Econ 301  
Intermediate Microeconomics  
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Solutions to midterm 1 (Group B)

Problem 1.

a) With $p_1 = $2, $p_2 = $10 and $m = $40 the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.

b) Tony’s marginal rate of substitution (MRS)

\[ MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1} \]

- The value of MRS at consumption bundle (1, 2) is

\[ |MRS| = \left| -\frac{2}{1} \right| = 2 \]

- Burrito ($x_2$) is less valuable than Mountain Dew ($x_1$)
- Tony’s indifference curve map is. (the slope of her indifferent curve that passes through bundle (1, 2) is $-2$.

c) Tony’s optimal choice is

- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:
  1. At the optimal bundle, the indifference curve is tangent to a budget set
  2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

\[ \begin{align*}
MRS &= -\frac{p_1}{p_2} \\
p_1x_1 + p_2x_2 &= m
\end{align*} \]
- the economic intuition behind the two conditions is:
The individual value of $x_1$ in terms of $x_2$ coincides with the market value
The income of a consumer is exhausted
- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows
From the MRS condition
\[ MRS = \frac{-x_2}{x_1} = \frac{p_1}{p_2} \]
hence
\[ x_2 = \frac{p_1}{p_2} x_1 \]
plugging in budget constraint
\[ p_1 x_1 + p_2 \left( \frac{p_1}{p_2} x_1 \right) = m \]
Solving for $x_1$ gives
\[ x_1 = \frac{1}{2} \frac{m}{p_1} \]
Plugging in
\[ x_2 = \frac{p_1}{p_2} \left( \frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2} \]
- the fraction of income spent on burritos is
\[ \frac{p_1 x_1}{m} = \frac{1}{2} = 50\% \]
- and the demand curve for burritos book (given $p_2 = $10, and $m = $40) and Engel curve (given $p_1 = $2, and $p_2 = $10)
  Demand curve
\[ x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{2} = \frac{20}{p_1} \]
and hence inverse demand is
\[ p_1 (x_1) = \frac{20}{x_1} \]
Geometrically
\[ x_1 = \frac{1}{2} \frac{m}{p_1} \]
Engel curve: Since
\[ x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m \]
hence
\[ m (x_1) = 4x_1 \]
Geometrically
- are they Giffen goods? Why? (yes/no answer + one sentence).
No, because the demand curve is downwardsloping on the whole domain.
e) The optimal consumption levels for \((x_1, x_2)\).
- at \(p_1 = \$2, p_2 = \$10\) and \(m = \$40\)

\[
x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{2} = 10
\]
and

\[
x_2 = \frac{1}{2} \frac{m}{p_2} = \frac{1}{2} \frac{40}{10} = 2
\]

and after the price of science-fiction book decreased, for \(p_1 = \$1, p_2 = \$10\) and \(m = \$40\)

\[
x_1 = \frac{1}{2} \frac{m}{p_1} = 20
\]
and

\[
x_2 = \frac{1}{2} \frac{m}{p_2} = \frac{1}{2} \frac{40}{10} = 2
\]

Hence the total change in consumption of \(x_1\) is

\[
\Delta x_1 = 20 - 10 = 10
\]

Geometrically

f) Substitution effect: auxiliary budget

\[
m' = 10 \times 1 + 10 \times 2 = 30
\]

and hence

\[
x_1 = \frac{1}{2} \frac{30}{1} = 15
\]

so \(SE\) is equal to

\[
SE = 15 - 10 = 5
\]

and income effect is

\[
IE = 10 - 5 = 5
\]

Problem 2.

a) Bill’s utility function is

\[
U(x_1, x_2) = \min(x_1, 2x_2)
\]

b) indifference curves, in the commodity space \((x_1, x_2)\) are

c) Bill’s demand for shoes is

\[
2x_2 = x_1
\]
\[
2x_1 + x_2 = 40
\]
\[4x_2 + x_2 = 40\]

\[x_2 = \frac{40}{5} = 8\]

**d)** geometrically Bill's optimal choice is

\[
x_1 = 16
\]

**e)** when the price of a left shoe goes down to \(p_1 = \$1\), the new demand is given by the system of equations

\[
2x_2 = x_1
\]

\[
x_1 + x_2 = 40
\]

and hence demand is

\[
x_1 = \frac{80}{3} = 26 \frac{2}{3}
\]

\[
x_2 = \frac{40}{3} = 13 \frac{1}{3}
\]

The substitution effect is zero (perfect complements) and the income effect is \(4.10 \frac{2}{3}\)

**Problem 3.**

a) the two secrets of happiness are

\[
-\frac{x_2}{10} = -\frac{1}{2}
\]

\[
x_1 + 2x_2 = 15
\]

and hence \(x_2 = 5\) and \(x_1 = 5\). Since both are positive, this is interior solution.

b) the two secrets of happiness are

\[
-\frac{x_2}{10} = -\frac{1}{2}
\]

\[
x_1 + 2x_2 = 8
\]

and hence secrets of happiness give \(x_2 = 5\) and \(x_1 = -5\). Since consumption must be non-negative the optimal consumption is \(x_1 = 0\) and \(x_2 = 4\), which is a corner solution.

**Problem 4.**

a) Jacob's budget set, with \(w = \$5\) and \(p_c = \$10\) is

Income is \(m = 5 \times 24 = \$120\)

b) They are perfect substitutes
c) \(|MRS| = 1 > \frac{w}{p_c} = \frac{1}{2}\) which implies that Jacob cares more about leisure than consumption, therefore he will spend the whole day at home.

\[ R = 24, LS = 0 \text{ and } C = 0 \]

d) Bonus Problem. (extra 10 points)

a) Monotone transformation \(\ln()\). Take a log of \(U\)

\[ \ln U() = \ln x_1 x_2^2 = \ln x_1 + \ln x_2^2 = \ln x_1 + 2 \ln x_2 = V() \]

where we used two properties of \(\ln\) function.

b) For \(U()\), marginal rate of substitution is

\[ MRS = \frac{MU_1}{MU_2} = -\frac{x_2^2}{2x_1 x_2} = -\frac{x_2}{2x_1} \]

and for \(V()\)

\[ MRS = -\frac{MU_1}{MU_2} = -\frac{1/x_1}{2/x_2} = -\frac{x_2}{2x_1} \]

and hence MRS coincides for all \((x_1, x_2)\). It follows that the slopes of indifference curves are the same at any point and hence they must be the same.