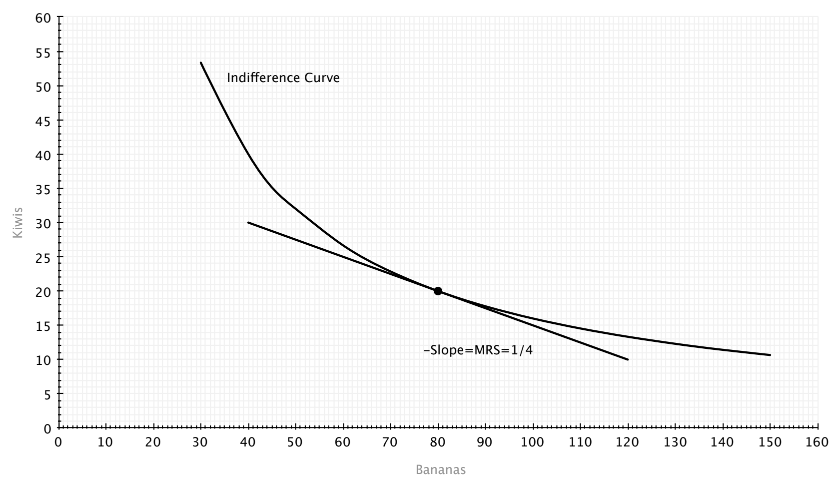
Final Solutions
ECON 301
May 13, 2012

Problem 1

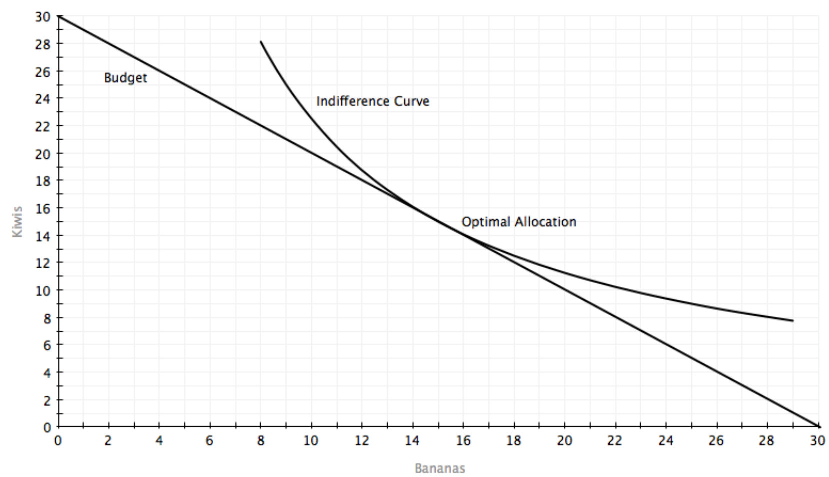
a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{1}{x_2}} = \frac{x_2}{x_1}$. At $(x_1, x_2) = (80, 20)$, $MRS = \frac{20}{80} = \frac{1}{4}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)

- Budget: $10x_1 + 10x_2 = 300$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

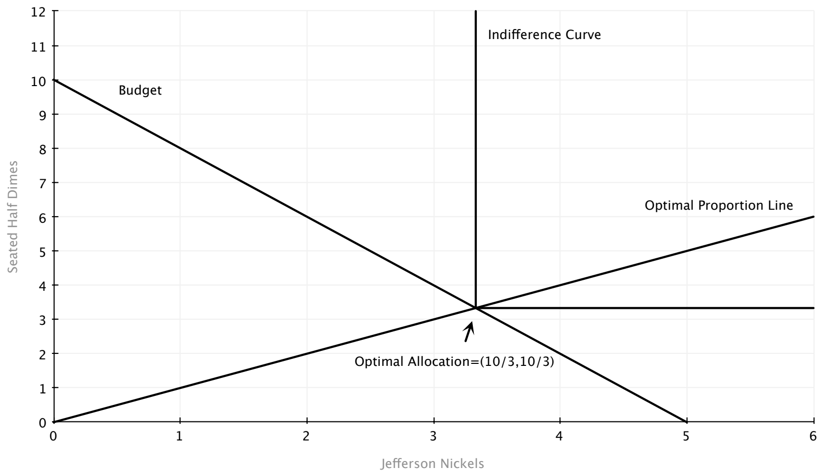
c) The optimal allocation is shown in the graph below



