Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)
Patrick spends his income on books \((x_1)\) and CD \((x_2)\).

a) Suppose the price of a book is \(p_1 = $10\), the price of a CD is \(p_2 = $5\), and Patrick’s daily budget is \(m = $40\). Show graphically Patrick’s budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick’s preferences are given by the following utility function
\[ U(x_1, x_2) = x_1 + 2 \ln(x_2) \]

- What is the value of MRS at consumption bundle \((5; 8)\) (give a number)?
- Suppose Patrick “consumes” 5 books and 8 CDs and one takes away 0.0001 of a book. What is compensation in terms of CDs is sufficient to make Patrick indifferent?
- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle \((5; 8)\).

c) From now on assume no gift. In the commodity space \((x_1, x_2)\), find (geometrically) Patrick’s optimal choice. Describe how the two “secrets of happiness” can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that \(p_1, p_2, m\) are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities \((x_1, x_2)\) for:
- \(p_1 = $10, p_2 = $5\) and \(m = $40\) (give two numbers).
- after the price of a book increases:
  - for \(p_1 = $30, p_2 = $5\) and \(m = $40\) (give two numbers).
  - Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael always consumes three hamburgers \(x_1\) along with one Coke \(x_2\) (this is the only healthy combination of the two products!).

a) Propose Michael’s utility function that represents his preferences over hamburgers and Coke (function \(U(x_1, x_2)\)).

b) In the commodity space \((x_1, x_2)\), carefully depict Michael’s indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters \(p_1\) and \(p_2\) and \(m\)). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels’s optimal choice of \(x_1\) and \(x_2\) as a function of \((p_1, p_2\) and \(m\)). Is the choice (solution) interior for any price and income? (Give formulas \(x_1(p_1, p_2, m)\) and a yes-no answer.)

e) Using \(x_1(p_1, p_2, m)\) derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption \(TCH\)? (You do not have to give any number. Just relate two effects to \(TCH\).)

Problem 3. (15 points) Adam spends all his income on food \((x_1)\) and clothing \((x_2)\). He is a fairly sophisticated fellow and his utility function is quite complicated

\[ U(x_1, x_2) = \left[ 700 \times \sqrt{\ln \left( 2 \ln x_1 + \ln x_2 \right)^2 + 10} \right]^{800}. \]
a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$ if the prices are $p_1 = 4$ and $p_2 = 4$ and $m = 1200$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 4$ and $m = 1200$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil’s Head Resort $C$.

a) Draw Frank’s budget set, given that the price of one ski pass is $p_c = $10 (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by $U(x_1, x_2) = RC^5$.

b) What is the real wage? (formula + nummer) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).
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Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

**Problem 1.** (50 points)
Patrick spends his income on books \((x_1)\) and CD \((x_2)\).

a) Suppose the price of a book is \(p_1 = $10\), the price of a CD is \(p_2 = $10\), and Patrick’s daily budget is \(m = $50\). Show graphically Patrick’s budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick’s preferences are given by the following utility function
\[U(x_1, x_2) = 2x_1 + 6 \ln x_2.\]

- Find Patrick’s marginal rate of substitution (MRS) for any bundle \((x_1, x_2)\) (give the formula for MRS).
- What is the value of MRS at consumption bundle \((3, 6)\) (give a number)?
- Suppose Patrick “consumes” 3 books and 6 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?
- Describe how the two “secrets of happiness” can be seen geometrically in the graph (two short sentences).
- Find Michaels’ s optimal choice of \(x_1\) and \(x_2\) as a function of \((p_1, p_2, m)\) (give formulas \(x_1(p_1, p_2, m)\) and a yes-no answer.)
- Using \(x_1(p_1, p_2, m)\) derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).
- Compare the substitution and income effects relative to a total change of consumption \(TCH\) (You do not have to give any number. Just relate two effects to \(TCH\).)

**Problem 2.** (25 points) Michael always consumes five hamburgers \(x_1\) along with one Coke \(x_2\) (this is the only healthy combination of the two products)!

a) Propose Michael’s utility function that represents his preferences over hamburgers and Coke (function \(U(x_1, x_2)\)).
b) In the commodity space \((x_1, x_2)\), carefully depict Michael’s indifference curves (and mark the optimal proportion line).
c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters \(p_1\) and \(p_2\) and \(m\)). Explain economic intuition behind the conditions (one sentence for each secret).
d) Find Michael’s optimal choice of \(x_1\) and \(x_2\) as a function of \((p_1, p_2\) and \(m)\). Is the choice (solution) interior for any price and income? (Give formulas \(x_1(p_1, p_2, m)\) and a yes-no answer.)
e) Using \(x_1(p_1, p_2, m)\) derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).
f) Compare the substitution and income effects relative to a total change of consumption \(TCH\)? (You do not have to give any number. Just relate two effects to \(TCH\).)

**Problem 3.** (15 points) Adam spends all his income on food \((x_1)\) and clothing \((x_2)\). He is a fairly sophisticated fellow and his utility function is quite complicated
\[U(x_1, x_2) = \left[12 \times \ln \left(\frac{6 \ln x_1 + 2 \ln x_2}{3} \right) + 3 \right]^{300}.\]
a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$ if the prices are $p_1 = 5$ and $p_2 = 10$ and $m = 80$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 10$ and $m = 80$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil’s Head Resort $C$.

a) Draw Frank’s budget set, given that the price of one ski pass is $p_c = \$10$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by $U(x_1, x_2) = R^{17} C^{34}$.

