Econ 301
Intermediate Microeconomics
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Problem set 10
(due Thursday, April 22th, before class)

Problem 1 (Monopoly and price discrimination)
Consider Microsoft Company from Problem Set 9, Problem 4: Microsoft Corporation decides how many packets of the new operating system - Windows Vista it is going to sell on the market. The research (fixed) costs associated with the development of the new system amounts to \( F = \$1000 \). The variable costs of the packet is negligible \( C(y) = 0 \). Microsoft’s inverse demand for the new operating system is given by

\[
p(y) = 100 - y
\]  
(1)

a) (1st-degree price discrimination) Suppose that Microsoft can perfectly discriminate. What is the profit of Microsoft in such case?

Let’s now assume more realistically that Microsoft can charge different prices on two segments of the market individual buyers and firms. The demands on two segments are

\[
\begin{align*}
y^I(p^I) &= 50 - \frac{4}{5} p^I \\
y^F(p^F) &= 50 - \frac{1}{5} p^F
\end{align*}
\]

b) Show that if Microsoft does not discriminate, the aggregate inverse demand is as in (1).

c) (3rd-degree price discrimination) Find the level of sales, prices, profits and elasticity of the demand each of the segments of the market.

d) Compare producer’s and consumers surplus in the three cases: uniform price (Problem 2), perfect discrimination (Problem 3 point a) and 3rd-degree price discrimination (Problem 3 point c).

Problem 2 (Demand elasticity)
Suppose the demand facing the monopolistic firm is given by

\[
y(p) = 1 - p
\]

a) Plot the inverse demand function, marking extreme points.

b) Mark the value of the demand elasticity for \( y = 0 \), \( y = 0.5 \) and \( y = 1 \).

c) Suppose the total cost is given by \( TC(y) = cy \) where \( c \) is some positive number \( c > 0 \). Show formally that the monopoly will chose a point on the elastic part of the demand. (that is the one for which elasticity is \( \varepsilon \in (-\infty, -1) \).

d) find the markup over a marginal cost \( c \).

Problem 3 (Monopoly and Labor Market)
a) Find a secret of happiness of a monopoly (on a good market) that determines the optimal demand for labor (in terms of elasticity of the demand). Hint: what is a derivative of \( p(y(L)) y(L) \) with respect to \( L \)?

b) Explain intuitively the condition from a)

c) Argue that the less elastic the demand (i.e., smaller \( |\varepsilon| \)) the smaller the demand for labor.

Problem 4 (Monopsony and Labor Market)
Consider a firm with the (short run) production function \( y(L) = 2L - L^2 \) and assume \( p = 1 \).

a) Find labor demand of a firm that is competitive on both labor and commodity market. Plot the demand for labor in the graph.

b) Suppose the (inverse) labor supply is \( w(L^S) = L^S \). Find a competitive level of employment and wage rate. Mark the equilibrium in the graph.
c) Find a total labor cost of a monopsony that recognizes its impact on wage rate \( C_L (L) = w(L)L \) and calculate marginal labor cost \( MC_L (L) \).

d) Explain why marginal cost of labor \( MC_L (L) \) is above \( w(L) \) for any \( L \). Plot the two curves in in the graph.

e) Write down a profit function of a monopsony and derive a secret of happiness \( MPL = MC_L \). Explain intuitively why monopsony does not hire one more or one less worker than the one determined by condition \( MPL = MC_L \).

f) Find optimal level of labor \( L^* \) and wage \( w^* \). Mark them in a graph and compare with competitive outcome from b). Is the level of labor efficient?

Minimal wage rate

g) Suppose minimal wage rate is \( w_{\text{min}} \) and hence the profit of the firm is given by \( \pi = y(L) - w_{\text{min}}L \). Find the labor demand of a firm as a function of \( w_{\text{min}} \).

h) Suggest minimal wage rate \( w_{\text{min}} \) that results in a competitive (and hence Pareto efficient) level of labor and wage rate.