Problem 1. (50 points)
a) With $p_1 = $2, $p_2 = $10 and $m = $40 the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.

b) Tony’s marginal rate of substitution (MRS)

$$MRS = - \frac{MU_1}{MU_2} = - \frac{x_2}{x_1}$$

- The value of MRS at consumption bundle (2,1) is

$$|MRS| = \left| - \frac{1}{2} \right| = \frac{1}{2}$$

- Burrito ($x_2$) is more valuable than Mountain Dew ($x_1$)
- Tony’s indifference curve map is. (the slope of her indifferent curve that passes through bundle (2,1) is $-\frac{1}{2}$).

c) Tony’s optimal choice is

- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:
1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = - \frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 = m \end{cases}$$
- the economic intuition behind the two conditions is:
  The individual value of $x_1$ in terms of $x_2$ coincides with the market value
  The income of a consumer is exhausted
- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows
  From the MRS condition
  \[ MRS = \frac{x_2}{x_1} = \frac{p_1}{p_2} \]
  hence
  \[ x_2 = \frac{p_1}{p_2} x_1 \]
  plugging in budget constraint
  \[ p_1 x_1 + p_2 \left( \frac{p_1}{p_2} x_1 \right) = m \]
  Solving for $x_1$ gives
  \[ x_1 = \frac{1}{2} \frac{m}{p_1} \]
  Plugging in
  \[ x_2 = \frac{p_1}{p_2} \left( \frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2} \]
  - the fraction of income spent on burritos is
  \[ \frac{p_1 x_1}{m} = \frac{1}{2} = 50\% \]
  - and the demand curve for burritos book (given $p_2 = $10, and $m = $40) and Engel curve (given
  $p_1 = $2, and $p_2 = $10)
  Demand curve
  \[ x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{p_1} = \frac{20}{p_1} \]
  and hence inverse demand is
  \[ p_1 (x_1) = \frac{20}{x_1} \]
  Geometrically
  Engel curve: Since
  \[ x_1 = \frac{1}{2} \frac{m}{p_1} \]
  at $p_1 = $2
  \[ x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m \]
  hence
  \[ m (x_1) = 4x_1 \]
  Geometrically
- are they Giffen goods? Why? (yes/no answer + one sentence).
No, because the demand curve is downwardsloping on the whole domain.
e) The optimal consumption levels for \((x_1, x_2)\).
- at \(p_1 = $2, p_2 = $10\) and \(m = $40\)
  \[
  x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{2} = 10
  \]
  and
  \[
  x_2 = \frac{1}{2} \frac{m}{p_2} = \frac{1}{2} \frac{40}{10} = 2
  \]
  and after the price of science-fiction book decreased, for \(p_1 = $1, p_2 = $10\) and \(m = $40\)
  \[
  x_1 = \frac{1}{2} \frac{m}{p_1} = 20
  \]
  and
  \[
  x_2 = \frac{1}{2} \frac{m}{p_2} = \frac{1}{2} \frac{40}{10} = 2
  \]
  Hence the total change in consumption of \(x_1\) is
  \[
  \Delta x_1 = 20 - 10 = 10
  \]
  Geometrically
  f) Substitution effect: auxiliary budget
  \[
  m' = 10 \times 1 + 10 \times 2 = 30
  \]
  and hence
  \[
  x_1 = \frac{1}{2} \frac{30}{1} = 15
  \]
  so \(SE\) is equal to
  \[
  SE = 15 - 10 = 5
  \]
  and income effect is
  \[
  IE = 10 - 5 = 5
  \]
Problem 2...
a) Bill’s utility function is
  \[
  U(x_1, x_2) = \min(2x_1, x_2)
  \]
b) Indifference curves in the commodity space \((x_1, x_2)\) are
  \[
  c) Bill’s demand for shoes is
  \[
  x_2 = 2x_1
  
  6x_1 + 2x_2 = 40
  \]
\[ 6x_1 + 2(2x_1) = 40 \]

\[ x_1 = \frac{40}{10} = 4 \]
\[ x_2 = 8 \]

d) geometrically Bills’s optimal choice is

e) when the price of a left shoe goes down to \( p_1 = \$1 \), the new demand is given by the system of equations

\[ x_2 = 2x_1 \]
\[ x_1 + 2x_2 = 40 \]

and hence demand is

\[ x_1 = 8 \]
\[ x_2 = 16 \]

The substitution effect is zero (perfect complements) and the income effect is 4.

Problem 3.
a) the two secrets of happiness are

\[ \frac{x_2}{100} = -1 \]
\[ x_1 + x_2 = 200 \]

and hence \( x_2 = 100 \) and \( x_1 = 100 \). Since both are positive, this is interior solution.

b) the two secrets of happiness are

\[ \frac{x_2}{100} = -1 \]
\[ x_1 + x_2 = 50 \]

and hence secrets of happiness give \( x_2 = 100 \) and \( x_1 = -50 \). Since consumption must be non-negative the optimal consumption is \( x_1 = 0 \) and \( x_2 = 50 \), which is a cornet solution.

Problem 4.
a) Jacob’s budget set, with \( w = \$10 \) and \( p_c = \$5 \) is

Income is \( m = 10 \times 24 = \$240 \)

b) They are perfect substitutes
c) $|MRS| = 1 < \frac{w}{p_c} = 2$ which implies that Jacob cares less about leisure than consumption, therefore he will spend the whole day at work

$$R = 0, LS = 24 \text{ and } C = 24 \frac{10}{5} = 48$$

d) Bonus Problem. (extra 10 points)
a) Monotone transformation $\ln()$. Take a log of $U$

$$\ln U() = \ln x_1^3 x_2 = \ln x_1^3 + \ln x_2 = 3 \ln x_1 + \ln x_2 = V()$$

where we used two properties of $\ln$ function.
b) For $U()$, marginal rate of substitution is

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_2^2 x_2}{x_1^3} = -\frac{3x_2}{x_1}$$

and for $V()$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3/ x_1}{1/x_2} = -\frac{3x_2}{x_1}$$

and hence MRS coincides for all $(x_1, x_2)$. It follows that the slopes of indifference curves are the same at any point and hence they must be the same.