Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (A)

You have 2h to complete the exam and the final consists of 6 questions (15+10+25+15+20+15=100).

Problem 1. (Consumer Choice)

Jeremy's favorite flowers are tulips x_1 and daffodils x_2. Suppose $p_1 = 2$, $p_2 = 4$ and $m = 40$.

a) Write down Jeremy's budget constraint (a formula) and plot all Jeremy's affordable bundles in the graph (his budget set). Find the slope of a budget line (number). Give an economic interpretation for the slope of the budget line (one sentence).

b) Jeremy's utility function is given by

$$U(x_1, x_2) = \sqrt{(\ln x_1 + \ln x_2)^2 + 7}$$

Propose a simpler utility function that represents the same preferences (give a formula). Explain why your utility represents the same preferences (one sentence).

c)[1] Plot Jeremy's indifference curve map (graph), find MRS analytically (give a formula) and find its value at bundle $(2, 4)$ (one number). Give economic interpretation of this number (one sentence). Mark its value in the graph.

d) Write down two secrets of happiness (two equalities) that allow determining the optimal bundle. Provide their geometric interpretation (one sentence for each). Find the optimal bundle (x_1, x_2) (two numbers). Is your solution interior? (a yes -no answer)

e) Hard: Find the optimal bundle given $p_1 = 2$, $p_2 = 4$ and $m = 40$ assuming $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers). Is your solution interior? (a yes -no answer)

Problem 2. (Producers)

Consider production function given by $F(K, L) = 3K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) Using the λ argument demonstrate that production function exhibits decreasing returns to scale.

b) Derive the cost function given $w_K = w_L = 9$.

c) Derive a supply function of a competitive firm, assuming the cost function from b) and fixed cost $F = 2$ (give a formula for $y(p)$). Plot the supply function in a graph, marking the threshold price below which a firm chooses inaction.

Problem 3. (Competitive Equilibrium)

Consider an economy with apples and oranges. Dustin's endowment of two commodities is given by $\omega^D = (8, 2)$ and Kate's endowment is $\omega^K = (2, 8)$. The utility functions of Dustin and Kate are the same and given by

$$U^i\left(x_1^i, x_2^i\right) = 5\ln x_1^i + 5\ln x_2^i$$

where $i = D, K$.

a) Plot the Edgeworth box and mark the point corresponding to the initial endowments.

b) Give a general definition of Pareto efficient allocation x (one sentence) and state its equivalent condition in terms of MRS (one sentence, you do not need to prove the equivalence).

c) Using the "MRS" condition verify that the initial endowments are not Pareto efficient.

d) Find a competitive equilibrium (six numbers). Provide an example of a competitive equilibrium with some other prices (six numbers).

e) Using MRS condition verify that the competitive allocation is Pareto efficient.

f) Hard: Find prices p_1, p_2 in a competitive equilibrium for identical preferences of two agents $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers, no calculations). Explain why any two prices that give rise to a relative price higher than p_1/p_2 cannot be equilibrium prices (which condition of equilibrium fails?)

[1] If you do not know the answer to b), to get partial credit in points c)-e) your can assume $U(x_1, x_2) = x_1x_2$.

Problem 4. (Short Questions)

a) Uncertainty: Find the certainty equivalent of a lottery which, in two equally likely states, pays $(0, 9)$. Bernoulli utility function is $u(c) = \sqrt{c}$ (one number). Is the certainty equivalent smaller or bigger than the expected value of a lottery 4.5. Why? (one sentence)

b) Market for lemons: In a market for racing horses one can find two types of animals: champions (Plums) and ordinary recreational horses (Lemons). Buyers can distinguish between the two types only long after they buy a horse. The values of the two types of horses for buyers and sellers are summarized in the table

	Lemon	Plum
Seller	1	4
Buyer	2	5

Are champions (Plums) going to be traded if probability of Lemons is $\frac{1}{2}$. (yes-no). Why? (a one sentence argument that involves the expected value of a horse to a buyer)

c) Signaling: The productivity of high ability workers (and hence the competitive wage rate) is 1000 while productivity of low ability workers is only 400. To determine the type, employer can, first offer an internship program with the length of x months, during which a worker has to demonstrate her high productivity. A low ability worker by putting extra effort can mimic high ability performance, which costs him $c(x) = 200x$. Find the minimal length x for which the internship becomes a credible signal of high ability. (one number)

d) PV of Perpetuity: You can rent an apartment paying 1000 per month (starting next month, till the "end of the world") or you can buy the apartment for 100.000. Which option are you going to chose if monthly interest rate is $r = 2\%$? (find the PV of rent and compare two numbers)

Problem 5. (Market Power)

Consider a monopoly facing the inverse demand $p(y) = 25 - y$, and with total cost $TC(y) = 5y$.

a) Find the marginal revenue of a monopoly, $MR(y)$ and depict it in a graph together with the demand (formula +graph). Which is bigger: price or marginal revenue? Why? (one sentence)

b) Find the optimal level of production and price (two numbers). Illustrate the optimal choice in a graph, depicting Consumer and Producer Surplus, and DWL (three numbers +graph).

c) Find equilibrium markup (one number).

d) First Degree Price Discrimination: Find Total Surplus, Consumer, Producer Surplus and DWL if monopoly can perfectly discriminate among buyers and quantities. (four numbers +graph)

e) Hard: find the individual level of production and price in a Cournot-Nash equilibrium with N identical firms with cost $TC(y) = 5y$, both as a function of N (two formulas). Argue that the equilibrium price converges to the marginal cost as N goes to infinity.