Problem 2

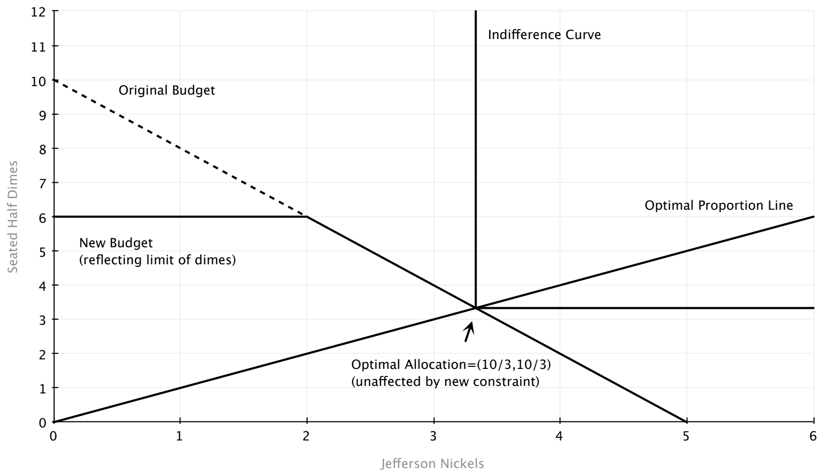
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * min(x_1, x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) The optimal bundle occurs where the optimal proportion line, $x_1 = x_2$, crosses the budget line, $4x_1 + 2x_2 = 20$. This happens when $(x_1, x_2) = (\frac{10}{3}, \frac{10}{3})$.



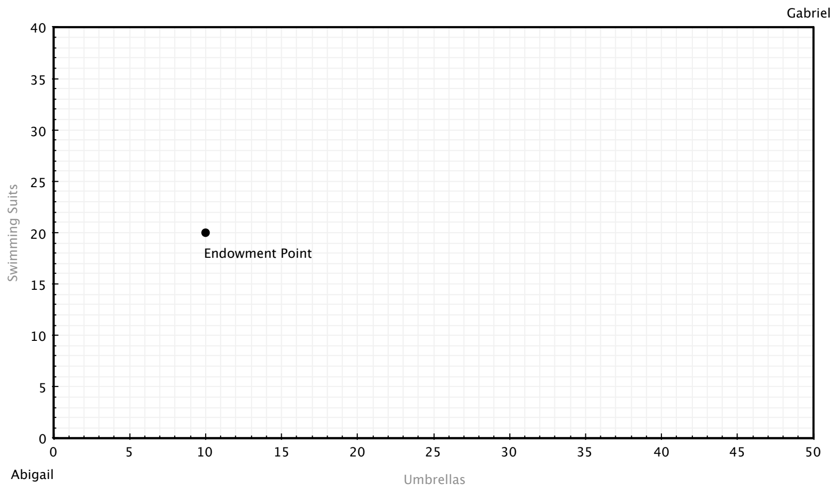
c) Giffen goods are goods that you consume more when their own price increases. Here we have $x_1 = x_2 = \frac{m}{p_1+p_2}$, so x_1 and x_2 are decreaseing in their own price: not Giffen goods.

d) The additional constraint is shown in the graph below, but it is not binding.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

