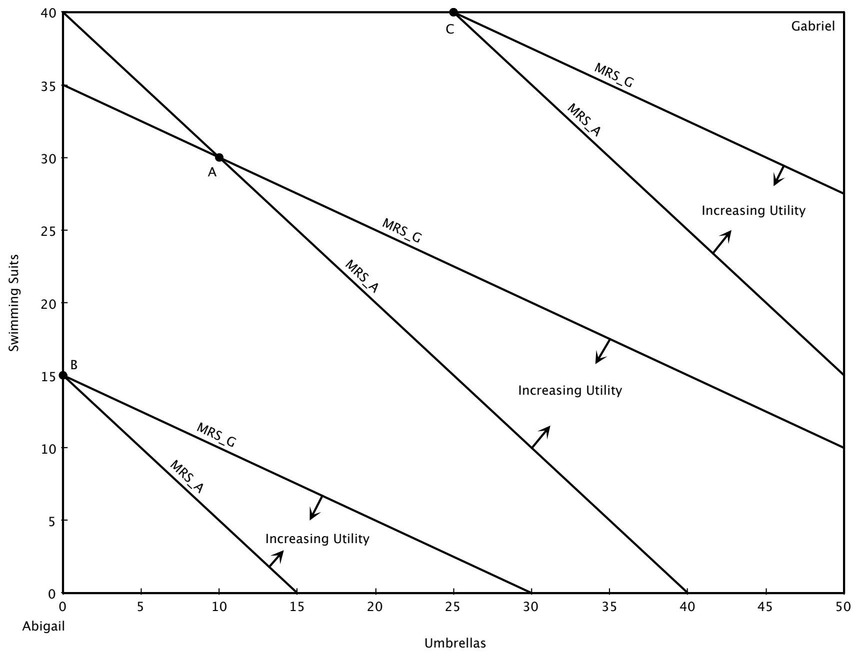
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

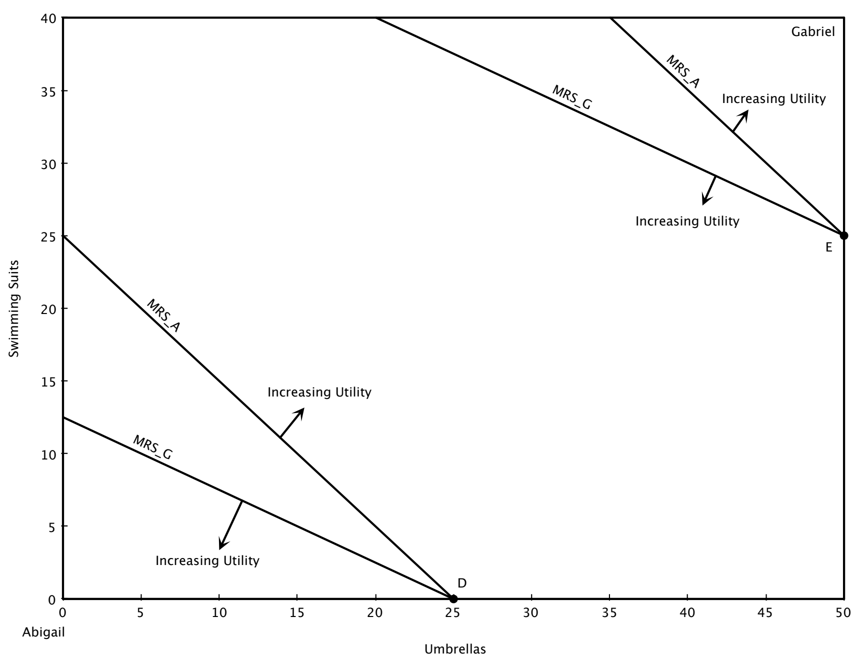
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



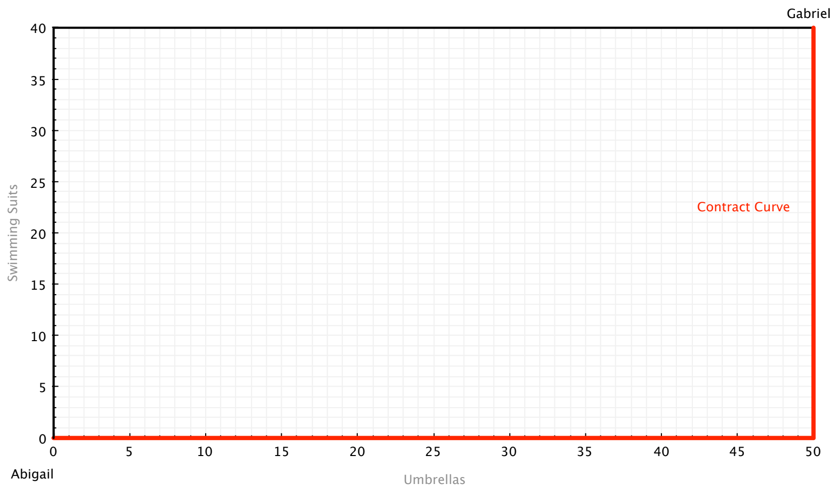
d) $MRS_A = 1$, and $MRS_G = 2$, so our condition for pareto optimality at an interior solution can never be satisfied. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider an interior point (A in the figure below), a point on the left border (B), and a point on the top border (C). In each case, both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning there are pareto improvements.