Problem 6. (Public good: Music downloads)

Freddy and Miriam share the same collection of songs downloaded from i-tunes (they have one PC). Each song costs 1. If Freddy downloads x^F and Miriam x^M, their collection contains $x^F + x^M$ and utility of Freddy (net of the cost) is given by

$$u^F\left(x^F\right) = 200\ln(x^F + x^M) - x^F,$$

while Miriam's utility (net of the cost) is

$$u^M\left(x^M\right) = 100\ln(x^F + x^M) - x^M,$$

(Observe that Freddy is more into music than Miriam.)

a) Find optimal number of downloads by Freddy x^F (his best response) for any choice of Miriam x^M (formula $x^F = R^F(x^M)$). Plot the best response in the coordinate system (x^F, x^M).

(Hint: You do not need prices. Utility functions are net cost and hence you just have to take the derivative with respect to x^F and equalize it to zero).

b) Find the optimal number of downloads by Miriam x^M, (her best response) for any choice of Freddy x^F and plot it in the coordinate system from point a).

c) Find the number of downloads in the Nash equilibrium (two numbers). Do we observe the free riding problem? (yes-no + one sentence)

d) Hard: Find Pareto efficient number of downloads $x = x^M + x^F$ (one number). Compare the Pareto efficient level of x with the equilibrium one. Which is bigger and why?

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Final Exam (B)

You have 2h to complete the exam and the final consists of 6 questions (15+10+25+15+20+15=100).

Problem 1. (Consumer Choice)

Jeremy's favorite flowers are tulips x_1 and daffodils x_2. Suppose $p_1 = 5$, $p_2 = 10$ and $m = 100$.

a) Write down Jeremy's budget constraint (a formula) and plot all Jeremy's affordable bundles in the graph (his budget set). Find the slope of a budget line (number). Give an economic interpretation for the slope of the budget line (one sentence).

b) Jeremy's utility function is given by

$$U(x_1, x_2) = \sqrt{(3 \ln x_1 + 3 \ln x_2)^4 + 8}$$

Propose a simpler utility function that represents the same preferences (give a formula). Explain why your utility represents the same preferences (one sentence).

c)[2] Plot Jeremy's indifference curve map (graph), find MRS analytically (give a formula) and find its value at bundle $(2, 4)$ (one number). Give economic interpretation of this number (one sentence). Mark its value in the graph.

d) Write down two secrets of happiness (two equalities) that allow determining the optimal bundle. Provide their geometric interpretation (one sentence for each). Find the optimal bundle (x_1, x_2) (two numbers). Is your solution interior? (a yes -no answer)

e) Hard: Find the optimal bundle given $p_1 = 5$, $p_2 = 10$ and $m = 100$ assuming $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers). Is your solution interior? (a yes -no answer)

Problem 2. (Producers)

Consider production function given by $F(K, L) = 5K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) Using the λ argument demonstrate that production function exhibits decreasing returns to scale.

b) Derive the cost function given $w_K = w_L = 25$.

c) Derive a supply function of a competitive firm, assuming the cost function from b) and fixed cost $F = 2$ (give a formula for $y(p)$). Plot the supply function in a graph, marking the threshold price below which a firm chooses inaction.

Problem 3. (Competitive Equilibrium)

Consider an economy with apples and oranges. Dustin's endowment of two commodities is given by $\omega^D = (20, 10)$ and Kate's endowment is $\omega^K = (10, 20)$. The utility functions of Dustin and Kate are the same and given by

$$U^i\left(x_1^i, x_2^i\right) = 4 \ln x_1^i + 4 \ln x_2^i$$

where $i = D, K$.

a) Plot the Edgeworth box and mark the point corresponding to the initial endowments.

b) Give a general definition of Pareto efficient allocation x (one sentence) and state its equivalent condition in terms of MRS (one sentence, you do not need to prove the equivalence).

c) Using the "MRS" condition verify that the initial endowments are not Pareto efficient.

d) Find a competitive equilibrium (six numbers). Provide an example of a competitive equilibrium with some other prices (six numbers).

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f) Hard: Find prices p_1, p_2 in a competitive equilibrium for identical preferences of two agents $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers, no calculations). Explain why any two prices that give rise to a relative price higher than p_1/p_2 cannot be equilibrium prices (which condition of equilibrium fails?)

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[2] If you do not know the answer to b), to get partial credit in points c)-e) your can assume $U(x_1, x_2) = x_1x_2$.

Problem 4. (Short Questions)

a) Uncertainty: Find the certainty equivalent of a lottery which, in two equally likely states, pays $(16, 0)$. Bernoulli utility function is $u(c) = \sqrt{c}$ (one number). Is the certainty equivalent smaller or bigger than the expected value of a lottery 8. Why? (one sentence)

b) Market for lemons: In a market for racing horses one can find two types of animals: champions (Plums) and ordinary recreational horses (Lemons). Buyers can distinguish between the two types only long after they buy a horse. The values of the two types of horses for buyers and sellers are summarized in the table

	Lemon	Plum
Seller	1	6
Buyer	2	8

Are champions (Plums) going to be traded if probability of Lemons is $\frac{1}{2}$. (yes-no). Why? (a one sentence argument that involves the expected value of a horse to a buyer)

c) Signaling: The productivity of high ability workers (and hence the competitive wage rate) is 1000 while productivity of low ability workers is only 400. To determine the type, employer can, first offer an internship program with the length of x months, during which a worker has to demonstrate her high productivity. A low ability worker by putting extra effort can mimic high ability performance, which costs him $c(x) = 200x$. Find the minimal length x for which the internship becomes a credible signal of high ability. (one number)

d) PV of Perpetuity: You can rent an apartment paying 1000 per month (starting next month, till the "end of the world") or you can buy the apartment for 30.000. Which option are you going to chose if monthly interest rate is $r = 2\%$? (find the PV of rent and compare two numbers)

