1 Problem 1

(a) His budget line without the CD gifts is straightforward: \(10x_1 + 10x_2 = 50\) and \(x_1\)-intercept \(\frac{50}{10} = 5\) and \(x_2\)-intercept \(\frac{50}{10} = 5\) represent his real incomes in terms of books and CDs, respectively. The red line in the figure below exhibits his budget line after those CD gifts.

(b) Since \(MU_1 = \frac{\partial U}{\partial x_1} = 2\) and \(MU_2 = \frac{\partial U}{\partial x_2} = \frac{6}{x_2}\), we can write his MRS as

\[
MRS_{1,2} = -\frac{2}{6/x_2} = -\frac{x_2}{3}. \tag{1.1}
\]

At \((3,6)\), its value is \(-2\) (or just its absolute value 2). Note that it does not rely upon \(x_1\), the quantity demanded for books. And he has to be compensated \(0.0001 \times 2\) CDs in order to preserve his current utility level. The indifference is described below, and the slope at \((3,6)\) should be the same as the value of MRS, 2.

(c) Two things are required to be an optimal; first, the tangency condition: the budget line should be a tangent line to the indifference curve. Second, the optimal point should be located on the budget line.

(d) The first Secret of Happiness (SOH) says that the MRS should coincide with (negative) the relative price at equilibrium: \(MRS = -\frac{p_1}{p_2}\). It means that rate of exchange at which the consumer is willing to stay put (MRS) must be equal to the price ratio. The second SOH is \(p_1x_1 + p_2x_2 = m\), which simply means that all the money should be exhausted by consuming both of the goods.

(e) Since \(MRS = -\frac{3x_2}{3}, x_2 = 3\) immediately follows from the first SOH. Note that he can afford \(x_2 = 3\). The extra money \$50 - \$10 \times 3 = \$20\) must be spent on good 1 by the second SOH.
and thus $x_1 = 2$. Now if $p_1$ increased to $20$, the first SOH says $\frac{x_2}{3} = \frac{20}{10} = 2$, that is, $x_2 = 6$ but it is not affordable since he needs $60$ for 6 CDs. Plugging $x_2 = 6$ into the budget line

$$20x_1 + 6 \times 10 = 50 \rightarrow x_1 = -\frac{1}{2} < 0.$$ (1.2)

Hence $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{50}{10} = 5$. While the first bundle $(x_1, x_2) = (2, 3)$ is interior, the second bundle $(x_1, x_2) = (0, 5)$ is at the corner.

(f) As depicted below, $(2, 3)$ is in the interior. Hence the marginal utilities from a dollar for books and for CDs at $(2, 3)$ must be the same. You can easily verify that

$$\text{when } (p_1, p_2) = (10, 10), \quad \frac{\mu_{U1}}{\mu_{p1}} = \frac{2}{10} = \frac{6/3}{10} = \frac{6/x_2}{p_2} = \frac{\mu_{U2}}{p_2}. \quad (1.3)$$

At $(x_1, x_2) = (0, 5)$, on the other hand, their marginal utilities from a dollar would not be the same. Observe that

$$\text{when } (p_1, p_2) = (20, 10), \quad \frac{\mu_{U1}}{\mu_{p1}} = \frac{4}{20} \text{ but } \frac{\mu_{U2}}{p_2} = \frac{6/x_2}{p_2} = \frac{6/5}{10} = \frac{6}{50}. \quad (1.4)$$

2 Problem 2

(a) Since hamburgers and cokes are perfect complements for Michael, his utility function over two goods must take a form of

$$U(x_1, x_2) = \min\{x_1, 5x_2\}. \quad (2.1)$$

(b) Recall that the indifference curve has a "L" shape in case of perfect complements. And the optimal preference line should be $x_1 = 5x_2$. 
(c) Two secrets of Happiness in case of perfect complements are as follows:

\( (i) \quad x_1 = 5x_2 \) \hspace{1cm} (2.2)

\( (ii) \quad p_1x_1 + p_2x_2 = m. \) \hspace{1cm} (2.3)

The condition \((i)\) provides us with the economic intuition that he is willing to consume two goods in the same proportion 2 : 1. The second condition says that his income must be exhausted.