In contrast, if we look at a point on the bottom border (D), or one on the right border (E), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the points are pareto optimal.



To summarize, the contract curve of pareto optimal allocations consists of the bottom and right borders of the Edgeworth box.



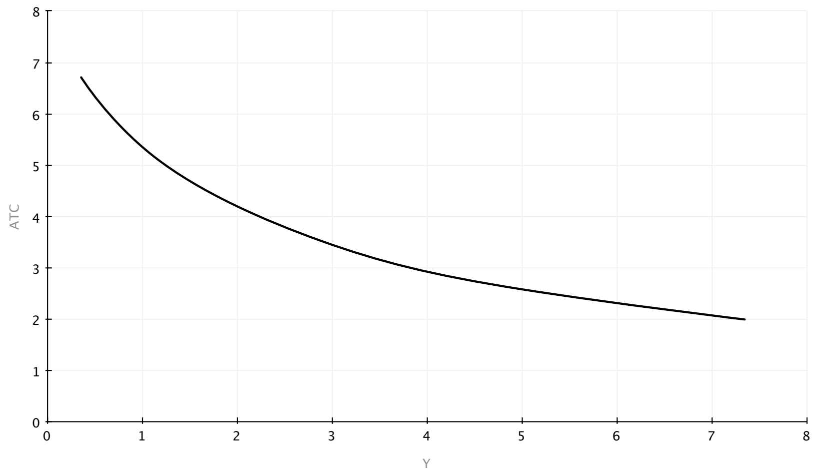
Alternative Argument: Let's normalize $p_2 = 1$ as usual, and then think about restrictions on p_1 that will allow the market to clear. If $p_1 < \frac{1}{2}$ then both Abigail and Gabriel only want to consume x_1, which is infeasible. If $p_1 > 1$, then both Abigail and Gabriel only want to consume x_2, which is also infeasible. If $\frac{1}{2} < p_1 < 1$ then Abigail only wants x_1, while Gabriel only wants x_2, so this corner solution will be feasible. If $p_1 = \frac{1}{2}$ Abigail only wants x_1, while Gabriel is indifferent between x_1 and x_2. Thus, the bottom border of the Edgeworth box (where Abigail has no x_2) is feasible. If $p_1 = 1$ Gabriel only wants x_2, while Abigail is indifferent between x_1 and x_2. Thus, the right border of the Edgeworth box (where Gabriel has no x_1) is feasible.

Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}x_w^2 + \frac{1}{2}x_l^2$. The certainty equivalent is defined by $ce^2 = \frac{1}{2}4^2 + \frac{1}{2}0^2 \Rightarrow ce = 2.83$. The expected value of the lottery is $\frac{1}{2}4 + \frac{1}{2}0 = 2$. The certainty equivalent is larger than the expected value because the bernouli utility function is convex, which is also the same thing as saying this person is risk loving.

c) $F(K, L) = K^a L^b$, with $1 < a$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



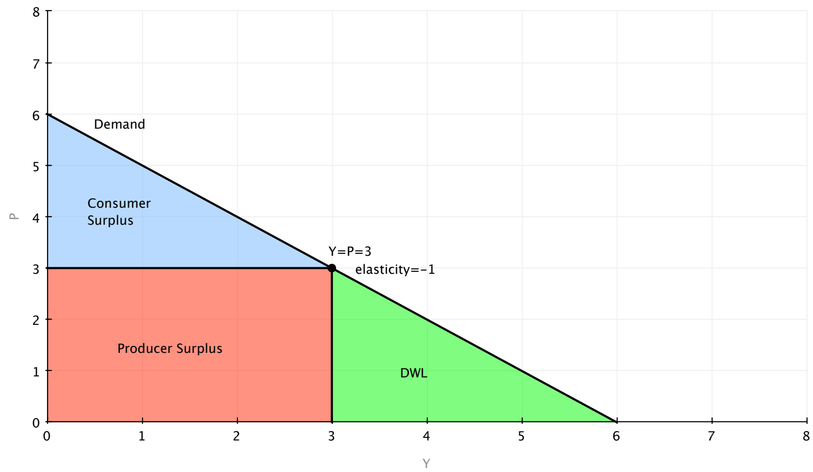
d) With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 2$, $D(p) = 2$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