Problem 5. (Market Power)

Consider a monopoly facing the inverse demand $p(y) = 40 - y$, and with total cost $TC(y) = 20y$.

a) Find the marginal revenue of a monopoly, $MR(y)$ and depict it in a graph together with the demand (formula +graph). Which is bigger: price or marginal revenue? Why? (one sentence)

b) Find the optimal level of production and price (two numbers). Illustrate the optimal choice in a graph, depicting Consumer and Producer Surplus, and DWL (three numbers +graph).

c) Find equilibrium markup (one number).

d) First Degree Price Discrimination: Find Total Surplus, Consumer, Producer Surplus and DWL if monopoly can perfectly discriminate among buyers and quantities. (four numbers +graph)

e) Hard: find the individual level of production and price in a Cournot-Nash equilibrium with N identical firms with cost $TC(y) = 5y$, both as a function of N (two formulas). Argue that the equilibrium price converges to the marginal cost as N goes to infinity.

Problem 6. (Public good: Music downloads)

Freddy and Miriam share the same collection of songs downloaded from i-tunes (they have one PC). Each song costs 1. If Freddy downloads x^F and Miriam x^M, their collection contains $x^F + x^M$ and utility of Freddy (net of the cost) is given by

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(Observe that Freddy is more into music than Miriam.)

a) Find optimal number of downloads by Freddy x^F (his best response) for any choice of Miriam x^M (formula $x^F = R^F(x^M)$). Plot the best response in the coordinate system (x^F, x^M).

(Hint: You do not need prices. Utility functions are net cost and hence you just have to take the derivative with respect to x^F and equalize it to zero).

b) Find the optimal number of downloads by Miriam x^M, (her best response) for any choice of Freddy x^F and plot it in the coordinate system from point a).

c) Find the number of downloads in the Nash equilibrium (two numbers). Do we observe the free riding problem? (yes-no + one sentence)

d) Hard: Find Pareto efficient number of downloads $x = x^M + x^F$ (one number). Compare the Pareto efficient level of x with the equilibrium one. Which is bigger and why?

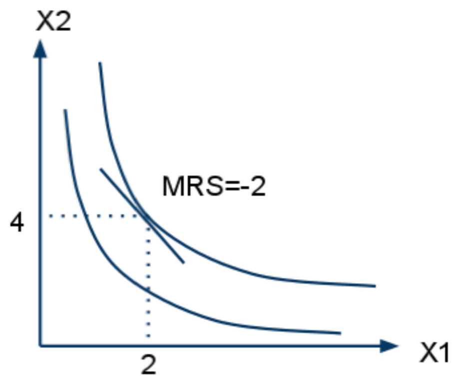
Econ 301 Final Exam Solutions - Spring 2011
Group A

Problem 1 - Consumer Choice

a) The budget constraint is $2x_1 + 4x_2 = 40$. The slope of the budget line is $-\frac{p_1}{p_2} = -\frac{2}{4} = -0.5$. Interpretation of the slope: the relative price of one tulip in the market is $\frac{1}{2}$ daffodil.

b) $U = \ln x_1 + \ln x_2$ or $U = x_1 x_2$ will represent the same preferences, since the operations of taking square root, adding a constant and taking the square of a function are all monotone transformations.

c)$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{x_2}{x_1}$. At $(2, 4)$ $MRS = -2$. Interpretation: given 2 tulips and 4 daffodils, Jeremy is willing to trade 1 tulip for 2 daffodils.



d) The two secrets of happiness are

$$2x_1 + 4x_2 = 40$$
$$-\frac{x_2}{x_1} = -0.5$$

The first condition implies the optimal bundle lies on the budget line. The second condition guarantees the indifference curve is tangent to the budget line at optimum. (MRS equals the slope of the budget line)
The optimal bundle is $(x_1, x_2) = (10, 5)$. The solution is interior since $x_1 \neq 0$ and $x_2 \neq 0$.

e) The function $U = 2x_1 + 3x_2$ represents preferences over perfect substitutes. $\frac{MU_1}{p_1} = \frac{2}{2} = 1 > \frac{MU_2}{p_2} = \frac{3}{4}$. Hence good 1 only will be consumed, and optimal $(x_1, x_2) = (20, 0)$. The solution is not interior.

Problem 2 - Producers

a) Take $\lambda > 1$. $F(\lambda K, \lambda L) = 3(\lambda K)^{\frac{1}{4}}(\lambda L)^{\frac{1}{4}} = \lambda^{\frac{1}{2}} \cdot 3K^{\frac{1}{4}}L^{\frac{1}{4}}$
$\lambda F(K, L) = \lambda \cdot 3K^{\frac{1}{4}}L^{\frac{1}{4}}$. Then $F(\lambda K, \lambda L) < \lambda F(K, L)$ and the production function exhibits decreasing returns to scale.

b) First apply the cost-minimization condition:

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L}$$

Given $w_K = w_L = 9$, $\frac{w_K}{w_L} = 1$. Plug in $MP_K = \frac{3}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}$ and $MP_L = \frac{3}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}$:

$$\frac{\frac{3}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}}{\frac{3}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}} = 1$$

