ECON. GROWTH

- FOCUS ON GROWTH OF GROSS DOMESTIC PRODUCT (GDP) AND GDP PER CAPITA
- WANT TO HAVE MODEL TO UNDERSTAND:
  1. DIFFERENT LEVELS OF GDP P. CAPITA ACROSS COUNTRIES
  2. DIFFERENT GROWTH RATES OF GDP & GDP p.c. ACROSS COUNTRIES & ACROSS TIME
  3. EFFECT OF GOVERNMENT POLICIES ON LEVELS OF GDP p.c. & GROWTH RATES

SOLOW MODEL:

FOCUS ON: CAPITAL, LABOR, TECHNOLOGY

MAIN CHARACTERISTICS:

- DYNAMICS
- SINGLE GOOD ⇆ CONSUMPTION / INVESTMENT
- VERY SIMPLE DEMAND/CONSUMER SIDE
- SUPPLY OF GOODS (Demand of Goods approach
- CLOSED ECONOMY
- TECHNOLOGY EVOLVES EXOGENOUSLY
- LABOR FORCE PARTICIPATION CONSTANT (MANY TIMES POPULATION = WORK FOR

△ SOLOW MODEL WITHOUT TECHNOLOGICAL CHANGE & WITH POPULATION GROWTH

OBJECTIVE: CHARACTERIZE MODEL BEHAVIOR TO GET IMPLICATIONS FOR POLICY AT:

(I) BGP (BALANCED GROWTH PATH)

(i) LOOK AT GROWTH RATES OF VARIABLES AND PER WORKER/CAPITA VARIABLES

$\left(\hat{K}, \hat{Y}, \hat{C} \;\&\; \widehat{(Y/L)} = \hat{y}, \widehat{(K/L)} = \hat{k}\right), \widehat{(C/L)} = \hat{c}$

(ii) LOOK AT LEVELS OF PER WORKER/CAPITA VARIABLES

$y, k, c,$

(II) OUTSIDE BGP

(i) LOOK AT GROWTH RATES OF VARIABLES & PER WORKER/CAPITA VARIABLES

(ii) LOOK AT ADJUSTMENT OF LEVELS OF PER WORKER/CAPITA VARIABLES.

(2) BASICS

PRODUCTION FUNCTION

(1) $Y = F(k, L) = k^{\alpha} L^{1-\alpha} \qquad 0<\alpha<1$

CONSUMPTION / SAVINGS DECISION

(2) $S = s \cdot Y \qquad 0<s<1$

s : SAVINGS RATE

SINCE $Y = C + I \Rightarrow C = (1-s)Y$ (3)

POPULATION = # WORKERS

(4) $\hat{L} = n$ (

LAW OF MOTION OF k

(5) $\dot{k} = I - \delta k \qquad 0<\delta<1$

δ : DEPRECIATION RATE

EQUILIBRIUM: SAVINGS = INVESTMENT

(6) $S = I$

USING (2) (5) & (6) WE GET

(7) $\boxed{\dot{k}} = S - \delta k = \boxed{s \cdot Y - \delta k}$

SINCE $\hat{k} = \Delta k / k$, USING (7)

(8) $\boxed{\hat{k}} = \dfrac{\dot{k}}{k} = s\dfrac{Y}{k} - \delta\dfrac{k}{k} = \boxed{s\dfrac{Y}{k} - \delta}$

BGP: ORIGINAL VARIABLES

$$Y = K^{\alpha} L^{1-\alpha}$$

ALONG BGP $\hat{Y}$, $\hat{K}$, $\hat{L}$ CONSTANT

TAKING GROWTH RATES

(i) $$\boxed{\hat{Y}} = \alpha \hat{K} + (1-\alpha)\underset{\overset{\parallel}{n}}{\hat{L}} = \boxed{\alpha \hat{K} + (1-\alpha) n}$$