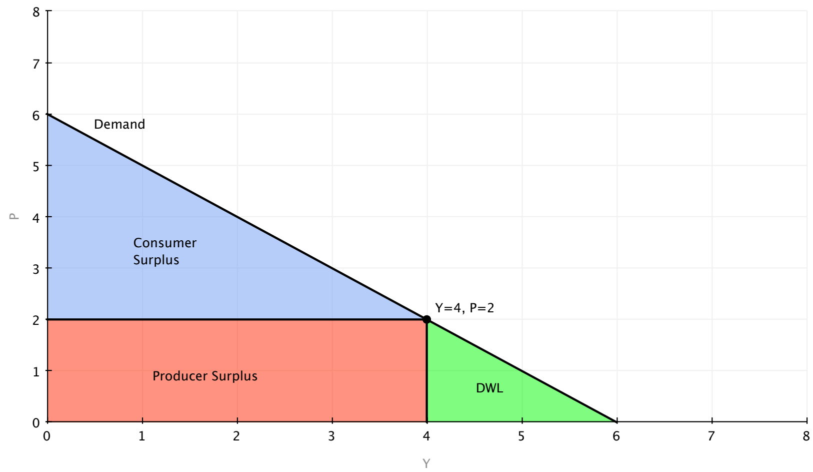
a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 0$, and make no profit. At $p = 0$ consumers purchase 6 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * 6 * 6 = 18$.

b) A monopolist chooses y to $\max(6 - y)y - 0$. The FOC of this problem is $6 - 2y = 0 \Rightarrow y = 3$. They charge price $p = 3$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium we have $\epsilon = -1 * \frac{3}{3} = -1$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 18.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6 - y_1 - y_2)y_1$. The FOC of this problem is $6 - 2y_1 - y_2 = 0$. Thus, the best response function for firm 1 is $y_1 = 3 - \frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 3 - \frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = 2$. Total production is 4, leaving $p = 2$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $2h - \frac{1}{2}h^2$. The FOC for this problem is $h = 2$. Given this choice of h, the orchard owner chooses t to max $5(t+2) - \frac{1}{2}t^2$. The FOC for this problem is $t = 5$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $5t + 7h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 7$, and the FOC for t is $t = 5$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final

You have 2h to complete the exam. The final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = 4, p_2 = 10$ and Ace's income is $m = 120$. His utility function is

$$U(x_1, x_2) = (x_1)^{20}(x_2)^{40}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (20, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Using magic forumula find the optimal bundle of Ace (two numbers), and show geometrically the .

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = x_1 + x_2$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2}\ln(x_1) + \frac{1}{2}\ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^i(x_1, x_2) = \min(x_1, x_2)$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$200 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 16 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = \sqrt{x}$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, decreasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Let the variable cost be $c(y) = y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). Given demand $D(p) = 8 - p$ determine a number of firms in the industry assuming free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 2y$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces one pound of honey which sells at the price of \$10 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 10h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$3 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 3(t + h) - \frac{1}{2}t^2$$

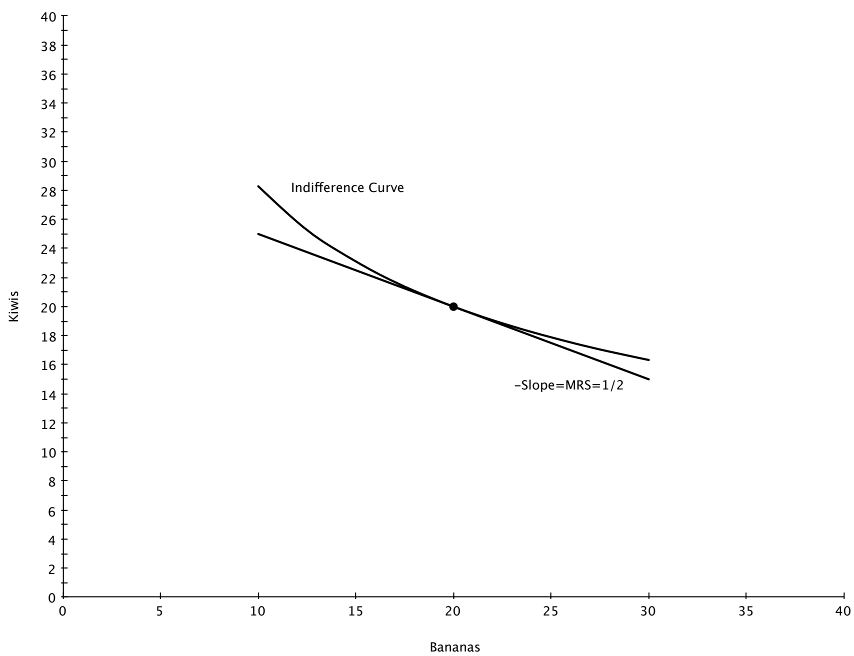
a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)

