Solutions to problem set 9
(due Thursday, April 15th, before class)

Problem 1 (Equilibrium with $N$ firms)

a) The individual supply with three symmetric firms is

$$y(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{p}{8} & \text{for } p \geq 8 \end{cases}$$

b) The aggregate supply with three symmetric firms is

$$y(p) = \begin{cases} 0 & \text{for } p < 8 \\ \frac{3p}{8} & \text{for } p \geq 8 \end{cases}$$

c) Price is determined from the market clearing condition

$$S(p) = D(p)$$

which gives

$$3 \times \frac{1}{8}p = S(p) = D(p) = 8 - \frac{1}{8}p$$

therefore equilibrium price is

$$p = 16$$

the level of production is

$$y = 2$$ and $$S(16) = 6$$

and the individual profit is

$$\pi = 16 \times 2 - 16 - 4 = 12 > 0$$

d) Maximally it will pay $12

Problem 2 (Free entry and market structure)

a) The price in equilibrium with entry is equal to $ATC^{MES} = 4\sqrt{F}$ and the level of individual production is $y^{MES} = \frac{1}{2}\sqrt{F}$. The number of firms can be determined from the market clearing condition

$$S(p) = D(p)$$

$$N \times \frac{1}{2}\sqrt{F} = 8 - \frac{1}{8}4\sqrt{F}$$

hence

$$N = \frac{16}{\sqrt{F}} - 1$$

therefore

$$N = \frac{16}{\sqrt{4}} - 1 = 7$$

b) The numbers of firms are

<table>
<thead>
<tr>
<th>$F$</th>
<th>64</th>
<th>16</th>
<th>4</th>
<th>1/4</th>
<th>1/16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>31</td>
<td>63</td>
</tr>
</tbody>
</table>

c) Market structures are

Monopoly $F = 64$

Oligopoly $F = 16$, $F = 4$

Perfect competition $F = \frac{1}{4}$, $F = \frac{1}{16}$
Problem 3 (Why monopolies exist)
Nuclear Power Plant - large fixed costs
Vista (Microsoft operating system) - patent
Casinos - a legal fiat
Niagara Falls State Park - a sole owner of the waterfalls

Problem 4 (Monopoly)
a) Total gains to trade are

\[ GTT = \frac{1}{2} \times 100 \times 100 - F = 5000 - 1000 = 4000 \]

Competitive producer sets the price to be equal to marginal cost \( p = MC = 0 \) and hence

\[ CS = 5000 \]

and

\[ PS = -1000 \]

Note that with price equal to zero firm has negative profit therefore it should exit the industry. The answer where the fixed cost was not subtracted is also correct. In such case \( F \) is considered as a sunk cost.

b) The total revenue is given by

\[ TR(y) = 100y - y^2 \]

therefore marginal revenue is

\[ MR(y) = 100 - 2y \]

Since marginal cost is zero, optimal production is

\[ y = 50 \]
\[ p = 100 - 50 = 50 \]
\[ \pi = 50 \times 50 - 1000 = 1500 \]

c) Such outcome is not Pareto efficient. DWL is

\[ DWL = \frac{1}{2} \times 50 \times 50 = 1250 \]

d) Consumer’s surplus is given by

\[ CS = \frac{1}{2} \times 50 \times 50 = 1250 \]

and producer’s surplus is

\[ PS = 50 \times 50 - 1000 = 1500 \]

Note that \( CS \) and \( PS \) sum up to 4000 - total gains to trade.

e) Find the elasticity of the demand at the optimal level of production. Is our monopoly operating on elastic or inelastic part of the demand?
Elasticity is defined as
\[ \varepsilon = \frac{\Delta y}{y} / \frac{\Delta p}{p} = \frac{\Delta y}{\Delta p} \times \frac{p}{y} \]
where \( \frac{\Delta y}{\Delta p} \) is a slope of the (not inverse!) demand function, which is equal to \(-1\). In optimum \( p = y = 50 \) hence
\[ \varepsilon = \frac{\Delta y}{\Delta p} \times \frac{p}{y} = -1 \times \frac{50}{50} = -1 \]

f) No because regulating Natural Monopoly in this case would force the firm to quit the market as it would have a negative profit.