CH 9

MODELS WITH NON RENEWABLE RESOURCES

R_t = STOCK OF NON-RENEWABLE RESOURCE AT TIME t

ASSUME IT IS EXTRACTED AS FOLLOWS:

$$E_t = a_E \cdot R_t \qquad (1)$$

E_t = EXTRACTION AT t; a_E = CONSTANT

AND

$$\dot{R}_t = -E_t \qquad (2)$$

THEN

$$(3) \qquad \boxed{\hat{R}_t = \frac{\dot{R}_t}{R_t} = \frac{-E_t}{R_t} = -a_E}$$

AGGR. OUTPUT:

$$(4) \quad Y = B K^{\alpha} E^{\gamma} L^{1-\alpha-\gamma}$$

$\alpha + \gamma < 1$

OR USING (1)

$$(5) \quad Y = B K^{\alpha} (a_E R_t)^{\gamma} L^{1-\alpha-\gamma}$$

LAW OF MOTION OF K:

$$(6) \quad \dot{K}_t = a Y - d K \Rightarrow \hat{K} = a\frac{Y}{K} - d$$

TECH. CHANGE: $\hat{B} = g_B$

BGP:

(6) $\Rightarrow \hat{y} = \hat{k}$

TAKING GROWTH RATES OF (5)

$$\hat{y} = \hat{B} + \alpha\hat{k} + \gamma(\hat{e}_E + \hat{h}_t) + (1-\alpha-\gamma)\hat{L}$$

$\hat{B} = g_B$; $\hat{k} = \hat{y}$; $\hat{e}_E + \hat{h}_t = -e_E$ BY (3); $\hat{L} = n$

$\Rightarrow$

$$\hat{y}(1-\alpha) = g_B - \gamma e_E + (1-\alpha-\gamma)n$$

$$\hat{y} = \underbrace{\frac{g_B}{1-\alpha}}_{g} - \underbrace{\frac{\gamma}{1-\alpha}}_{\bar{\gamma}} e_E + \underbrace{\left(\frac{1-\alpha-\gamma}{1-\alpha}\right)}_{1-\bar{\gamma}} n$$

TO SIMPLIFY NOTATION

$\Rightarrow \hat{y} = g - \bar{\gamma} e_E + n - \bar{\gamma} n$

(7) $\boxed{\hat{y} = g + n - \bar{\gamma}[e_E + n]}$ GROWTH RATE OF OUTPUT AT BGP

$\Rightarrow \boxed{\hat{y}} = \hat{y} - n = \boxed{g - \bar{\gamma}[e_E + n]}$

GROWTH RATE OF OUTPUT PER WORKER AT BGP