Makeup Final Solutions
ECON 301
May 15, 2012

Problem 1

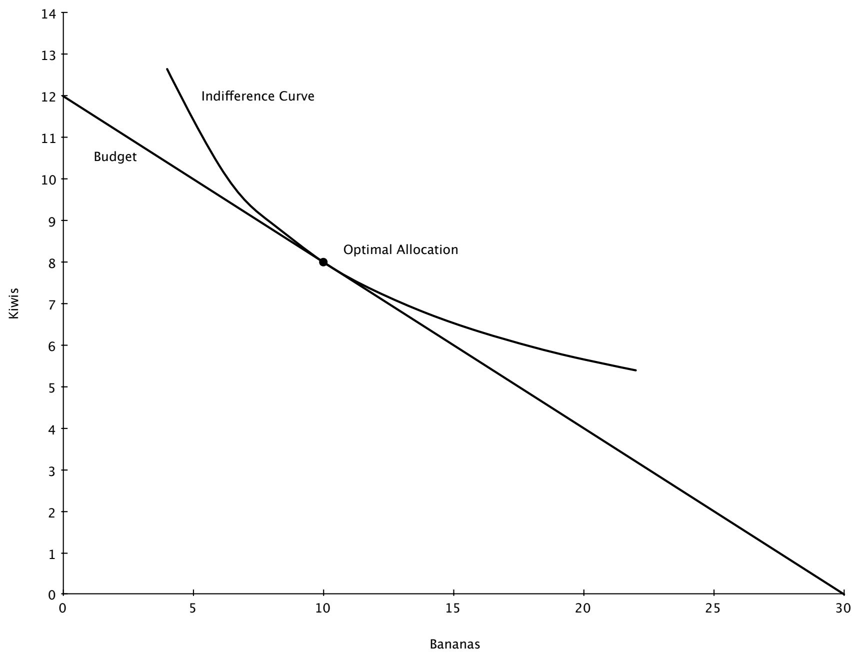
a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + 2 \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{2}{x_2}} = \frac{x_2}{2x_1}$. At $(x_1, x_2) = (20, 20)$, $MRS = \frac{20}{40} = \frac{1}{2}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)

- Budget: $4x_1 + 10x_2 = 120$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

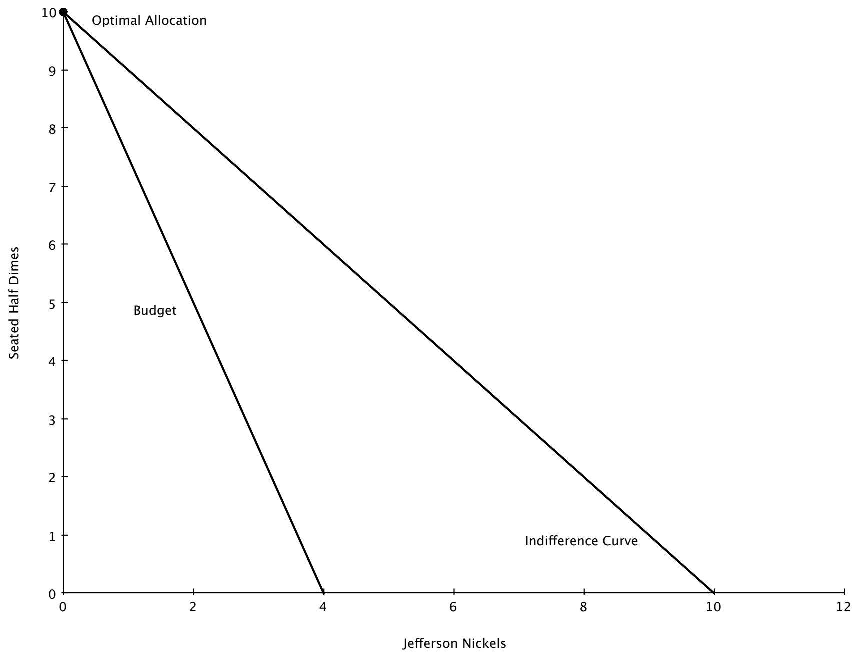
c) The optimal allocation is shown in the graph below