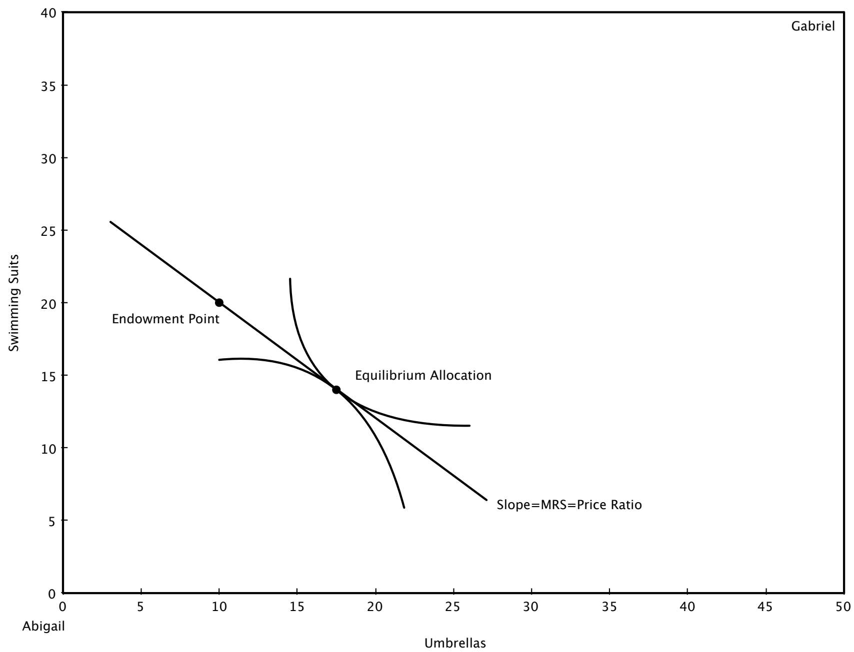
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

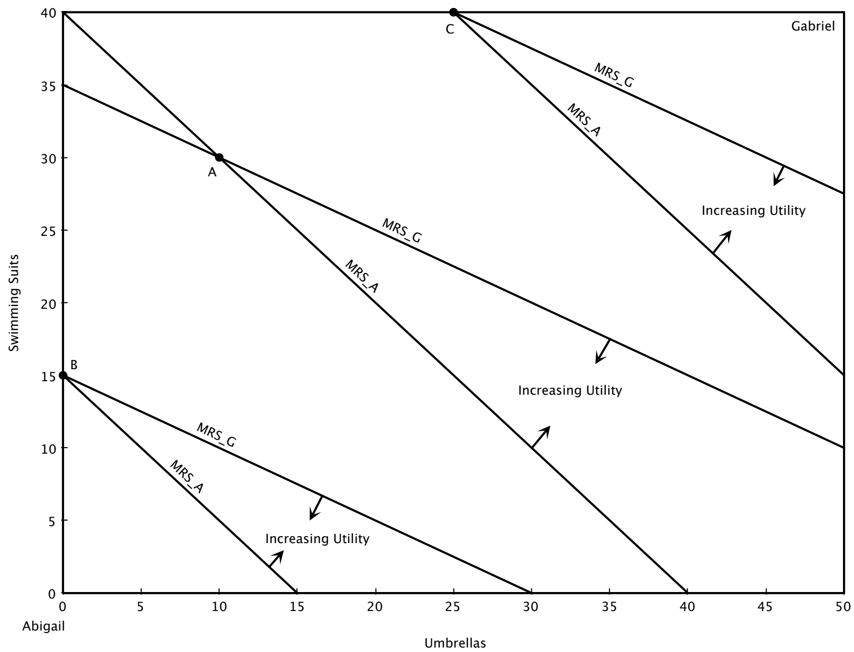
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

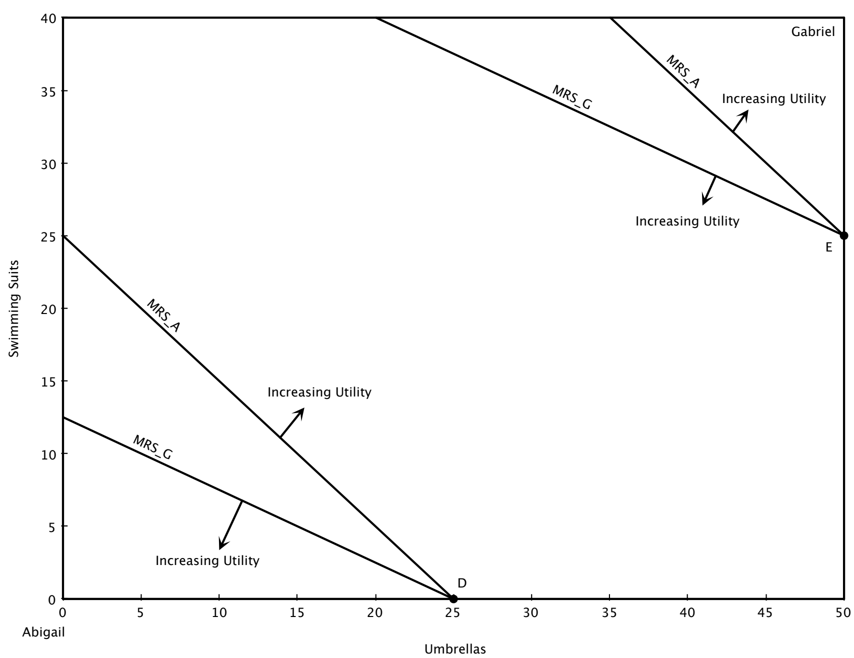
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



