**Econ 301**  
Intermediate Microeconomics  
Prof. Marek Weretka

**Midterm 1 (Group B)**

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (10 extra "e" points). Make sure you answer the first four questions before working on the bonus one!

**Problem 1.** (50 points) To reenergize for Econ 301 class, in the morning, Tony always drinks Mountain Dew \( x_1 \) and eats Burritos \( x_2 \).

a) Suppose Mountain Dew costs \( p_1 = $2 \), burrito costs \( p_2 = $10 \), and his daily budget is \( m = $40 \). Show graphically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set is affected by inflation of 100% that affects prices of both commodities but does not affect income (so his income stays the same \( m = $40 \))?

b) Tony’s preferences are given by the following utility function

\[ U(x_1, x_2) = x_1^3 x_2^3. \]

Find Tony’s marginal rate of substitution (MRS) as a function of \( x_1, x_2 \) (give a formula for MRS).

- What is the value of MRS at consumption bundle \( (1, 2) \) (give a number)?

- Which of the two goods is more valuable, soda or burrito, if Tony drinks one Mountain Dew and consumes two burritos?

- Depict his indifference curve map in a commodity space. Mark the slope of the indifference curve at the bundle \((1, 2)\).

c) In the commodity space \((x_1, x_2)\), find (geometrically) Tony’s optimal choice, assuming pre-inflation prices \( p_1 = $2 \), and \( p_2 = $10 \). Describe how the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences) can be seen in the graph.

d) Write down mathematically two secrets of happiness, assuming that \( p_1, p_2, m \) are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).

- Derive the optimal consumption of \( x_1 \) and \( x_2 \) as a function of \( p_1, p_2, m \) (show the derivation).

- What fraction of income is spent on burritos (give the percentage)?

- Find analytically and geometrically the demand curve for Mountain Dew (given \( p_2 = $10 \) and \( m = $40 \)) and Engel curve (given \( p_1 = $2 \), and \( p_2 = $10 \))

- Are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels of both types of commodities \((x_1, x_2)\) for:

- \( p_1 = $2, p_2 = $10 \) and \( m = $40 \) (give two numbers).

and after the price of Mountain Dew decreased:

- for \( p_1 = $1, p_2 = $10 \) and \( m = $40 \) (give two numbers).

What is the total change in consumption of Mountain Dew? (give a number). Illustrate the change on the graph.

f) decompose the total change in consumption of \( x_1 \) from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2.** (20 points) Bill is a wild-animal lover. From his recent trip to Galapagos Islands he brought a small Iguana. His new pet has only three legs: two left and one right. (Iguanas use magma heated soil to warm their eggs and his favorite pet lost one right leg during the last volcano eruption). To survive the famous Madisonian winter, the iguana has to wear shoes, left \((x_1)\) and right ones \((x_2)\).

a) Write down Bill’s utility function representing his preferences over right and left shoe (function \( U(x_1, x_2) \)).

b) In the commodity space \((x_1, x_2)\), carefully depict Bill’s indifference curves.
c) Find analytically Bill’s demand for shoes if \( p_1 = \$2 \) and \( p_2 = \$1 \) and Bill’s budget for iguana shoes is \( m = \$15 \). Is the solution interior? (give two numbers and a yes/no answer).

d) Illustrate Bill’s optimal choice on the graph including the indifference curves and the budget set.

e) Suppose the price of left shoe goes down to \( p_1 = \$1 \). Find Bill’s new demand for shoes. What can you say about the substitution effect? How about the income effect? (Answer the latter question without any calculations, using only a graph).

Problem 3. (15 points) Ramon decides about his new collections of postage stamps. He is interested in two themes: "the birds of the world", \( x_1 \), (measures the number of stamps in the subcollection with birds) and "the famous mathematicians", \( x_2 \). The utility derived from the collection is given by

\[
U(x_1, x_2) = x_1 + 10 \times \ln x_2.
\]

a) What is the optimal collection of stamps if the prices are \( p_1 = 1 \) and \( p_2 = 2 \) and \( m = 15 \). (find two numbers \( x_1 \) and \( x_2 \)). Is your solution interior, or corner?

b) Find the optimal collection if the prices are still \( p_1 = 1 \) and \( p_2 = 2 \), but the income is only \( m = 8 \). Depict Ramon’s optimal choice in the commodity space.

Problem 4. (15 points) Jacob can use his 24h for leisure \( R \), or work. The hourly wage rate is \( w = 5 \). Jacob spends all his money on cheese curds \( C \).

a) Draw Jacob’s budget set, given the price of cheese is \( p_c = \$10 \) (mark the endowment point).

Let the utility function be given by

\[
U(x_1, x_2) = R + C.
\]

b) Are leisure and cheese curds perfect complements, perfect substitutes or none of them?

c) What is the optimal choice of leisure, cheese curds and labor supply? (Find geometrically and give three numbers).

d) Harder: Plot a graph with labor supply (horizontal axis) and wage rate (vertical one) assuming \( p_c = \$10 \) (hint: for what value of \( w \) do we go from one "bang" to the other "bang" solution?)

Bonus Problem. (extra 10 points) Let \( U(x_1, x_2) = x_1 x_2^2 \), and \( V(x_1, x_2) = \ln x_1 + 2 \ln x_2 \) be two utility functions.

a) Show that \( U(\cdot) \) is a monotone transformation of \( V(\cdot) \), and hence they define the same preferences.

b) Derive MRS for each of the two functions. Using the two formulas for MRS, argue that the functions define the same indifference curve maps.