Problem 2

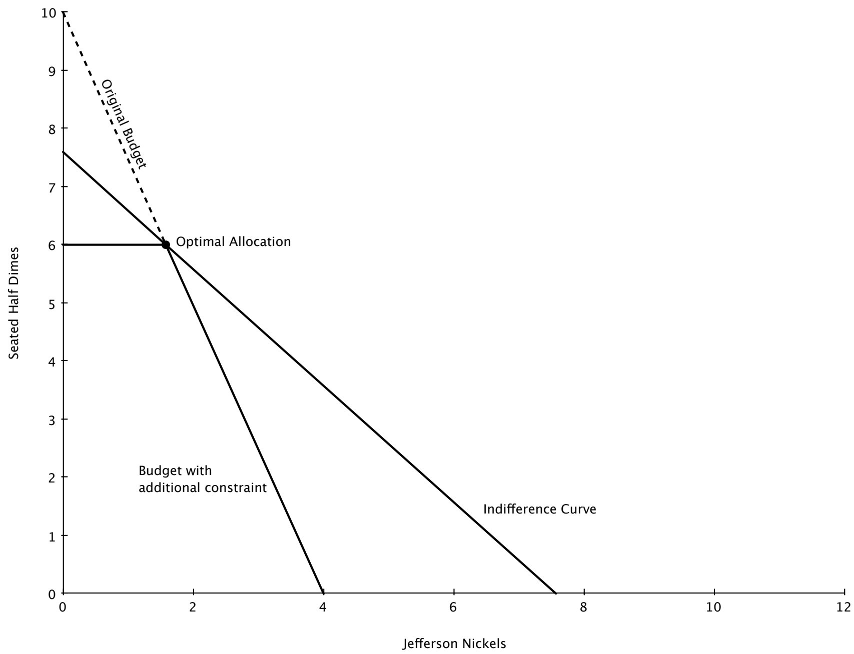
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * (x_1 + x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) Since we are dealing with perfect substitues we know we will have a corner solution. We will choose only the good that delivers utility in the least expensive manner. Because each unit of x_1 and x_2 give the same amount of utility, this will be the cheaper good, x_2. At $p_2 = 2$ and $m = 20$ we can afford $x_2 = 10$.



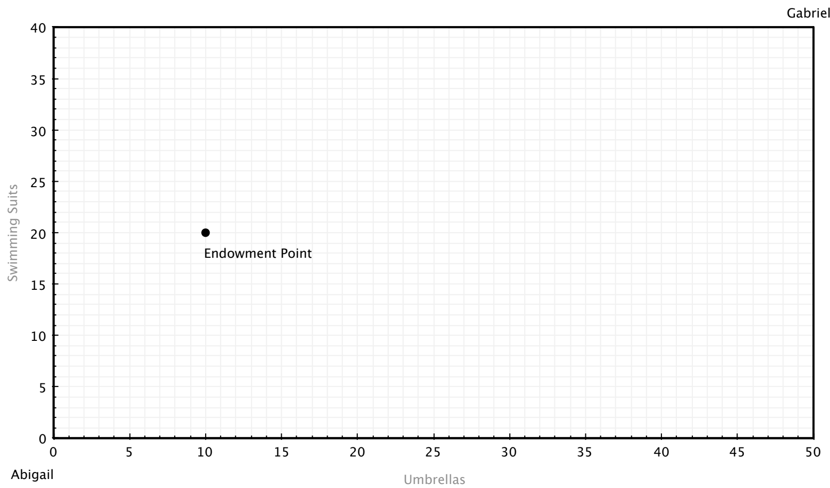
c) Giffen goods are goods that you consume more when their own price increases. Here you spend all your money on the cheaper good. As the price of that good increases you can buy less of it, until it becomes the more expensive good at which point you switch entirely to the other good: not Giffen goods.

d) As shown in the graph below, the additional constraint forces you to start buying Jefferson Nickels after all 6 Seated Half Dimes have been purchased.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

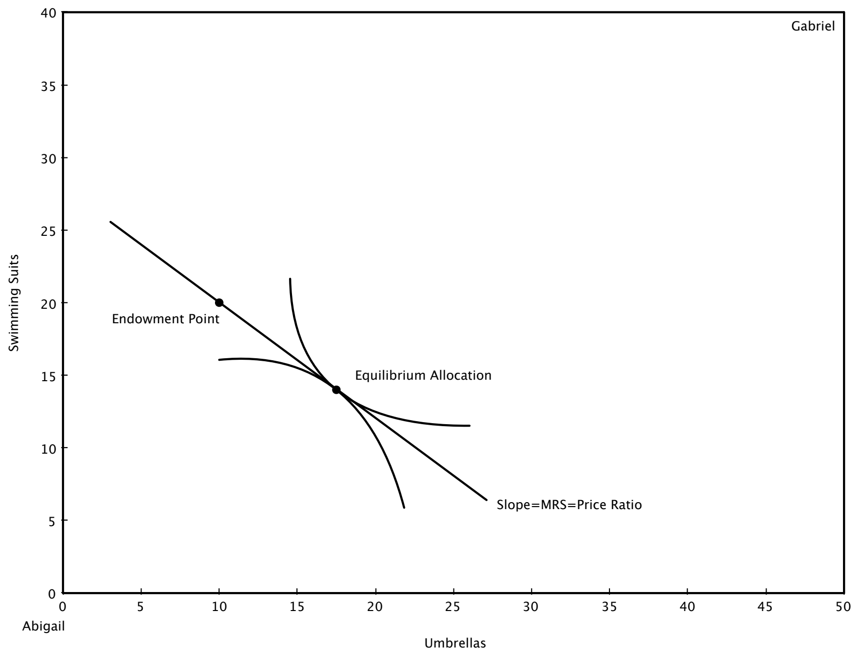
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

