CH. 9

MODEL WITH LAND (T)

(1) $Y = B\, k^{\alpha}\, T^{\beta}\, L^{1-\alpha-\beta}$

ASSUME

- $\alpha + \beta < 1 \Rightarrow$ CRS
- $\hat{B} = g_B$ (TECH. CHANGE)
- $\hat{L} = n$ POP. GROWTH

LAW OF MOTION OF k:

(2) $\dot{k} = sY - d\,k \Rightarrow \boxed{\hat{k} = s\frac{Y}{k} - d}$ (3)

$\underset{\text{dep. RATE} < 1}{d}$

BGP: $\hat{Y}$, $\hat{k}$, ... CONSTANT

Eq. (3) $\Rightarrow$ ALONG BGP $\frac{Y}{k}$ CONSTANT

$\Rightarrow \hat{Y} = \hat{k}$ (4)

TAKING GROWTH RATES OF (1):

$$\hat{Y} = \underset{g_B}{\hat{B}} + \alpha \underset{\hat{Y} \text{ BY (4)}}{\hat{k}} + \beta \underset{0}{\hat{T}} + (1-\alpha-\beta)\underset{n}{\hat{L}}$$

$$\Rightarrow \hat{Y}(1-\alpha) = g_B + (1-\alpha-\beta)n$$

$$\hat{Y} = \frac{g_B}{1-\alpha} + \left(\frac{1-\alpha}{1-\alpha} - \underbrace{\frac{\beta}{1-\alpha}}_{\tilde{\beta}}\right) n$$

(TO SIMPLIFY EXPOSITION)

$$\Rightarrow \quad (5) \quad \boxed{\hat{Y} = \frac{g_B}{1-\alpha} + (1-\bar{\beta})\, n}$$

THEN OUTPUT PER WORKER GROWS AT:

$$(6) \quad \widehat{\left(\frac{Y}{L}\right)} = \boxed{\hat{y}} = \hat{Y} - n = \boxed{\frac{g_B}{1-\alpha} - \bar{\beta} n}$$

REMARKS

(1) IF $\beta = 0$ (i.e. NO LAND) $\Rightarrow \bar{\beta} = 0$
WE ARE BACK AT BASIC SOLOW

(2) IF $\beta > 0$, A LARGER β (i.e. LAND IS MORE IMPORTANT FOR PRODUCTION) RESULTS IN LOWER $\hat{y}$

SO LAND INTRODUCES A DRAG ON GROWTH

(3) POPULATION GROWTH HAS A SIGNIFICANT ROLE: $\uparrow n \Rightarrow \downarrow \hat{y}$ (POP. GROWTH IS REALLY BAD BECAUSE OF THE FIXED RESOURCE) (CONTRAST THIS WITH ROLE IN CH. 6)

(4) EQ. (6) SHOWS THAT THERE IS A "RACE" BETWEEN TECH.

PROGRESS & THE DIMINISHING RETURNS INTRODUCED BY THE FIX FACTOR –

IF $g_B = 0$ (i.e NO TECH. PROGRESS)

$\Rightarrow \hat{y} = -\bar{\beta} n$ (i.e $y \to 0$: EXTREME VERSION OF MALTHUS THEORY!) –

<u>COMPARATIVE STATICS</u> (EX. 1, JONES CH. 9)

WE FOCUS ON THE RATIO : $\frac{K}{Y}$

(CAPITAL OUTPUT RATIO THAT WE CALL z, i.e.

$$z = \frac{K}{Y}$$
)

USING (1), WE HAVE (DIVIDING BY K)

$$\frac{Y}{K} = B\, K^{\alpha-1} T^{\beta} L^{1-\alpha-\beta}$$

$$\hat{y} - \hat{k} = \underset{\substack{\| \\ g_B}}{\hat{B}} + (\alpha-1)\hat{k} + (1-\alpha-\beta)n$$

NOW:

$$\hat{z} = \widehat{(K/Y)} = \hat{k} - \hat{y} = -(\hat{y} - \hat{k})$$

$$(7) \quad \Rightarrow \hat{z} = -g_B + (1-\alpha)\hat{k} - (1-\alpha-\beta)n$$

EQ (3) $\Rightarrow$

$$\hat{k} = a\,\frac{Y}{K} - d = a z^{-1} - d$$

THEN (7) CAN BE WRITTEN AS:

$$\hat{z} = -\rho_B - (1-\alpha-\sigma)\,n + (1-\alpha)\left[a z^{-1} - d\right]$$