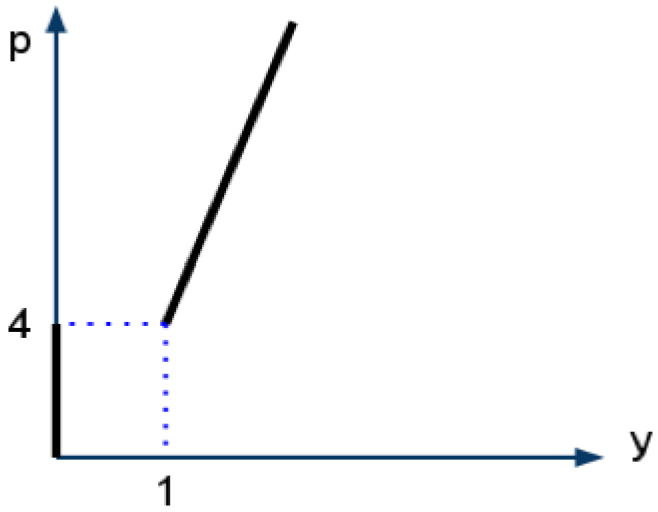
Therefore in optimum $K = L$, which implies $y = 3\sqrt{K}$ and $y = 3\sqrt{L}$, so $K = L = \frac{y^2}{9}$.
Plug the result into the cost function: $c(y) = w_K K + w_L L = 9\frac{y^2}{9} + 9\frac{y^2}{9} = 2y^2$.

c) A competitive firm facing price p, variable costs $2y^2$ and fixed costs 2 maximizes

$$\pi(y) = py - 2y^2 - 2$$

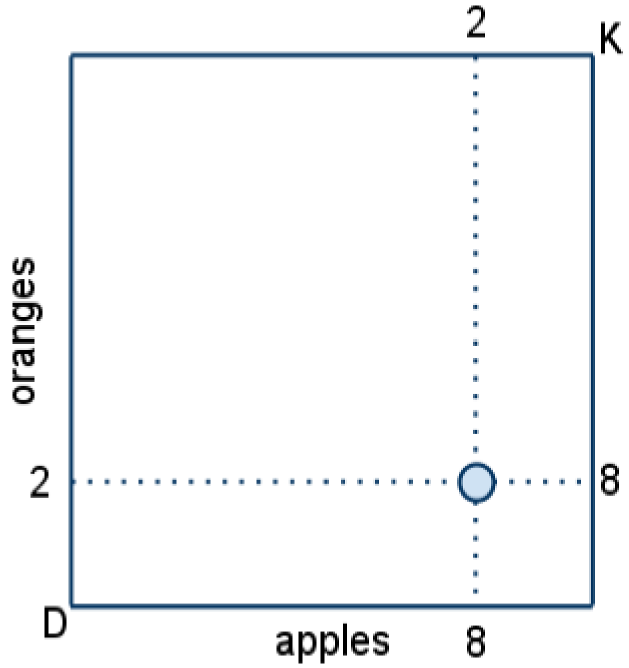
The profit-maximizing output solves $\pi' = 0 : p - 4y = 0$. So the supply function is $y(p) = \frac{p}{4}$ provided $\pi \geq 0$. The profit is non-negative as long as $MC \geq ATC$, or $4y \geq \frac{2y^2+2}{y}$, so $y \geq 1$ and thus the threshold price is 4. The answer is

$$y(p) = \begin{cases} p/4 & if \quad p \geq 4 \\ 0 & if \quad p \leq 4 \end{cases}$$



Problem 3 - Competitive Equilibrium

a) The total endowment in the economy is $w = (10, 10)$.



b) An allocation is Pareto efficient if there is no way to make one agent better off without hurting the other one. The condition for Pareto efficiency is $MRS^D = MRS^K$.

c) $MRS^i = -\frac{MU_1^i}{MU_2^i} = -\frac{x_2^i}{x_1^i}$. Given the initial endowments are $w^D = (8, 2)$, $w^K = (2, 8)$

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{2}{8} = -0.25$$

$$MRS^K = -\frac{x_2^K}{x_1^K} = -\frac{8}{2} = -4$$

$MRS^D \neq MRS^K$, hence the initial allocation is not Pareto efficient.

d) A competitive equilibrium is an allocation $(x_1^D, x_2^D, x_1^K, x_2^K)$ and a vector of prices (p_1, p_2) such that
- consumption bundles are optimal given the prices
- markets clear

If we normalize $p_2 = 1$, the incomes (the cost of the initial endowments) are:

$$m^D = 8p_1 + 2$$

$$m^K = 2p_1 + 8$$

Using the formula for Cobb-Douglas utility function, the optimal choices for good 1 are:

$$x_1^D = \frac{1}{2}\frac{8p_1 + 2}{p_1}$$

$$x_1^K = \frac{1}{2}\frac{2p_1 + 8}{p_1}$$

Since the markets must clear, it must be that $x_1^D + x_1^K = 10$, so

$$\frac{1}{2}\frac{8p_1 + 2}{p_1} + \frac{1}{2}\frac{2p_1 + 8}{p_1} = 10$$

therefore $p_1 = 1$ and hence the equilibrium allocation is $(x_1^D, x_2^D, x_1^K, x_2^K) = (5, 5, 5, 5)$.
Since what matters is the price ratio, not the prices, the same allocation with different prices such that $\frac{p_1}{p_2} = 1$ will constitute a different competitive equilibrium.

e)

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{5}{5} = -1 = MRS^K$$

f) In order for consumption bundles to be optimal given the prices, it must be that

$$MRS^D = MRS^K = -\frac{p_1}{p_2}$$

Hence $\frac{p_1}{p_2} = \frac{2}{3}$ in equilibrium. If $\frac{p_1}{p_2} > \frac{2}{3}$, both agents will consume good 2 only, and the market clearing condition fails.