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).
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Midterm 1 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)

Patrick spends his income on books \(x_1\) and CD \(x_2\).

a) Suppose the price of a book is \(p_1 = $6\), the price of a CD is \(p_2 = $2\), and Patrick’s daily budget is \(m = $18\). Show graphically Patrick’s budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick’s preferences are given by the following utility function

\[ U(x_1, x_2) = 4x_1 + 4 \ln x_2. \]

Find Patrick’s marginal rate of substitution (MRS) for any bundle \((x_1, x_2)\) (give the formula for MRS).

- What is the value of MRS at consumption bundle \((3, 8)\) (give a number)?
- Suppose Patrick “consumes” 3 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?
- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle \((3, 8)\).

d) Write down mathematically two secrets of happiness, assuming that \(p_1, p_2, m\) are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities \((x_1, x_2)\) for:

- for \(p_1 = $6, p_2 = $2\) and \(m = $18\) (give two numbers).
- and after the price of a book increases:
  - for \(p_1 = $20, p_2 = $2\) and \(m = $18\) (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael always consumes one hamburger \(x_1\) along with two Cokes \(x_2\) (this is the only healthy combination of the two products) .

a) Propose Michael’s utility function that represents his preferences over hamburgers and Coke (function \(U(x_1, x_2)\)).

b) In the commodity space \((x_1, x_2)\), carefully depict Michael’s indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters \(p_1\) and \(p_2\) and \(m\)). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels’s optimal choice of \(x_1\) and \(x_2\) as a function of \((p_1, p_2, m)\). Is the choice (solution) interior for any price and income? (Give formulas \(x_1(p_1, p_2, m)\) and a yes-no answer.)

e) Using \(x_1(p_1, p_2, m)\) derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption \(TCH\)? (You do not have to give any number. Just relate two effects to \(TCH\).)

Problem 3. (15 points) Adam spends all his income on food \((x_1)\) and clothing \((x_2)\). He is a fairly sophisticated fellow and his utility function is quite complicated

\[ U(x_1, x_2) = \ln \left[ 0.5 \times \sqrt{(10 \ln x_1 + 2 \ln x_2)^2 + 3} \right]^{300}. \]
a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of \( x_1 \) and \( x_2 \) if the prices are \( p_1 = 2 \) and \( p_2 = 2 \) and \( m = 120 \) (find two numbers \( x_1 \) and \( x_2 \)). Is your solution interior, or corner?

c) Assume \( p_2 = 2 \) and \( m = 100 \). Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure \( R \) or work. The hourly wage rate is \( w = $120 \). Frank is a committed skier and uses all his income on ski passes in Devil’s Head Resort \( C \).

a) Draw Frank’s budget set, given that the price of one ski pass is \( p_c = $30 \) (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

\[ U(x_1, x_2) = R^3C. \]

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).
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Midterm 1 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)

Patrick spends his income on books \((x_1)\) and CD \((x_2)\).

a) Suppose the price of a book is \(p_1 = 10\), the price of a CD is \(p_2 = 2\), and Patrick’s daily budget is \(m = 30\). Show graphically Patrick’s budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 1 CDs (assume that he can always dispose the gift).

b) Patrick’s preferences are given by the following utility function
\[ U(x_1, x_2) = 8x_1 + 8 \ln x_2. \]
Find Patrick’s marginal rate of substitution (MRS) for any bundle \((x_1, x_2)\) (give the formula for MRS). 
- What is the value of MRS at consumption bundle \((4, 8)\) (give a number)?
- Suppose Patrick “consumes” 4 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent? 
- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle \((4, 8)\).
- From now on assume no gift. In the commodity space \((x_1, x_2)\), find (geometrically) Patrick’s optimal choice. Describe how the two “secrets of happiness” can be seen geometrically in the graph (two short sentences).
- Write down mathematically two secrets of happiness, assuming that \(p_1, p_2, m\) are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).
- Using the two conditions from d) find the optimal consumption levels of both types of commodities \((x_1, x_2)\) for:
- for \(p_1 = 10, p_2 = 2\) and \(m = 30\) (give two numbers).
- for \(p_1 = 40, p_2 = 2\) and \(m = 30\) (give two numbers).
Is each of the solutions interior? Illustrate the change on the graph.
- Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael always consumes one hamburger \(x_1\) along with four Cokes \(x_2\) (this is the only healthy combination of the two products!).

a) Propose Michael’s utility function that represents his preferences over hamburgers and Coke (function \(U(x_1, x_2)\)).

b) In the commodity space \((x_1, x_2)\), carefully depict Michael’s indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters \(p_1\) and \(p_2\) and \(m\)). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels’s optimal choice of \(x_1\) and \(x_2\) as a function of \((p_1, p_2\) and \(m\)). Is the choice (solution) interior for any price and income? (Give formulas \(x_1(p_1, p_2, m)\) and a yes-no answer.)

e) Using \(x_1(p_1, p_2, m)\) derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption \(TCH\)? (You do not have to give any number. Just relate two effects to \(TCH\).)

Problem 3. (15 points) Adam spends all his income on food \((x_1)\) and clothing \((x_2)\). He is a fairly sophisticated fellow and his utility function is quite complicated
\[ U(x_1, x_2) = \ln \left( 0.5 \times \sqrt{ \left( 12 \ln x_1 + 6 \ln x_2 \right)^2 + 17 \times 21 - 7 } \right)^{300}. \]
a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$ if the prices are $p_1 = 1$ and $p_2 = 1$ and $m = 60$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 1$ and $m = 60$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = $120. Frank is a committed skier and uses all his income on ski passes in Devil’s Head Resort $C$.

a) Draw Frank’s budget set, given that the price of one ski pass is $p_c = $30 (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^3 C.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).