LAW OF MOTION OF K OR $\hat{K}$

$$\hat{K} = s \cdot \frac{Y}{K} - (\delta + n)$$

IF $\hat{K}$ CONSTANT ALONG BGP $\Rightarrow$

$s\frac{Y}{K}$ CONSTANT, $\Rightarrow \widehat{\left(\frac{Y}{K}\right)} = 0$

(ii) $$\Longrightarrow \hat{Y} = \hat{K}$$

USING (ii) INTO (i) $\Rightarrow$

$$\hat{K} = \alpha \hat{K} + (1-\alpha) n$$

$\Longrightarrow$

$$(1-\alpha)\hat{K} = (1-\alpha) n$$

$$\Longrightarrow \boxed{\hat{K} = n} = \hat{Y} = \hat{L}$$

SO AT BGP

K, Y, L GROW AT THE SAME RATE n

$\Longrightarrow$

$\boxed{\frac{Y}{L}}$, $\boxed{\frac{K}{L}}$ CONSTANT AT BGP

THEN IF WE USE MODIFIED VARIABLES $y = Y/L$, $k = K/L$

THE SYSTEM WILL HAVE A S. STATE !!

3. CHANGE OF VARIABLE (NEW VARIABLES $y = \frac{Y}{L}$, $k = \frac{K}{L}$, $c = \frac{C}{L}$)

LOOK AT MODEL IN THESE NEW VARIABLES
NEED TO CALCULATE LAW OF MOTION OF k.

DIVIDE BOTH SIDES OF (1) BY L

$$\frac{Y}{L} = \frac{K^{\alpha} L^{1-\alpha}}{L} = K^{\alpha} \cancel{L^{1-\alpha-1}} = \left(\frac{K}{L}\right)^{\alpha}$$

i.e.

(10) $$y = \underbrace{k^{\alpha}}_{\text{WE CALL THIS IN GENERAL } f(k)}$$

LAW OF MOTION OF k

SINCE $k = K/L$ USING HAT ALGEBRA

(11) $$\hat{k} = \hat{K} - \underbrace{\hat{L}}_{n \text{ BY (4)}} = \hat{K} - n$$

USING (8) & (11)

$$\hat{k} = s\,(Y/K) - \delta - n = s\left(\frac{Y}{K}\right) - (\delta + n)$$

DIVIDING & MULTIP. THE 1ST TERM BY L

$$= s\left[\frac{Y/L}{K/L}\right] - (\delta + n)$$

THEN

(12) $$\boxed{\hat{k}} = \boxed{s\,\frac{y}{k} - (\delta + n)}$$

USING (10)

(12') $$\hat{k} = s\,(k^{\alpha}/k) - (\delta + n) = s\,k^{\alpha-1} - (\delta + n)$$

4. SINCE $\hat{k}_n = \dot{k}_n / k_n$, WE HAVE $\dot{k}_n = \hat{k}_n \cdot k_n$

WE MULTIPLY (12') BY k_n AND GET:

$$\boxed{\dot{k}_n} = \hat{k}_n \cdot k_n = s \cdot k_n^{\alpha-1} k_n - (\delta+n) k_n \tag{14}$$

$$= \boxed{s\, k_n^{\alpha} - (\delta+n)\, k_n}$$

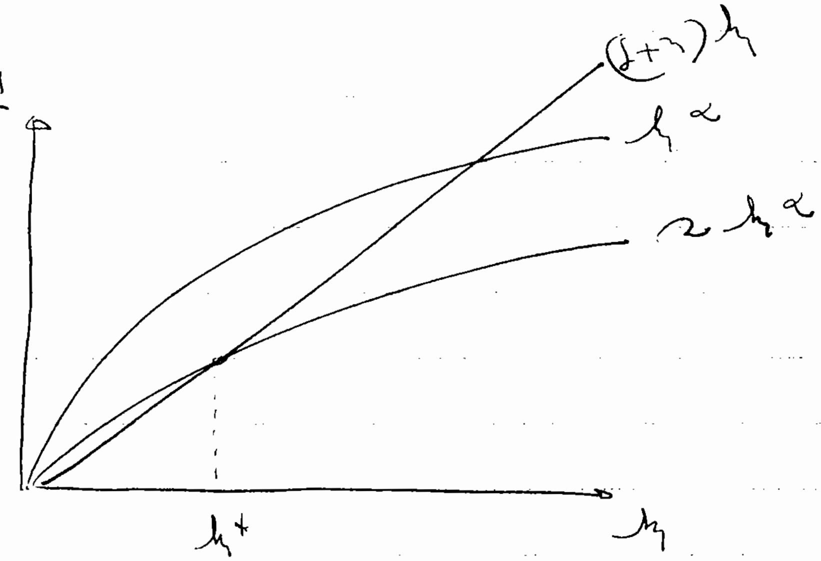
REMARK:

