Econ 301  
Intermediate Microeconomics  
Prof. Marek Weretka  

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 3 questions (60+25+15=100 points) + bonus (10 "e" points). Make sure you answer the first three questions before working on the bonus one!

**Problem 1.** (60 points) Maggie likes to read science fiction ($x_1$) and romance ($x_2$) novels. Her preferences over the two types of books are represented by a utility function $U(x_1, x_2) = (x_1)^{10}(x_2)^{20}$

a) Find Maggie’s marginal rate of substitution (MRS) as a function of $x_1, x_2$ (give a formula).
- what is the value of MRS at consumption bundle (2, 2) (give a number).
- complete the sentence: "The Marginal Rate of Substitution is a (marginal) value of a... in terms of...
" - how much one must compensate Maggie in terms of romance books, after taking away 0.00001 of a science-fiction book, in order to keep her indifferent? (give a number, assume she consumes bundle (2, 2)).
- depict her indifference curve map in a commodity space. Mark the slope of her indifferent curve that passes through bundle (2, 2).

b) Suppose the price of a science-fiction book is $p_1 = $10, a romance book costs $p_2 = $5 and her total monthly spending on books is $m = $300. Show graphically her budget constraint in the commodity space. Mark the two extreme consumption bundles (give values). On the same graph, show how the budget set would be affected by the introduction of an ad valorem tax on romance books ($x_2$) at rate 100%?

c) In the commodity space $(x_1, x_2)$, find (geometrically) Maggie’s optimal choice.
- describe the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).
- provide some economic intuition behind the two conditions (ca. two sentences for each).
- derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).
- what fraction of income is spent on science-fiction novels (give the percentage).
- find analytically and geometrically the demand curve for science-fiction book (given $p_2 = $5, and $m = $300) and Engel curve (given $p_1 = $10, and $p_2 = $5)
- are science-fiction books normal goods? Why? (yes/no answer + one sentence).
- are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels for both types of books $(x_1, x_2)$.
- for $p_1 = $10, $p_2 = $5 and $m = $300 (give two numbers).
- and after the price of science-fiction book decreased:
- for $p_1 = $5, $p_2 = $5 and $m = $300 (give two numbers).
What is the total change in consumption of $x_1$? (give one number). Illustrate this change on the graph.

f) decompose the total change in $x_1$ from f) into a substitution and income effect. (Calculate the two numbers, show how you found them on the graph.) Complete the two sentences:
"The substitution effect is attributed to the pure change in... induced by the decrease of nominal price $p_1$"

"The income effect can be attributed to the pure change in... induced by the decrease of nominal price $p_1$"

g) which of the following alternative utility functions represents Maggie’s preferences (there are two such
functions)?

\[
V(x_1, x_2) = 30 (x_1^{10} \times x_2^{20}) + 3 \\
V(x_1, x_2) = 10x_1 \times 20x_2 \\
V(x_1, x_2) = 10 \ln x_1 \times 20 \ln x_2 + 2 \\
V(x_1, x_2) = 10 \ln x_1 + 20 \ln x_2 + 7
\]

Explain why the utility functions you have selected represent Maggie’s preferences (one sentence). Suggest the transformation of \( U() \) function that makes the two \( V() \) functions equivalent.

**Problem 2.** (20 points) Jimmy’s favorite hobby is slot car racing. He assembles slot cars from parts, by adding four wheels \((x_1)\) to an engine \((x_2)\) (these are supertrucks, with five wheels on each side). He purchases the parts on the market.

a) write down Jimmy’s utility function representing his preferences over wheels and engines (function \( U(x_1, x_2) \)).

b) in the commodity space \((x_1, x_2)\), carefully depict Jimmy’s indifference curves.

c) find analytically Jimmy’s demand for parts if one wheel costs \( p_1 = $50 \), an engine is \( p_2 = $100 \) and Jimmy’s budget for slot cars is \( m = $600 \). Is the solution interior (give two numbers and yes/no answer).

d) illustrate Jimmy’s optimal choice on the graph including the indifference curves and the budget set.

e) suppose the price of one wheel goes down to \( p_1 = $25 \). Find Jimmy’s new demand for the parts. What can you say about the substitution effect? How about the income effect? (you can answer the last question without any calculations, using only a graph).

**Problem 3.** (20 points) Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 10 cigar packs \((x_1)\), or in singles \((x_2)\). Ramon has no income. Instead he is initially endowed with \( \omega_1 = 5 \) packs and \( \omega_2 = 50 \) cigars.

a) draw Ramon’s budget set, given the price of a pack is equal to \( p_1 = $5 \) and a single cigar is \( p_2 = $1 \) (mark the endowment point).

b) Illustrate geometrically Ramon’s optimal demand for packs and single cigars, given his utility function

\[
U(x_1, x_2) = 10x_1 + x_2
\]

(Give two numbers \((x_1, x_2)\), and mark them on the graph, including budget set and the indifference curves.)

c) What is your answer to b) when prices are \( p_1 = 20 \) and \( p_2 = 1 \). (Give two numbers \((x_1, x_2)\), and plot the graph.)

d) Harder: Give the formula for the demands \( x_1, x_2 \) as a function of \( p_1, p_2 \) and endowments \( \omega_1 \) and \( \omega_2 \). Show the demand curve for \( x_1 \) on the graph, assuming \( p_2 = 1, \omega_1 = 5 \) and \( \omega_2 = 50 \).

**Bonus Problem.** (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods

b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)