Problem 4 - Short Questions

a) The expected utility of the lottery is $EU = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{9} = 1.5$. The certainty equivalent is a number that gives the same utility as the lottery in expectation, so it solves $\sqrt{CE} = 1.5$. Hence $CE = 2.25$. The certainty equivalent is less than the expected payoff of the lottery since the agent is risk-averse.

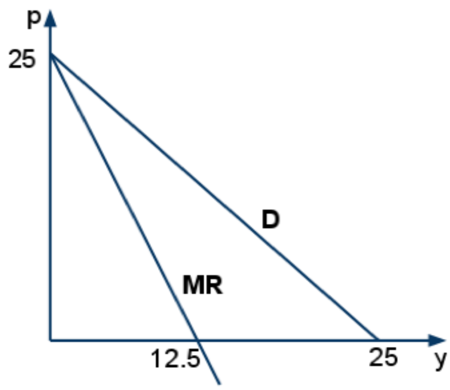
b) The expected value of a horse to a buyer equals $\frac{1}{2}2 + \frac{1}{2}5 = 3.5$, which is less than 4 - the value of a Plum horse to a seller. Hence Plums won't be traded in the market.

c) The internship becomes a credible signal of high ability if the low ability workers choose not to accept the internship offer: $1000 - 200x \leq 400$. So minimal length should be 3.

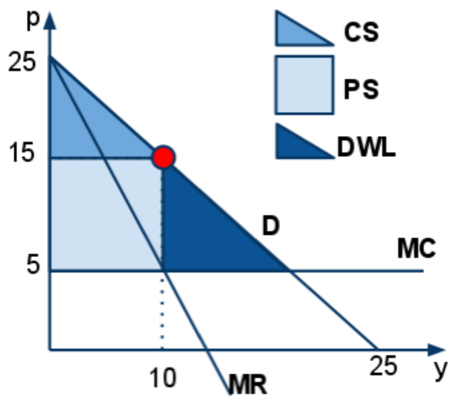
d) The present value of renting is $PV = \frac{1000}{0.02} = 50000$. It's less than the price of the apartment (100 000), so renting is cheaper.

Problem 5 - Market Power

a) Marginal revenue is the derivative of the total revenue. $TR(y) = p(y)y = (25 - y)y$. So $MR(y) = 25 - 2y$. Marginal revenue is smaller than the price because in order to sell an additional unit of output, the monopolist has to decrease price for all the units he is willing to sell.



b) In optimum $MR = MC$, so $25 - 2y^M = 5$, and $y^M = 10$. $p^M = 25 - y^M = 15$.
$CS = \frac{1}{2} \cdot (25 - 15) \cdot 10 = 50$, $PS = (15 - 5) \cdot 10 = 100$, $DWL = \frac{1}{2} \cdot (15 - 5) \cdot (20 - 10) = 50$.



c) Markup is determined by the formula $\frac{P}{MC} = \frac{15}{5} = 3$. Another way to calculate it is via elasticity: $\frac{1}{1+\frac{1}{e}} = \frac{1}{1-\frac{1}{3/2}} = 3$.

d) Under perfect price discrimination the monopolist sells as long as $P \geq MC$ and extracts full surplus. Hence $TS = PS = \frac{1}{2} \cdot (25 - 5) \cdot 20 = 200$. $CS = DWL = 0$.

e) Let Y denote total output in the industry and y be output of an individual firm. An individual firm chooses y to maximize

$$\pi = (25 - Y)y - 5y$$

$\pi' = 0$ gives $25 - Y - y - 5 = 0$. In equilibrium every firm anticipates the same behavior from every other firm, so $Y = ny$. Thus $(25 - 5) - (n + 1)y = 0$ and $y = \frac{20}{n+1}$.
$p = 25 - Y = 25 - ny = 25 - \frac{20n}{n+1}$. As $n \to \infty$, $\frac{20n}{n+1} \to 20$ and hence $p \to 5$.

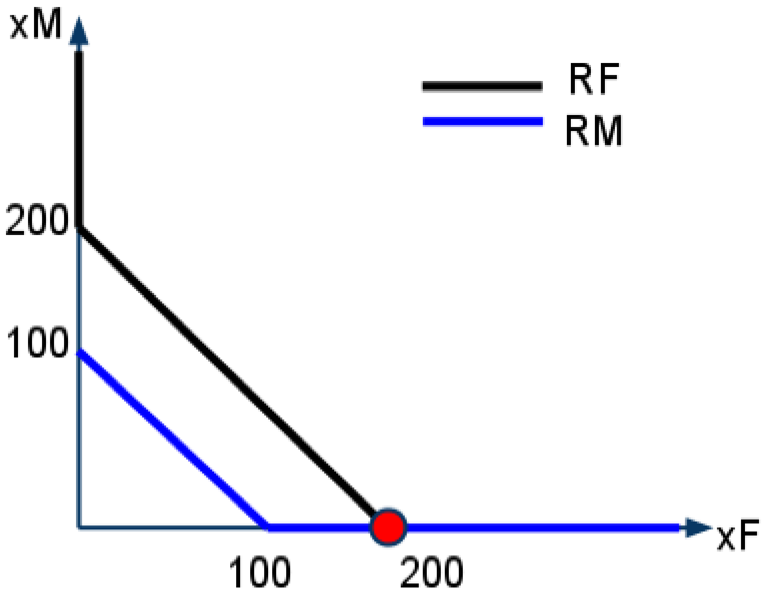
Problem 6 - Public Goods

a) Freddy's best response solves $\frac{du^F}{dx^F} = 0$:
$\frac{200}{x^F + x^M} - 1 = 0$, so Freddy's best response is