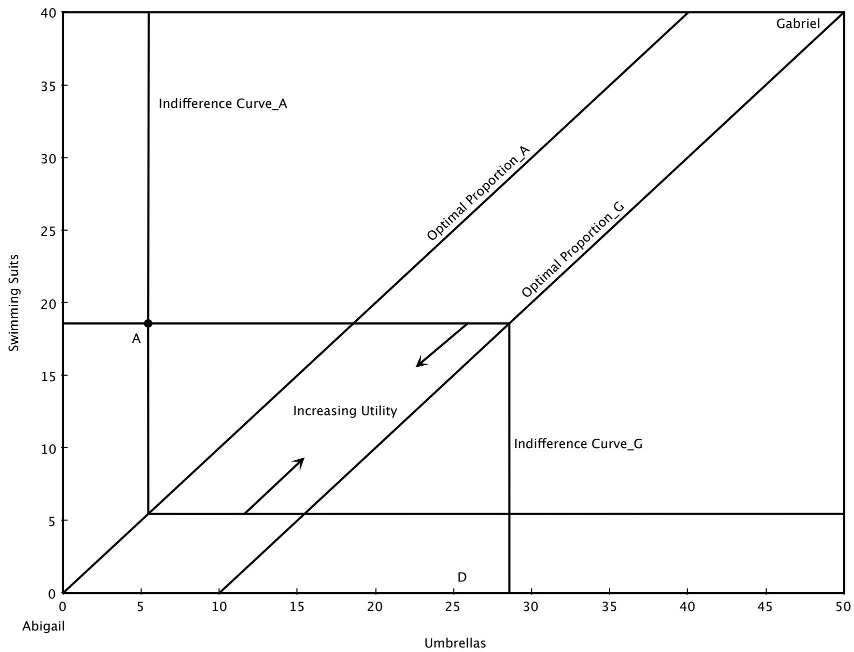
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

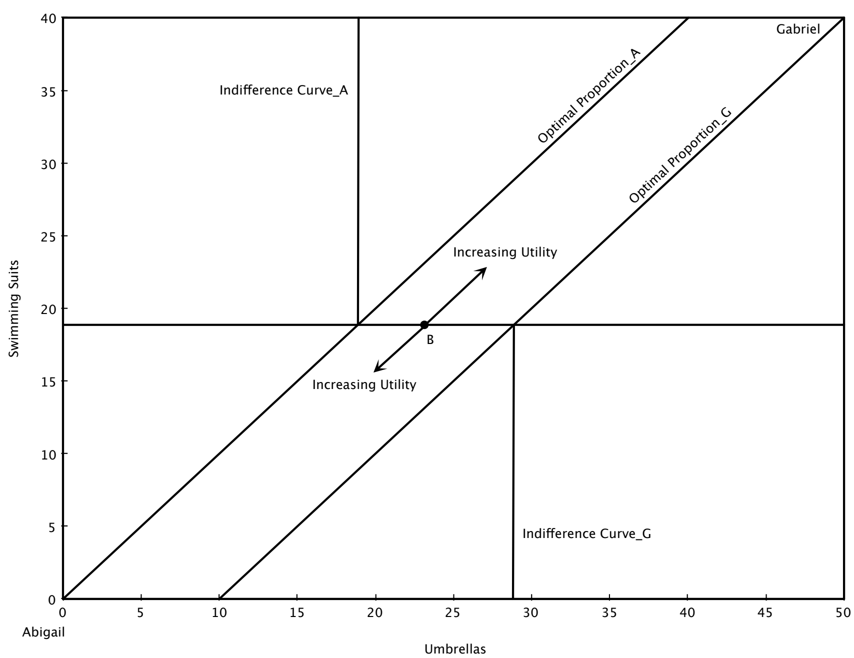
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



