COMPARATIVE STATICS: SOLOW MODEL WITH TECH. CHANGE

$$Y = K^{\alpha}(AL)^{1-\alpha} \quad \text{and} \quad \hat{A} = g$$

IN THIS CASE WE MAKE A CHANGE OF VARIABLE AND ALL MAGNITUDE ARE NOW IN "EFFECTIVE UNIT OF LABOR": $\tilde{y} = \frac{Y}{AL} = \frac{y}{A}$

$$\tilde{k} = \frac{K}{AL} = \frac{k}{A}$$

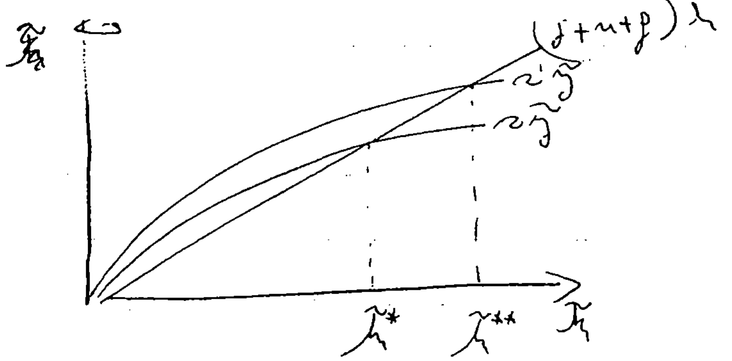
$$\Rightarrow \tilde{y} = \tilde{k}^{\alpha}$$

THE LAW OF MOTION OF $\tilde{k}$ IS:

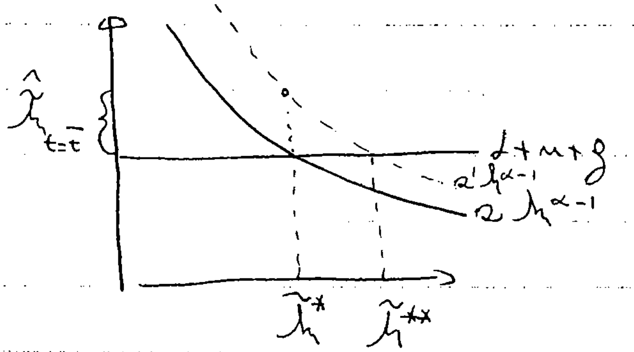
$$\hat{\tilde{k}} = s \cdot \frac{\tilde{y}}{\tilde{k}} - (\delta + n + g) \quad \text{or} \quad \dot{\tilde{k}} = s\tilde{y} - (\delta + n + g)\tilde{k}$$

AT S.S.: $\hat{\tilde{k}} = 0$

1) $\uparrow s \Rightarrow \tilde{k}^{*} \uparrow$ to a new level: $\tilde{k}^{**}$



Transitional DYNAMICS: $\hat{\tilde{k}} = \frac{s\tilde{y}}{\tilde{k}} - (\delta + n + g) = s k^{\alpha - 1} - (\delta + n + g)$



Assume $s \uparrow$ to s' at $t = \bar{t}$

$\Rightarrow \hat{\tilde{k}}_{t=\bar{t}} > 0$

as $t \uparrow$ $\hat{\tilde{k}} > 0$ but $\downarrow$

until NEW S.S. REACHED

(i) PATH OF $\hat{y} = \widehat{\left(\frac{Y}{L}\right)}$

SINCE $\frac{Y}{L} = y = k^{\alpha} A^{1-\alpha}$ taking logs:

$$\log y = \alpha \log k + (1-\alpha) \log A$$

taking derivatives $\Rightarrow$

$$\frac{d \log y}{dt} = \alpha \frac{d \log k}{dt} + (1-\alpha) \frac{d \log A}{dt}$$

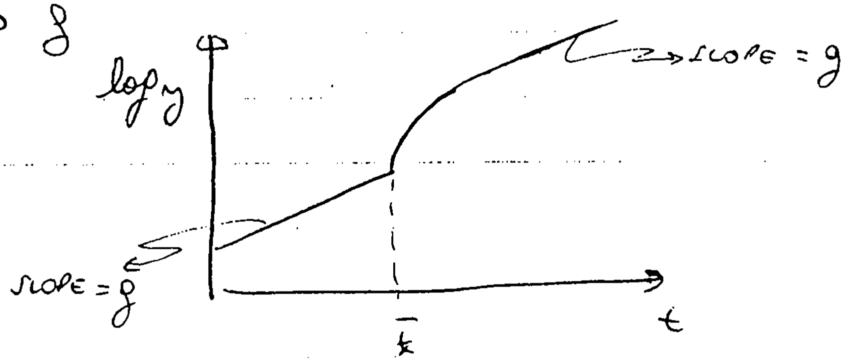
$$\boxed{\hat{y}} = \alpha \hat{k} + (1-\alpha) \hat{A} = \boxed{\alpha \hat{k} + (1-\alpha) g}$$

