Econ 301
Intermediate Microeconomics
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Final Exam

You have 2h to complete the exam. The final consists of 6 questions. The last question (about the equilibrium) is harder than the other ones.

Problem 1. (Consumer Choice)
Jeremy reads books, $x_1$, while drinking coffee, $x_2$. His utility function is given by $U(x_1, x_2) = (x_1)^{48} (x_2)^{24}$

a) Plot Jeremy’s indifference curve map (graph), find his MRS analytically (give a formula). Depict his MRS in the graph at the consumption bundle $(3, 3)$.

b) Using the "magic formula,” find the optimal level of consumption $x_1$ and $x_2$ if $p_1 = p_2 = 2$ and $m = 30$ and show it in the graph. Plot carefully the budget line and indifference curve passing through the optimal point (a graph + two numbers).

c) Suppose Jeremy’s preferences change and now they are given by $U(x_1, x_2) = \min (x_1, x_2)$. In a separate graph, plot his indifference curves, and find his $MRS$ for consumption bundles $(2, 1)$ and $(1, 2)$ (a graph + two numbers).

d) Find Jeremy’s optimal choice given $p_1 = 2 = 2$ and $m = 30$ and new preferences (two numbers).

Problem 2. (Technology)
Suppose a producer has access to the technology given by the Cobb-Douglass production function $y = K^{1/4}L^{1/4}$.

a) What can you say about the returns to scale (chose: IRS, CRS or DRS) and MPK (chose: increasing, constant, or decreasing).

b) Find a (variable) cost function $C(y)$ given the prices of inputs $w_K = 2, w_L = 2$ (give a function).

c) Suppose that in order to have access to the technology, the producer first has to pay the fixed cost $F = 4$, and hence the total cost is given by $TC = 4 + C(y)$. Find the supply function of the individual firm and plot it in the graph (give the formula, in the graph mark the prices for which the market will not open).

d) Assume that producers are competitive and there is free entry. Determine the number of firms operating in the industry with demand $D(p) = 10 - p$ (a number).

e) What is the microstructure of the industry (choose: competitive, oligopolistic, duopolistic or monopolistic).

Problem 3. (Short Questions)
a) You are leasing a car for which you are going to pay $8000 in each of the following three years. Find the present value of your payment if interest is equal to $r = 100\%$ (number).

b) A Bernoulli utility function is $u(x) = \sqrt{x}$, and two states of the world are equally likely. Which of the two options will be preferred: lottery ($0, $16$) or $8$ for sure (choose one)? Explain why (one sentence). Find the certainty equivalent of lottery ($0, 16$) (a number).

c) Signalling: Suppose there are two types of managers: talented with productivity $1$ and not talented with productivity $0$. The types are unobservable to employers and wages are competitive - they correspond to the expected productivity. Is an MBA program that takes $e = 2$ to complete a sufficient signal to separate the two types if the cost of effort of a not talented manager is $c(e) = 0.25e$

(yes/no + one sentence explaining why).

d) Find the minimal $e$ that is sufficient for separation (one number).

Problem 4. (Market Power)
Consider an industry with the inverse demand equal to \( p(y) = 10 - y \), and suppose that the total cost function is \( TC = 2y \).

a) Find the level of production in the industry and the price if there is only one firm (monopoly) charging a uniform price (give two numbers). Illustrate the choice using a graph, depicting Consumer, Producer Surplus and DWL (give three numbers and mark them on the graph).

b) Is the outcome Pareto efficient (yes/no + one sentence explaining why or why not). If not, which strategy would you suggest to increase both profit and restore efficiency?

c) Find the elasticity of the demand at the equilibrium level of production (number). Is a monopoly operating on elastic or inelastic part of the supply? (chose elastic or inelastic)

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are three firms operating in the industry (give three numbers). Show DWL in the graph. (Hint: use the symmetry of the firms.)

Problem 5. (Provision of Public Good)

There are two railroad transportation companies \( A \) and \( B \) whose profit depends on how the system of high-speed railroad system is developed. \( t^A \) denotes the miles of railroads built by \( A \) and \( t^B \) are the railroads constructed by firm \( B \). Suppose that the railroads are public good: once constructed none of the firms can prevent the other firm from using the whole railroad system, \( t = t^A + t^B \).

The profit of firm \( A \) is given by

\[
\pi^A(t^A, t^B) = 100 \times (t^A + t^B) - \frac{1}{2} (t^A)^2,
\]

where 100 \( \times t \) is the revenue from selling tickets and \( \frac{1}{2} (t^A)^2 \) is the cost of constructing the railroad track. Similarly, the profit of firm \( B \) is given by

\[
\pi^B(t^A, t^B) = 100 \times (t^A + t^B) - \frac{1}{2} (t^B)^2.
\]

a) Find the number of miles of railroads in the country if each firms individually chooses the level of construction to maximize profit (two numbers).

b) Find Pareto efficient level of \( t^A \) and \( t^B \). Are the two values higher or smaller then the ones in a) ? Why? (two numbers + one sentence)

c) Should the railroad track system in our example be determined by free market or a government (one sentence)?

Problem 6. (Difficult: Equilibrium With Intertemporal Choice)

Consider an intertemporal choice problem in which Jey is a manager who earns 0 today, and 100, tomorrow, \( \omega^J = (0, 100) \), while Kate is an athlete with an income of 100 and 0 tomorrow. \( \omega^K = (100, 0) \). The utility function of Jey and Kate is the same and given by

\[
U^i(x_1^i, x_2^i) = x_1^i + x_2^i
\]

where \( i = J, K \)

a) Plot the Edgeworth box and mark the point corresponding to endowments of Jey and Kate (graph).

b) Give a general definition of Pareto efficiency (one sentence), give the condition in terms of MRS (one sentence + a formula). Is the endowment Pareto efficient?

c) In the Edgeworth box, find all the allocations that are Pareto Efficient (contract curve).

d) Find (one) competitive equilibrium. Calculate borrowing and savings for both agents in your equilibrium. (Hint: in the intertemporal choice \( 1 + r = \frac{P_1}{P_2} \)).

e) Is a competitive equilibrium (allocation) unique (yes or no answer)? If not characterize the set of all allocations observed in competitive equilibria.