EQ. (14) IS ALWAYS TRUE (i.e. HOLDS AT BGP & OUTSIDE)

$$\dot{k} = s\, f(k) - (\delta+n) k \tag{14}$$

$$= s\, k_n^{\alpha} - (\delta+n)\, k_n$$

DIAGRAM

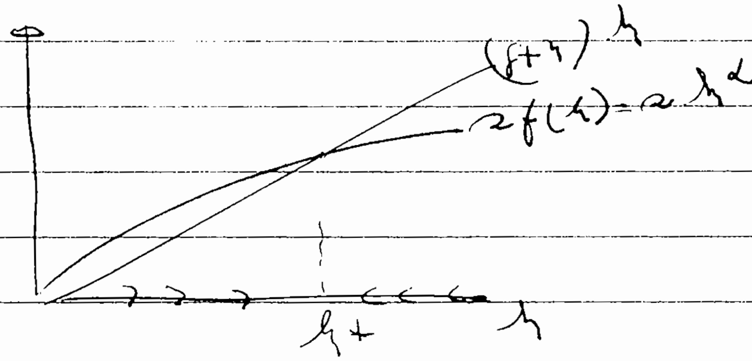


5 ANALYSIS

(I) BGP (ORIGINAL VARIABLES)

(II) STEADY ~~STATE~~ (MODIFIED ")

(II) MODIFIED VARIABLES (14) $\Delta k = s k^{\alpha} - (\delta + n) k$



System has a steady state at k^*

IF $k < k^* \Rightarrow \Delta k > 0 \Rightarrow k \uparrow$ $(\hat{k} > 0)$

IF $k > k^* \Rightarrow \Delta k < 0 \Rightarrow k \downarrow$ $(\hat{k} < 0)$

IF $k = k^* \Rightarrow \Delta k = 0 \Rightarrow k$ CONSTANT $(\hat{k}^* = 0)$

REMARKS ABOUT y:

$$y = k^{\alpha} \Rightarrow \hat{y} = \alpha \hat{k}$$

so $y^* = k^{*\alpha}$ is the steady state value for y

IF $y < y^* \Rightarrow \Delta y > 0$, $y \uparrow$ $(\hat{y} > 0)$

$y > y^* \Rightarrow \Delta y < 0$, $y \downarrow$ $(\hat{y} < 0)$

$y = y^* \Rightarrow \Delta y = 0$, y CONSTANT $(\hat{y}^* = 0)$

(I) $\hat{k}^* = \hat{y}^* = 0 \Rightarrow$

$\hat{Y} = \hat{K} = \hat{C} = n$ THIS IS BGP

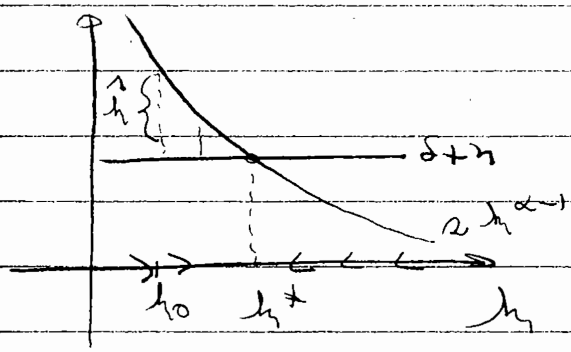
ADVANCED ANALYSIS

(A) PATHS & GROWTH RATES

(14) $\Delta k = s k^{\alpha} - (\delta + n) k$

DIV. BY k

$$\frac{\Delta k}{k} = \hat{k} = s\, k^{\alpha - 1} - (\delta + n)$$



NOTICE THE SHAPES

$z(k) = s k^{\alpha - 1} = s \frac{1}{k^{1-\alpha}} > 0$

$z' = s(\alpha - 1) k^{\alpha - 2} < 0$

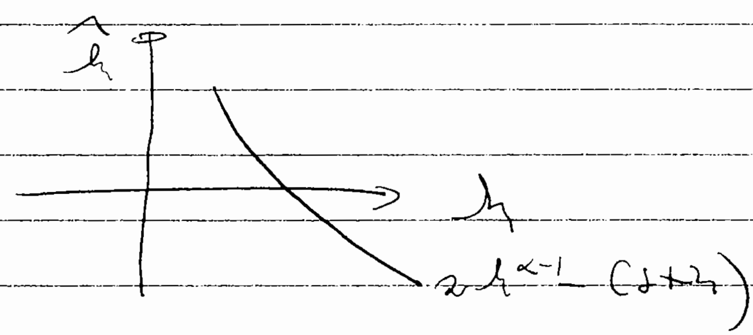
$z'' = s(\alpha - 1)(\alpha - 2) k^{\alpha - 3} > 0$