$$R^F(x^M) = \begin{cases} 200 - x^M & if \quad x^M \leq 200 \\ 0 & if \quad x^M > 200 \end{cases}$$

b) In the same way Miriam's best response is derived from $\frac{du^M}{dx^M} = 0$:
$\frac{100}{x^F + x^M} - 1 = 0$, hence

$$R^M(x^F) = \begin{cases} 100 - x^F & if \quad x^F \leq 100 \\ 0 & if \quad x^F > 100 \end{cases}$$



c) The equilibrium is the intersection of best responses: $(x^F, x^M) = (200, 0)$. Miriam free rides because she values the collection less than Freddy does.

d) In Pareto efficient case the sum of utilities is maximized with respect to $x^F + x^M$:

$$u^F + u^M = 300 \ln(x^F + x^M) - (x^F + x^M)$$

The efficient number of downloads is $x^F + x^M = 300$. It's greater than the equilibrium one because one agent's downloads create a positive externality for the other agents.

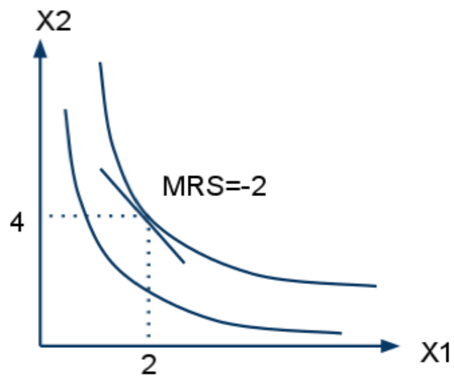
Econ 301 Final Exam Solutions - Spring 2011
Group B

Problem 1 - Consumer Choice

a) The budget constraint is $5x_1+10x_2 = 100$. The slope of the budget line is $-\frac{p_1}{p_2} = -\frac{5}{10} = -0.5$. Interpretation of the slope: the relative price of one tulip in the market is $\frac{1}{2}$ daffodil.

b) $U = \ln x_1 + \ln x_2$ or $U = x_1x_2$ will represent the same preferences, since the operations of taking square root, adding a constant and taking the square of a function are all monotone transformations.

c)$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{x_2}{x_1}$. At $(2, 4)$ $MRS = -2$. Interpretation: given 2 tulips and 4 daffodils, Jeremy is willing to trade 1 tulip for 2 daffodils.



d) The two secrets of happiness are

$$5x_1 + 10x_2 = 100$$
$$-\frac{x_2}{x_1} = -0.5$$

The first condition implies the optimal bundle lies on the budget line. The second condition guarantees the indifference curve is tangent to the budget line at optimum. (MRS equals the slope of the budget line)
The optimal bundle is $(x_1, x_2) = (10, 5)$. The solution is interior since $x_1 \neq 0$ and $x_2 \neq 0$.

e) The function $U = 2x_1 + 3x_2$ represents preferences over perfect substitutes. $\frac{MU_1}{p_1} = \frac{2}{5} > \frac{MU_2}{p_2} = \frac{3}{10}$. Hence good 1 only will be consumed, and optimal $(x_1, x_2) = (20, 0)$. The solution is not interior.

Problem 2 - Producers

a) Take $\lambda > 1$. $F(\lambda K, \lambda L) = 5(\lambda K)^{\frac{1}{4}}(\lambda L)^{\frac{1}{4}} = \lambda^{\frac{1}{2}} \cdot 5K^{\frac{1}{4}}L^{\frac{1}{4}}$
$\lambda F(K,L) = \lambda \cdot 5K^{\frac{1}{4}}L^{\frac{1}{4}}$. Then $F(\lambda K, \lambda L) < \lambda F(K,L)$ and the production function exhibits decreasing returns to scale.

b) First apply the cost-minimization condition:

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L}$$

Given $w_K = w_L = 25$, $\frac{w_K}{w_L} = 1$. Plug in $MP_K = \frac{5}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}$ and $MP_L = \frac{5}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}$:

$$\frac{\frac{5}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}}{\frac{5}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}} = 1$$

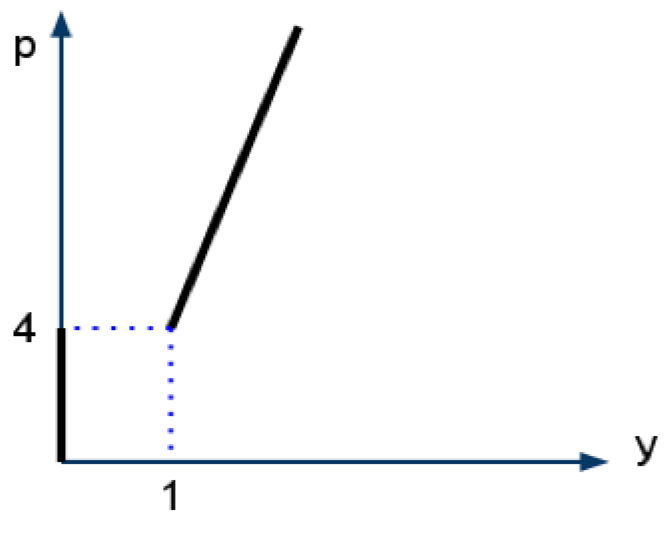
Therefore in optimum $K = L$, which implies $y = 5\sqrt{K}$ and $y = 5\sqrt{L}$, so $K = L = \frac{y^2}{25}$.
Plug the result into the cost function: $c(y) = w_K K + w_L L = 25\frac{y^2}{25} + 25\frac{y^2}{25} = 2y^2$.