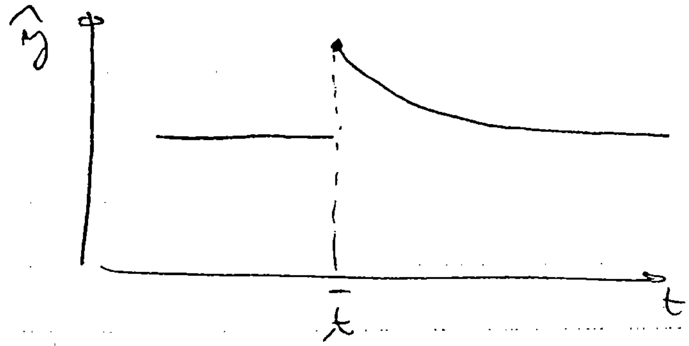
At old s.s. $\hat{\tilde{h}} = 0 \Rightarrow \hat{h} = \hat{A} = g$ so $\hat{y} = \alpha g + (1-\alpha)g = g$

Notice then that $\hat{y} = \frac{d \log y}{dt}$ = constant $= g$

At $\bar{t}$, $\hat{\tilde{h}} > 0$, then $\hat{y}_{t=\bar{t}} = \frac{d \log y}{dt} > g$

Since $\hat{\tilde{h}}$ is $\downarrow$ as $t \uparrow$ $\Rightarrow$ $\hat{y}$ is $\downarrow$ until it eventually gets back to g





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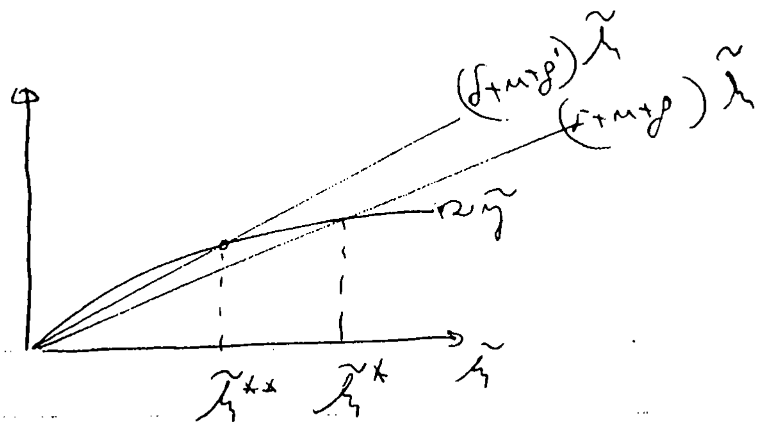
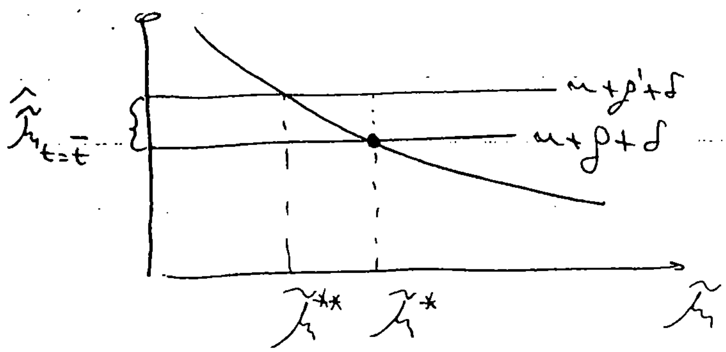
2) ↑ g





notice that at $t=t'$

(1) $\hat{\tilde{k}}_{t=\bar{t}} \equiv -(g'-g) = g-g'$

(i) Path of $\hat{k}$ and $\tilde{y}$

At $t=\bar{t}$, (1) $\Rightarrow$ $\hat{\tilde{k}}_{t=\bar{t}} = \hat{k}_{t=\bar{t}} - \underset{\substack{\| \\ g'}}{\hat{A}} = g-g' < 0$

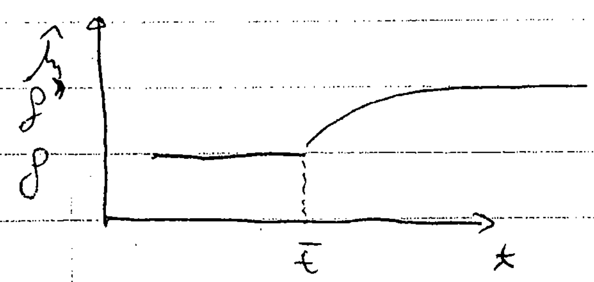
$\Longrightarrow$ $\boxed{\hat{k}_{t=\bar{t}}} = g-g'+g' = \boxed{g}$

At old s.s., $\hat{k}^{*} = g$

" new s.s., $\hat{k}^{**} = g'$

Since $\hat{\tilde{k}}_t < 0$ ~~but~~ ↓ in ABSOLUTE VALUE AS t↑,

$\hat{k}_t$ is INCREASING AS t↑ until it reaches g'.



NOTICE $\tilde{y} = \tilde{k}^{\alpha} \Rightarrow \hat{\tilde{y}} = \alpha \hat{\tilde{k}}$

and $\hat{\tilde{y}} = \hat{y} - \hat{A}$. At old s.s. $\hat{\tilde{y}} = 0 \Rightarrow \hat{y} = g$

AT NEW s.s. $\hat{\tilde{y}} = 0 \Rightarrow \hat{y} = g'$

THEN:

$$\hat{\tilde{y}} = \hat{y} - \hat{A} \implies \hat{y} = \hat{\tilde{y}} + A = \alpha \hat{\tilde{\lambda}} + \hat{A}$$

At $t = \bar{t}$, $\hat{\tilde{\lambda}}_{t=\bar{t}} = -[g' - g] \implies \hat{y}_{t=\bar{t}} = \alpha[g - g'] + g'$
$$= \boxed{\alpha g + (1-\alpha) g'}$$