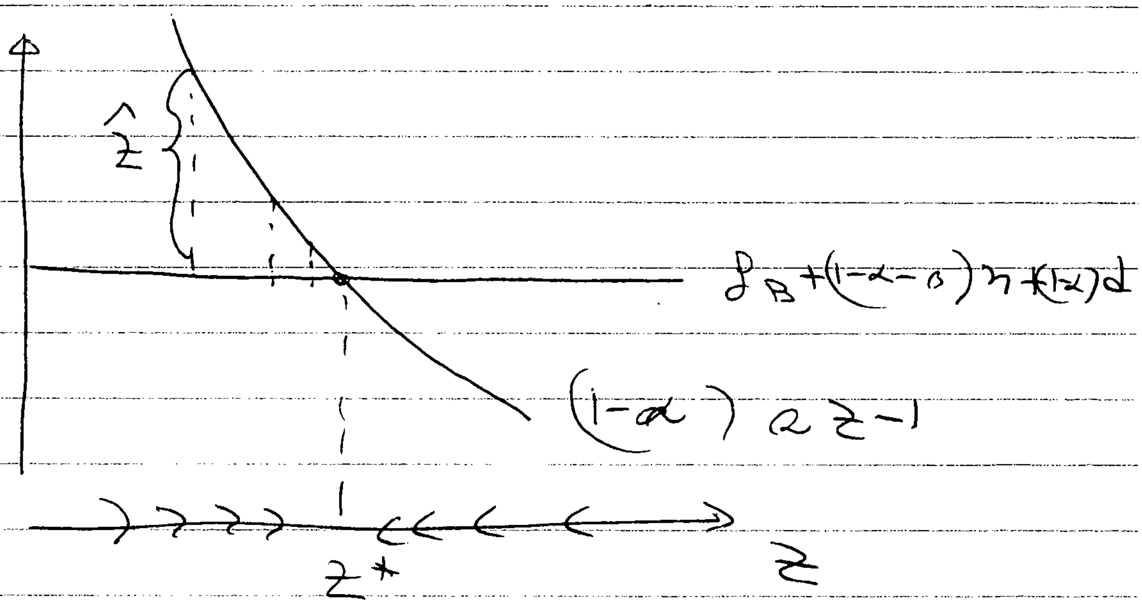
i.e.

$$(8) \quad \hat{z} = (1-\alpha)\,a z^{-1} - \left[\rho_B + (1-\alpha-\sigma)\,n + (1-\alpha)\,d\right]$$

EQ. (8) HOLDS ALWAYS

AT BGP $\hat{z} = 0$ SINCE $\hat{Y} = \hat{K}$

THEN:



IF $z < z^*$, BY (8): $\hat{z} > 0$ (i.e. $z \uparrow$)

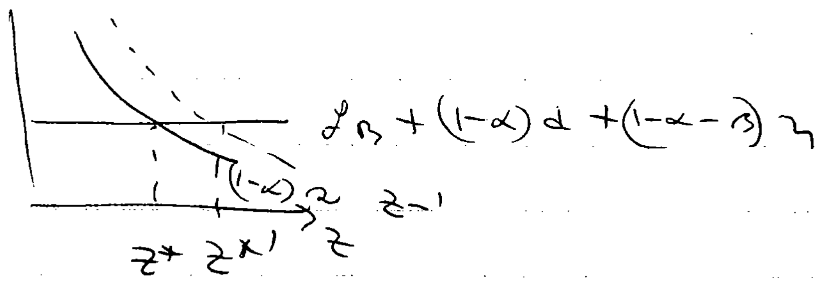
IF $z > z^*$, " ": $\hat{z} < 0$ (i.e. $z \downarrow$)

IF $z = z^*$, $\hat{z} = 0$

IT IS EASY TO SEE FROM THE DIAGRAM THAT z CONVERGES TO z^* AND THAT AS WE GET CLOSER TO z^*, $\dot{z}$ DECREASES (IN ABSOLUTE VALUE)

(a) • CHANGES IN Ω:

$$\uparrow \Omega \Rightarrow \uparrow z^*$$



OTHER CHANGES SIMILAR —