ALSO

$z \to 0$ AS $k \to \infty$

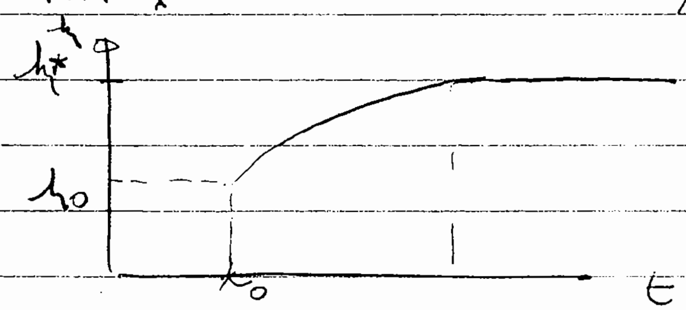
$z \to \infty$ AS $k \to 0$

- THE FURTHER IS k FROM k^* THE LARGER IN ABSOLUTE VALUE $\hat{k}$ IS.

- 

- PATHS

AT $t = t_0$, $k = k_0$



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B) STEADY STATE CALCULATION
WHAT IS k^*, y^*?

AT S.S. $\Delta k = 0 \Rightarrow \Delta k = 0 = s k^\alpha - (\delta + n) k$

USING (14)

DIVIDING BOTH SIDES BY k^α

$$0 = s\frac{\cancel{k^\alpha}}{\cancel{k^\alpha}} - (\delta+n)\frac{k}{k^\alpha}$$

$$= s - (\delta+n)\, k^{1-\alpha}$$

$\Longrightarrow$

$$k^{*\,1-\alpha} = \frac{s}{\delta+n}$$

OR

$$\boxed{k^* = \left[\frac{s}{\delta+n}\right]^{\frac{1}{1-\alpha}}}$$

THEN

$$\boxed{y^* = k^{*\alpha} = \left[\frac{s}{\delta+n}\right]^{\frac{\alpha}{1-\alpha}}}$$

SO $\uparrow s \Rightarrow \uparrow$ S.S. k^*, y^*

$\uparrow \delta, n \Rightarrow \downarrow$ S.S. k^*, y^*

REMARKS / RESULTS OF SOLOW WITHOUT TECH. CHANGE

1 • OUTPUT PER WORKER AT THE S.S.
INCREASES WITH ↑ s (SAVINGS RATE)
DECREASES WITH ↑ δ (Depreciation)
↑ n (rate of population growth)

2 GROWTH RATES

• GROWTH RATE OF OUTPUT PER WORKER AT THE S.S. IS ZERO

• IF CAPITAL PER WORKER IS BELOW S.S. THEN IT GROWS & SO DOES OUTPUT PER WORKER

• IF CAPITAL PER WORKER IS ABOVE THE S.S. LEVEL THEN IT SHRINKS & SO DOES OUTPUT PER WORKER

• THE FURTHER THE CAPITAL PER WORKER IS FROM THE S.S. LEVEL IN ABSOLUTE VALUE THE HIGHER THE GROWTH RATE OF OUTPUT PER WORKER IN ABSOLUTE VALUE

(i.e. GROWTH RATE ↓ AS WE APPROACH THE S.S.)

OTHER REMARK

AT S.S. $\hat{y} = 0 \Rightarrow \widehat{\left(\frac{Y}{L}\right)} = 0$

$\Rightarrow \boxed{\hat{Y} = \hat{L} = n}$

AND $\hat{k} = 0 \Rightarrow \widehat{\left(\frac{K}{L}\right)} = 0$

$\Rightarrow \boxed{\hat{K} = \hat{L} = n}$

THEN $\hat{Y} = \hat{K} = \hat{L} = n$

i.e. OUTPUT, CAPITAL & LABOR GROW AT THE SAME RATE (RATE OF POPULATION GROWTH)

SO:

THE STEADY STATE OF THE MODIFIED SYSTEM IS THE BALANCED GROWTH PATH OF THE ORIGINAL SYSTEM.