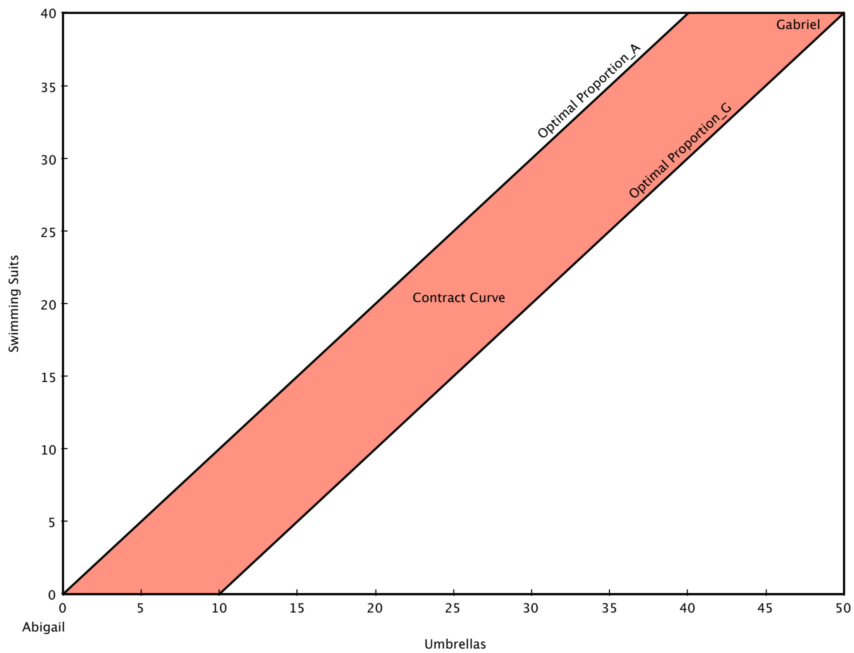
d) With perfect complements the MRS is not defined at the optimal point, so we can't equate them to find the contract curve. The optimal proportion line for both Abigail and Gabriel is where $x_1 = x_2$, but because the Edgeworth box is not square these lines do not coincide. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider a point outside the two optimal proportion lines (A in the figure below). Both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning is a pareto improvement.



In contrast, if we look at a point in between the two optimal proportion lines (B), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the point is pareto optimal.



To summarize, the contract curve of pareto optimal allocations is the space in between the two optimal proportion lines.

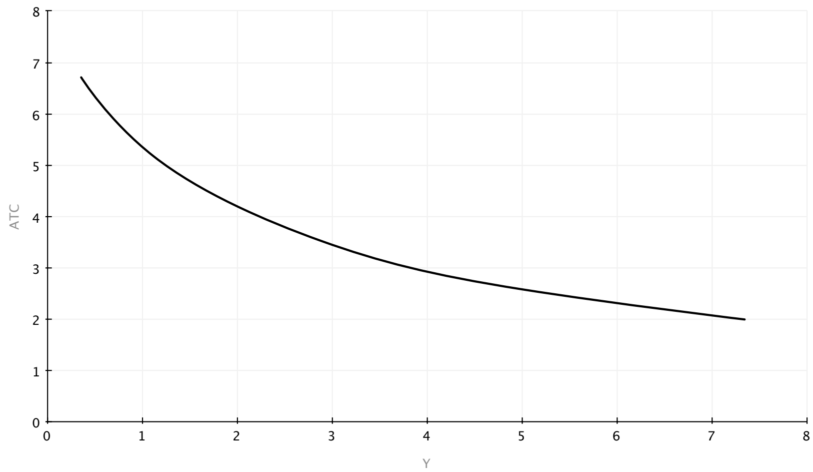


Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}\sqrt{x_w} + \frac{1}{2}\sqrt{x_l}$. The certainty equivalent is defined by $\sqrt{ce} = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{0} \Rightarrow ce = 4$. The expected value of the lottery is $\frac{1}{2}16 + \frac{1}{2}0 = 8$. The certainty equivalent is smaller than the expected value because the bernouli utility function is concave, which is also the same thing as saying this person is risk averse.

c) $F(K, L) = K^a L^b$, with $0 < a < 1$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