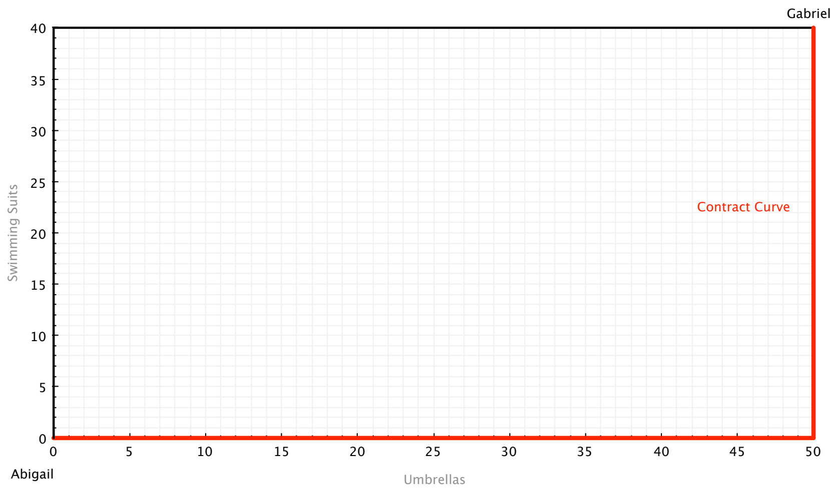
d) $MRS_A = 1$, and $MRS_G = 2$, so our condition for pareto optimality at an interior solution can never be satisfied. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider an interior point (A in the figure below), a point on the left border (B), and a point on the top border (C). In each case, both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning there are pareto improvements.



In contrast, if we look at a point on the bottom border (D), or one on the right border (E), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the points are pareto optimal.



To summarize, the contract curve of pareto optimal allocations consists of the bottom and right borders of the Edgeworth box.



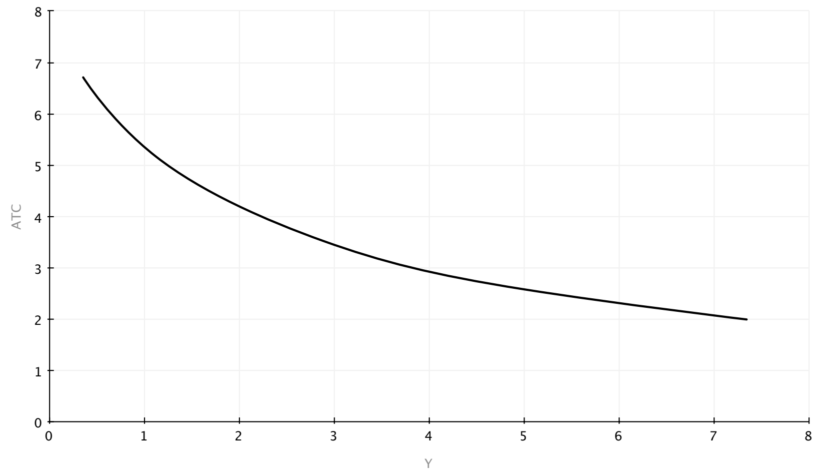
Alternative Argument: Let's normalize $p_2 = 1$ as usual, and then think about restrictions on p_1 that will allow the market to clear. If $p_1 < \frac{1}{2}$ then both Abigail and Gabriel only want to consume x_1, which is infeasible. If $p_1 > 1$, then both Abigail and Gabriel only want to consume x_2, which is also infeasible. If $\frac{1}{2} < p_1 < 1$ then Abigail only wants x_1, while Gabriel only wants x_2, so this corner solution will be feasible. If $p_1 = \frac{1}{2}$ Abigail only wants x_1, while Gabriel is indifferent between x_1 and x_2. Thus, the bottom border of the Edgeworth box (where Abigail has no x_2) is feasible. If $p_1 = 1$ Gabriel only wants x_2, while Abigail is indifferent between x_1 and x_2. Thus, the right border of the Edgeworth box (where Gabriel has no x_1) is feasible.

Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}x_w^2 + \frac{1}{2}x_l^2$. The certainty equivalent is defined by $ce^2 = \frac{1}{2}4^2 + \frac{1}{2}0^2 \Rightarrow ce = 2.83$. The expected value of the lottery is $\frac{1}{2}4 + \frac{1}{2}0 = 2$. The certainty equivalent is larger than the expected value because the bernouli utility function is convex, which is also the same thing as saying this person is risk loving.

c) $F(K, L) = K^a L^b$, with $1 < a$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



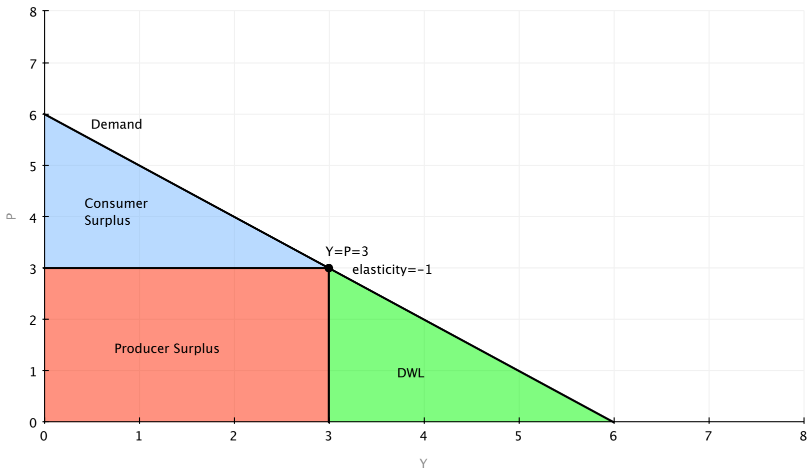
d) With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 2$, $D(p) = 2$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

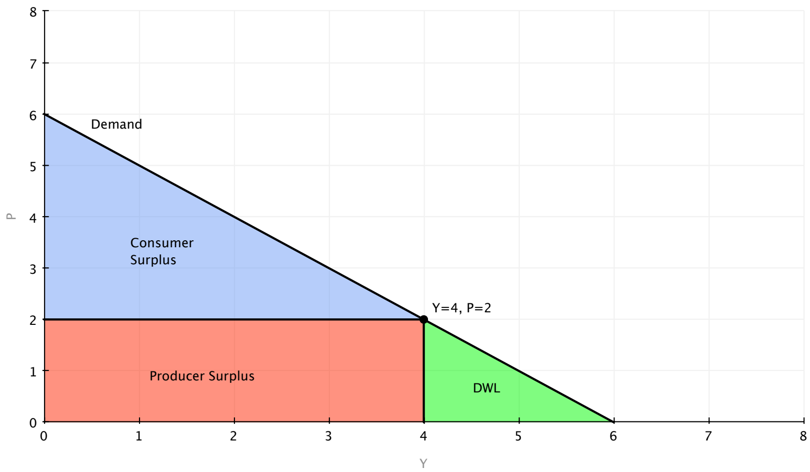
a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 0$, and make no profit. At $p = 0$ consumers purchase 6 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * 6 * 6 = 18$.

b) A monopolist chooses y to $\max(6 - y)y - 0$. The FOC of this problem is $6 - 2y = 0 \Rightarrow y = 3$. They charge price $p = 3$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium we have $\epsilon = -1 * \frac{3}{3} = -1$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 18.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6 - y_1 - y_2)y_1$. The FOC of this problem is $6 - 2y_1 - y_2 = 0$. Thus, the best response function for firm 1 is $y_1 = 3 - \frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 3 - \frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = 2$. Total production is 4, leaving $p = 2$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $2h - \frac{1}{2}h^2$. The FOC for this problem is $h = 2$. Given this choice of h, the orchard owner chooses t to max $5(t+2) - \frac{1}{2}t^2$. The FOC for this problem is $t = 5$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $5t + 7h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 7$, and the FOC for t is $t = 5$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.