Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25+35+15+25=100 points) + a bonus (10 "extra" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (25p). (Labor supply)
Eric’s total available time is 24h (per day). He works as a waiter with the wage rate $w$ and he spends his money on consuming New York Steaks $C$, that cost $p$ each.

a) on a graph with leisure time ($R$) measured on the horizontal axis and consumption ($C$) on the vertical one plot Eric’s budget set assuming $w = 100$, $p = 5$. Provide some economic interpretation of the slope of the budget line.

b) suppose his utility is given by

$$U(C; R) = R^3 \times C$$

where $R$ is leisure and $C$ is consumption of New York Steaks. Find his optimal time at work (labor supply LS), the relaxation time $R$ and the steak consumption $C$ as a function of $w$ and $p$ (parameters). Calculate the values of the three variables for $w = 100$, and $p = 5$.

c) on a graph with labor supply $LS$ measured on the horizontal axis and real wage $w/p$ on the vertical one plot the entire labor supply curve (marking the three points that you have found analytically); what can you say about the sensitivity (elasticity) of labor supply to changes in real wage rate? explain in 2 short sentences.

Problem 2 (35p). (Edgeworth box - Irving Fisher interest rate determination)
Consumption can take place in two periods: today ($C_1$) and tomorrow ($C_2$). Peter has income of $100 today and $300 tomorrow. (hence his endowment is $\omega^p = (100; 300)$). Amanda today’s income is $100 in both periods ($\omega^a = (100, 100)$). They both have the same utility function

$$U^i (C_1, C_2) = \frac{1}{2} \ln (C_1) + \frac{1}{2} \ln (C_2)$$

a) mark the allocation corresponding to the endowment point in the Edgeworth box

b) argue whether the endowment allocation is (or is not) Pareto efficient (use values of $MRS$ at the endowment point in your argument). Illustrate your argument geometrically in the Edgeworth Box from a)

c) find analytically the equilibrium interest rate and allocation and show it in the Edgeworth box. (Hint: Instead of working with "intertemporal" model, you can first find equilibrium prices $p_1$ and $p_2$, and then use the formula:

$$\frac{p_1}{p_2} = 1 + r$$

d) who among the two traders is borrowing and who is lending? How much? (one sentence + two numbers)

e) argue that the "invisible hand of financial markets" works perfectly, that is, the equilibrium outcome is Pareto efficient. (one sentence, two numbers, use values of $MRS$)

f) find $PV$ (in today’s $), and $FV$ (in tomorrow’s $) of Amanda’s income, given the equilibrium interest rate. (give two numbers)

Problem 3 (15p). (Short questions)
Answer the following three questions a), b) and c)

a) Find the constant payment $x$ you have to make in three consecutive periods (one, two, and three), in order to pay back a loan worth $2800 taken in period zero, given that the interest rate is 100%? (one number)
b) Consider a lottery that pays $36 when it rains and $25 when it does not, and both states are equally likely \( \pi_R = \pi_{NR} = \frac{1}{2} \). Find the expected value of the lottery and the certainly equivalent of the lottery, given Bernoulli utility function \( u(c) = \sqrt{c} \). Which is bigger? Explain why. (two numbers + one sentence)

c) Consider a pineapple tree that every year produces fruits worth $500 (starting next year), forever. How much are you willing to pay for such a tree now, given the interest rate of 10%? (one number)

**Problem 4 (25p). (Producers)**

Consider a producer that has the following technology
\[
y = K^\frac{1}{6} L^\frac{1}{6}
\]

a) what returns to scale are represented by this production function? (choose: CRS, IRS or DRS and support your choice with a mathematical argument).

b) find analytically the level of capital \( (K) \), labor \( (L) \) and output \( (y) \) that maximizes profit, and the value of maximal profit, given \( p = 6 \) and \( w_K = w_L = 1 \).

c) find the average cost function \( AC (y) \), and plot it on a graph (prices are as in b). On the same graph show geometrically the level of maximal profit from b) (Hint: for the second part, take the value \( y \) from b)).

**Bonus Problem.** (extra 10 points)

Derive (not just give!) the formula for PV of annuity (explain each step, starting with deriving PV for perpetuity).