CH 6

GROWTH & DEV. MODEL

TECHNOLOGY MOTIVATION

$$(0) \quad Y = L^{1-\alpha} \int_0^h x_j^{\alpha} \, dj$$

WHERE $0 < \alpha < 1$

h = # OF INT. GOODS THAT CAN BE USED (REFLECT SKILL LEVEL OF LABOR FORCE)

AGG. CAPITAL STOCK:

$$K = \int_0^h x_j \, dj$$

IF $x_j = x$ FOR ALL j: (TRUE DUE TO SYMMETRY OF MODEL)

$$K = x[h-0] = h \cdot x \Rightarrow x = K/h$$

THEN (0) CAN BE REWRITTEN.

$$Y = L^{1-\alpha} x^{\alpha} [h-0] = L^{1-\alpha} h \cdot \left[\frac{K}{h}\right]^{\alpha}$$

$$= L^{1-\alpha} h^{1-\alpha} K^{\alpha}$$

i.e.

$$\boxed{(1) \quad Y = K^{\alpha} (hL)^{1-\alpha}}$$

ALSO $\hat{L} = n$ (POP. GROWTH)

LAW OF MOTION OF K-STOCK:

(2) $\dot{k} = s_K Y - dk$

LAW OF MOTION OF h

$$(3) \quad \dot{h} = \mu e^{\psi u} A^{\gamma} h^{1-\gamma}$$

A = # OF INT. GOODS AVAILABLE IN THE WORLD (OR TECH. LEVEL OF LEADER)

$0<\gamma<1$, $\mu>0$
↳ REPRESENTS HOW EASY IT IS TO GET TECHNOLOGY FROM ABROAD

u = TIME DEVOTED TO SKILL ACCUMULATION (≈ YEARS OF SCHOOLING)

DESIGNS ARE AVAILABLE FREE.

(3) ⟹

$$(4) \quad \frac{\dot{h}}{h} = \boxed{\hat{h} = \mu e^{\psi u} \left(\frac{A}{h}\right)^{\gamma}}$$

EVOLUTION OF FOREIGN TECHNOLOGY:

$$(5) \quad \hat{A} = g$$

BGP: $\hat{Y}, \hat{k}, \hat{h}, \ldots$ CONSTANT

GROWTH RATES (2) ⟹ Y/k CONSTANT ⟹ $\hat{Y} = \hat{k}$
↳ USUAL ARGUMENT

(4) ⟹ $\hat{A} = \hat{h}$

TAKING GROWTH RATES OF (1)

$$\hat{Y} = \alpha\hat{k} + (1-\alpha)[\hat{h} + \underbrace{\hat{L}}_{= n}] \Rightarrow \hat{Y}(\cancel{1-\alpha}) = \cancel{(1-\alpha)}[\hat{h}+n]$$

$$\Rightarrow \quad \hat{Y} = \hat{K} = \hat{h} + n = \hat{A} = g$$

$$\Rightarrow \widehat{\left(\frac{Y}{hL}\right)} = \hat{Y} - \hat{h} - n = 0$$

$$\Rightarrow \hat{y} = \widehat{\left(\frac{Y}{L}\right)} = \hat{h} = \hat{A} = g$$

OUTPUT per worker grows at rate of tech. change

($\hat{A} = g$ given overseas!)

PATH OF OUTPUT PER WORKER AT BGP

(2) $\Rightarrow$

$$\hat{K} = s_K \frac{Y}{K} - d = s_K \frac{Y/L}{K/L} - d$$

$$= s_K \frac{y}{k} - d$$

then

$$\hat{k} = \hat{K} - n = s_K \frac{y}{k} - (d + n)$$

ALONG BGP: $\hat{k} = \hat{K} - \hat{L} = g$

$$\Rightarrow \hat{k} = s_K \frac{y}{k} - (d + n) = g$$

$$\Rightarrow \boxed{\frac{y}{k} = \frac{d + n + g}{s_K}} \quad (5)$$

(1) $\Rightarrow$

$$Y/L = y = k^{\alpha} h^{1-\alpha} \quad (6.)$$

$$(5) \Rightarrow k = \frac{2k}{d+n+g}\, y \quad (7)$$

then (6) & (7) ⇒

$$y = \left[\frac{2k}{d+n+g}\right]^{\alpha} y^{\alpha}\, h^{1-\alpha}$$

$$\Rightarrow y^{1-\alpha} = \left[\frac{2k}{d+n+g}\right]^{\alpha} h^{1-\alpha}$$

⇒

AT BGP $\boxed{y^{*} = \left[\frac{2k}{d+n+g}\right]^{\frac{\alpha}{1-\alpha}} h}$ (8)

AT BGP $\hat{h} = g$, then (4) ⇒

$$g = \mu\, e^{\psi u} \frac{A^{\gamma}}{h^{\gamma}}$$

$$\Rightarrow \quad h = \left[\mu \frac{e^{\psi u}}{g}\right]^{\frac{1}{\gamma}} A \quad (9)$$

then (8) & (9)

$$\boxed{y^{*}_{t} = \left[\frac{2k}{d+n+g}\right]^{\frac{\alpha}{1-\alpha}} \left[\mu \frac{e^{\psi u}}{g}\right]^{\frac{1}{\gamma}} A_t}$$

similar to Solow with H-K!! in CH 3

$\uparrow \mu \Rightarrow \uparrow y^{*}_{t}$