c) A competitive firm facing price p, variable costs $2y^2$ and fixed costs 2 maximizes

$$\pi(y) = py - 2y^2 - 2$$

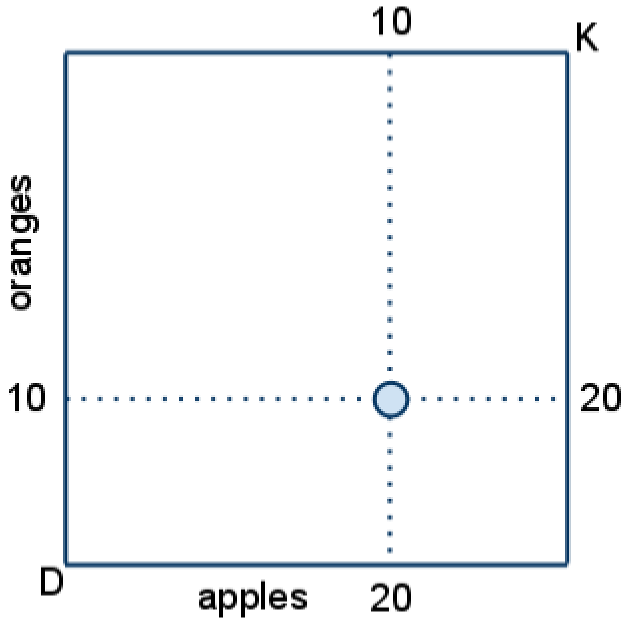
The profit-maximizing output solves $\pi' = 0 : p - 4y = 0$. So the supply function is $y(p) = \frac{p}{4}$ provided $\pi \geq 0$. The profit is non-negative as long as $MC \geq ATC$, or $4y \geq \frac{2y^2+2}{y}$, so $y \geq 1$ and thus the threshold price is 4. The answer is

$$y(p) = \begin{cases} p/4 & if \quad p \geq 4 \\ 0 & if \quad p \leq 4 \end{cases}$$



Problem 3 - Competitive Equilibrium

a) The total endowment in the economy is $w = (30, 30)$.



b) An allocation is Pareto efficient if there is no way to make one agent better off without hurting the other one. The condition for Pareto efficiency is $MRS^D = MRS^K$.

c) $MRS^i = -\frac{MU_1^i}{MU_2^i} = -\frac{x_2^i}{x_1^i}$. Given the initial endowments are $w^D = (20, 10)$, $w^K = (10, 20)$

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{10}{20} = -0.5$$

$$MRS^K = -\frac{x_2^K}{x_1^K} = -\frac{20}{10} = -2$$

$MRS^D \neq MRS^K$, hence the initial allocation is not Pareto efficient.

d) A competitive equilibrium is an allocation $(x_1^D, x_2^D, x_1^K, x_2^K)$ and a vector of prices (p_1, p_2) such that
- consumption bundles are optimal given the prices
- markets clear

If we normalize $p_2 = 1$, the incomes (the cost of the initial endowments) are:

$$m^D = 20p_1 + 10$$

$$m^K = 10p_1 + 20$$

Using the formula for Cobb-Douglas utility function, the optimal choices for good 1 are:

$$x_1^D = \frac{1}{2}\frac{20p_1 + 10}{p_1}$$

$$x_1^K = \frac{1}{2}\frac{10p_1 + 20}{p_1}$$

Since the markets must clear, it must be that $x_1^D + x_1^K = 30$, so

$$\frac{1}{2}\frac{20p_1 + 10}{p_1} + \frac{1}{2}\frac{10p_1 + 20}{p_1} = 30$$

therefore $p_1 = 1$ and hence the equilibrium allocation is $(x_1^D, x_2^D, x_1^K, x_2^K) = (15, 15, 15, 15)$. Since what matters is the price ratio, not the prices, the same allocation with different prices such that $\frac{p_1}{p_2} = 1$ will constitute a different competitive equilibrium.

e)

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{15}{15} = -1 = MRS^K$$

f) In order for consumption bundles to be optimal given the prices, it must be that

$$MRS^D = MRS^K = -\frac{p_1}{p_2}$$

Hence $\frac{p_1}{p_2} = \frac{2}{3}$ in equilibrium. If $\frac{p_1}{p_2} > \frac{2}{3}$, both agents will consume good 2 only, and the market clearing condition fails.

Problem 4 - Short Questions

a) The expected utility of the lottery is $EU = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{16} = 2$. The certainty equivalent is a number that gives the same utility as the lottery in expectation, so it solves $\sqrt{CE} = 2$. Hence $CE = 4$. The certainty equivalent is less than the expected payoff of the lottery since the agent is risk-averse.

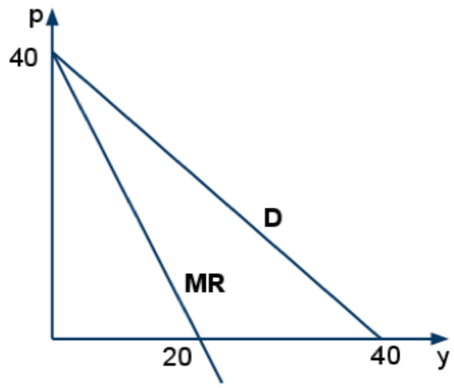
b) The expected value of a horse to a buyer equals $\frac{1}{2}2 + \frac{1}{2}8 = 5$, which is less than 6 - the value of a Plum horse to a seller. Hence Plums won't be traded in the market.

c) The internship becomes a credible signal of high ability if the low ability workers choose not to accept the internship offer: $1000 - 200x \leq 400$. So minimal length should be 3.