(d) Plugging (2.2) into (2.3) gives us the equation of \( x_1 \) only;

\[ p_15x_2 + p_2x_2 = x_2(5p_1 + p_2) = m. \] \hspace{1cm} (2.4)

Hence \( x_2 = \frac{m}{5p_1+p_2} \) and \( x_1 = \frac{5m}{5p_1+p_2}. \) Since \( x_1, x_2 > 0 \) (without loss of generality, his income \( m > 0 \)) they are interior.

(e) They are ordinary since as \( x_1 \) and \( x_2 \) are downward sloping. They are normal since as \( m \) goes up, so do \( x_1 \) and \( x_2. \) Finally, they are gross complements since as the price of one good goes up the quantity of the other good will decrease.

(f) By Slutsky equation we can decompose the total change in demand into two effects; substitution and income effects. However, its substitution effects would be zero because they are perfect complements. Its main intuition follows from the fact that he is willing to consume in the same proportion so his optimal choice will be determined by the set of kinks in his indifference curve.

3 Problem 3

(a) Let a function \( V(x_1, x_2) = 6 \log x_1 + 2 \log x_2. \) You can easily see that Adam’s complicated utility function is just a monotone transformation of \( V. \) In fact, we shall rewrite his original
utility function in terms of $V$ as

$$U(x_1, x_2) = \log \left[ 12 \times \sqrt{\log V^2 + 3} \right]^{300}.$$  \hspace{1cm} (3.1)

(b) Part (a) enables us to analyze his consumption behavior using the simple function $V(x_1, x_2)$. Since it is a Cobb-Douglas utility, the magic formulae lead us to his demand function:

$$x_1 = \frac{a \cdot m}{a + b \cdot p_1} = \frac{6 \cdot 80}{6 + 2 \cdot 5} = 12$$

$$x_2 = \frac{b \cdot m}{a + b \cdot p_2} = \frac{2 \cdot 80}{6 + 2 \cdot 10} = 2.$$  \hspace{1cm} (3.2)

(c) Assume that $p_2 = 10$ and $m = 80$. Now we can think of $x_1$’s demand function which displays how the quantity of $x_1$ changes as $p_1$ varies, as well as the $p_1$-offer curve which displays how the set of optimal bundles $(x_1, x_2)$ changes as $p_1$ varies. When $p_2 = 10$ and $m = 80$, the above demand function comes down to

$$x_1 = \frac{a \cdot m}{a + b \cdot p_1} = \frac{6 \cdot 80}{6 + 2 \cdot p_1} = \frac{60}{p_1}$$

$$x_2 = \frac{b \cdot m}{a + b \cdot p_2} = \frac{2 \cdot 80}{6 + 2 \cdot 10} = 2.$$  \hspace{1cm} (3.3)

(3.2) addresses his demand function which is depicted below. With both of them (3.2) and (3.3), you can draw $p_1$-offer curve which must be flat.

4 Problem 4

(a) If Frank enjoys leisure during $R$ hours, his labor income would be $w(24 - R) = 20 \times (24 - R)$. Those money will be totally spent on ski passes($C$). Denoting its price by $p_C$, we can write his budget line as

$$p_C C = w(24 - R).$$  \hspace{1cm} (4.1)
Hence you end up with \(10C + 20R = 20 \times 24\) when \(p_C = 10\) and \(w = 20\). Note that he is just endowed with his daily hours, 24 hours. Point \(A = (24, 0)\) in the figure indicates his endowment.

(b) The real wage is simply \(\frac{w}{p_C} = 2\). It is the slope of his budget line.

(c) Since \(U(R, C) = R^{17}C^{34}\) is Cobb-Douglas, his demand for \(R\) and \(C\) are

\[
R = \frac{a \ m}{a + b \ w} = \frac{17 \times 480}{17 + 34 \times 20} = 8
\]

\[
\text{and} \quad C = \frac{b \ m}{a + b \ p_C} = \frac{34 \times 480}{17 + 34 \times 10} = 32, \]

respectively. From \(R = 8\), his labor supply is immediate; \(24 - 8 = 16\) hours.