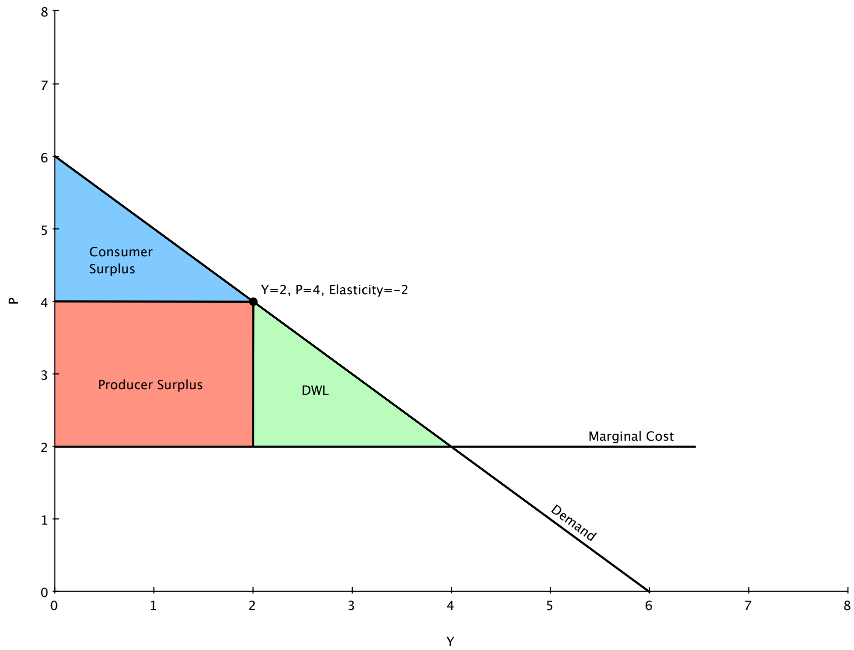
d) Total cost is given by $y^2 + 4$, which makes $ATC = y + \frac{4}{y}$. We minimize this function to find ATC^{MES} and y^{MES}. Since it is a convex function the FOC will find the minimum. The FOC is $1 - \frac{4}{y^2} = 0 \Rightarrow y^{MES} = 2$. Then, $ATC^{MES} = 2 + \frac{4}{2} = 4$. With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 4$, $D(p) = 4$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

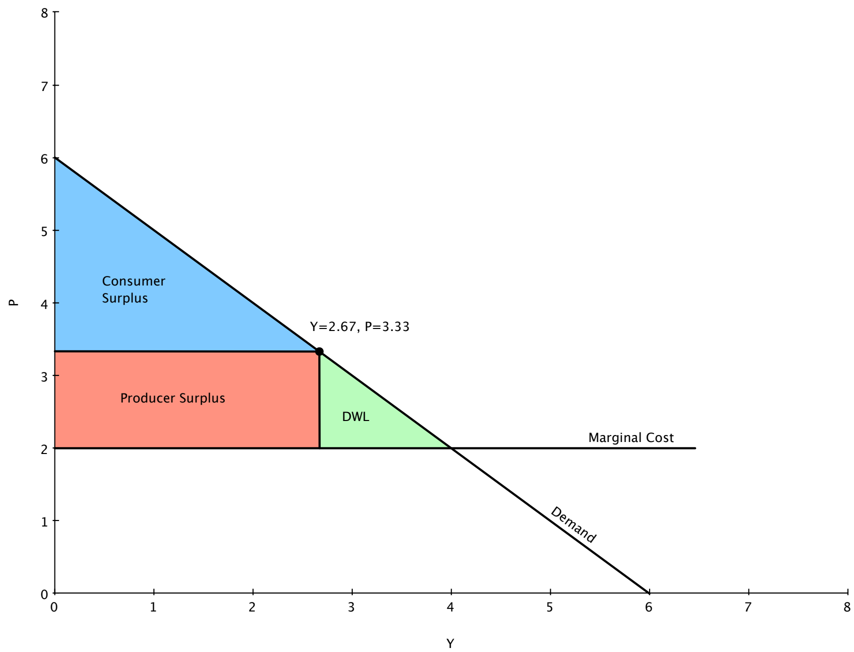
a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 2$, and make no profit. At $p = 2$ consumers purchase 4 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * (6 - 2) * 4 = 8$.

b) A monopolist chooses y to $\max(6 - y)y - 2y$. The FOC of this problem is $6 - 2y = 2 \Rightarrow y = 2$. They charge price $p = 4$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium we have $\epsilon = -1 * \frac{4}{2} = -2$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay, and will do so as long as that price is larger than the marginal cost of 2. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 8.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6-y_1-y_2)y_1 - 2y_1$. The FOC of this problem is $4 - 2y_1 - y_2 = 0$. Thus, the best response function for firm 1 is $y_1 = 2 - \frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 2 - \frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = \frac{4}{3}$. Total production is $2\frac{2}{3}$, leaving $p = 3\frac{1}{3}$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $10h - \frac{1}{2}h^2$. The FOC for this problem is $h = 10$. Given this choice of h, the orchard owner chooses t to max $3(t + 10) - \frac{1}{2}t^2$. The FOC for this problem is $t = 3$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $3t + 13h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 13$, and the FOC for t is $t = 3$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.