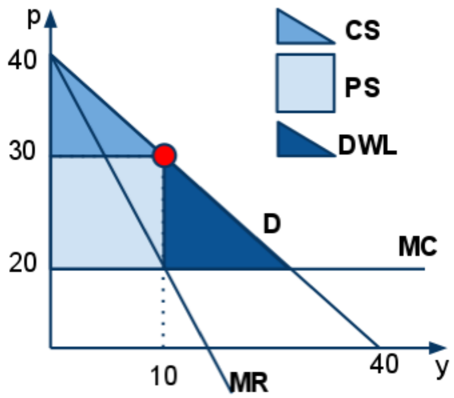
d) The present value of renting is $PV = \frac{1000}{0.02} = 50000$. It's more than the price of the apartment (30 000), so purchasing the apartment is cheaper.

Problem 5 - Market Power

a) Marginal revenue is the derivative of the total revenue. $TR(y) = p(y)y = (40 - y)y$. So $MR(y) = 40 - 2y$. Marginal revenue is smaller than the price because in order to sell an additional unit of output, the monopolist has to decrease price for all the units he is willing to sell.



b) In optimum $MR = MC$, so $40 - 2y^M = 20$, and $y^M = 10$. $p^M = 40 - y^M = 30$.
$CS = \frac{1}{2} \cdot (40 - 30) \cdot 10 = 50$, $PS = (30 - 20) \cdot 10 = 100$, $DWL = \frac{1}{2} \cdot (30 - 20) \cdot (20 - 10) = 50$.



c) Markup is determined by the formula $\frac{P}{MC} = \frac{30}{20} = 1.5$. Another way to calculate it is via elasticity: $\frac{1}{1+\frac{1}{e}} = \frac{1}{1-\frac{1}{3}} = 1.5$.

d) Under perfect price discrimination the monopolist sells as long as $P \geq MC$ and extracts full surplus. Hence $TS = PS = \frac{1}{2} \cdot (40 - 20) \cdot 20 = 200$. $CS = DWL = 0$.

e) Let Y denote total output in the industry and y be output of an individual firm. An individual firm chooses y to maximize

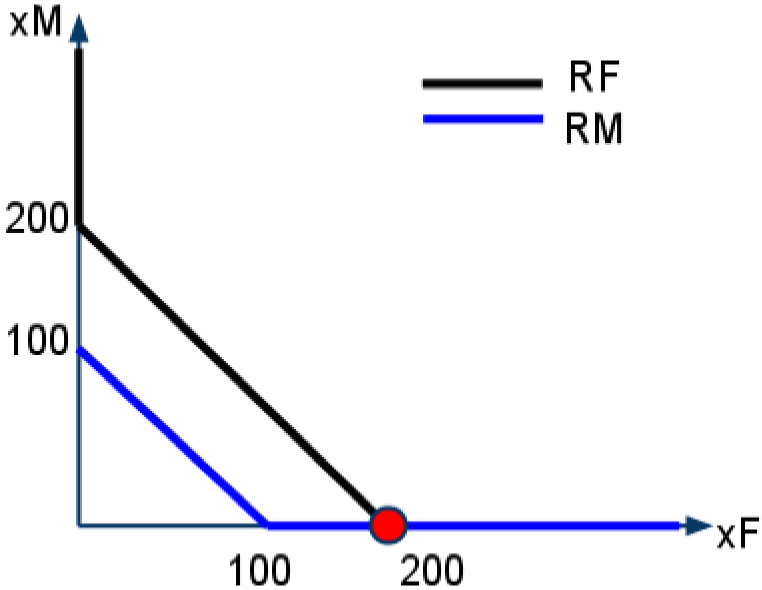
$$\pi = (40 - Y)y - 20y$$

$\pi' = 0$ gives $40 - Y - y - 20 = 0$. In equilibrium every firm anticipates the same behavior from every other firm, so $Y = ny$. Thus $(40 - 20) - (n + 1)y = 0$ and $y = \frac{20}{n+1}$.
$p = 40 - Y = 40 - ny = 40 - \frac{20n}{n+1}$. As $n \to \infty$, $\frac{20n}{n+1} \to 20$ and hence $p \to 20$.

Problem 6 - Public Goods

a) Freddy's best response solves $\frac{du^F}{dx^F} = 0$:
$\frac{200}{x^F+x^M} - 1 = 0$, so Freddy's best response is

$$R^F(x^M) = \begin{cases} 200 - x^M & if \quad x^M \leq 200 \\ 0 & if \quad x^M > 200 \end{cases}$$



b) In the same way Miriam's best response is derived from $\frac{du^M}{dx^M} = 0$:
$\frac{100}{x^F+x^M} - 1 = 0$, hence

$$R^M(x^F) = \begin{cases} 100 - x^F & if \quad x^F \leq 100 \\ 0 & if \quad x^F > 100 \end{cases}$$

c) The equilibrium is the intersection of best responses: $(x^F, x^M) = (200, 0)$. Miriam free rides because she values the collection less than Freddy does.

d) In Pareto efficient case the sum of utilities is maximized with respect to $x^F + x^M$:

$$u^F + u^M = 300\ln(x^F + x^M) - (x^F + x^M)$$

The efficient number of downloads is $x^F + x^M = 300$. It's greater than the equilibrium one because one agent's downloads create a positive externality for the other agents.