6-8-09

CH.5 ROMER MODEL

TECHNOLOGY:

(1) $Y = K^{\alpha}(A \cdot L_Y)^{1-\alpha}$ $\alpha < 1$

(2) $\dot{K} = s_K \cdot Y - dK$

(3) $\dot{A} = \delta A^{\phi} L_A^{\lambda}$

(4) $\hat{A} = \dfrac{\delta \; L_A^{\lambda}}{A^{1-\phi}}$

L_Y = LABOR USED IN FINAL GOOD PRODUCTION

A_t = STOCK OF IDEAS INVENTED UP TO TIME t

d = DEPRECIATION PHYSICAL CAPITAL

s_K = SAVINGS RATE

L_A = LABOR USED IN R&D SECTOR

ADDITIONAL PARAMETERS:

$\delta > 0$

$0 < \phi < 1$ → POSITIVE: "STANDING ON SHOULDERS"
→ BETWEEN 0 & 1: "FISHING"

$0 < \lambda < 1$: STEPPING ON TOES

ADDITIONAL ASSUMPTIONS / IMPLICATIONS

$\hat{L} = n$

FEASIBILITY / EQUILIBRIUM:

$L = L_Y + L_A$

FROM DATA: $\hat{L}_A = \hat{L}_Y = \hat{L} = n$

POPULATION = WORKFORCE

$$\Longrightarrow \frac{L_A}{L} = \text{CONSTANT} = \%\text{ WORKERS IN R\&D SECTOR} = s_R$$

$$\frac{L_Y}{L} = \text{CONSTANT} = \%\text{ WORKERS IN FINAL GOODS SECTOR} = s_Y = (1-s_R)$$

NEXT STEPS :

→ (I) CHARACTERIZE BGP

→ (II) COMPARATIVE STATICS

(I) BGP $\Longrightarrow \hat{Y}, \hat{K}, \hat{A} \ldots$ CONSTANT

Eq. (3) $\Longrightarrow$

$$\hat{K} = s_K \frac{Y}{K} - d$$

THEN $\hat{Y} = \hat{K}$

TAKING GROWTH RATES OF (1)

$$\hat{Y} = \alpha \hat{K} + (1-\alpha)\left[\hat{A} + \hat{L}_Y\right]$$

$$\hat{Y} = \hat{K}, \qquad \hat{K} = \hat{Y}$$

$$\hat{Y} = \alpha \hat{Y} + \cancel{(1-\alpha)}\left[\hat{A} + \hat{L}_Y\right]$$

$$\Longrightarrow \boxed{\hat{Y} = \hat{A} + \hat{L}_Y}$$

$$\Rightarrow \boxed{\hat{Y} = \hat{K} = \hat{A} + n}$$

SINCE

$$y = \frac{Y}{L} \Rightarrow \boxed{\hat{y}} = \hat{Y} - \underset{\substack{\| \\ n}}{\hat{L}} = \hat{A} + n - n = \boxed{\hat{A}}$$

REMARK: OUTPUT PER WORKER GROWS AT THE RATE OF TECH. CHANGE

QUESTION: WHAT IS $\hat{A}$ AT BGP?

(4) $\hat{A} = \dfrac{\delta L_A^{\lambda}}{A^{1-\phi}}$

ALONG BGP, $\hat{A}$ CONSTANT $\Rightarrow$

$$\widehat{\left(\delta L_A^{\lambda}\right)} = \widehat{\left(A^{1-\phi}\right)}$$

$$\underset{\substack{\| \\ 0}}{\hat{\delta}} + \lambda \underset{\substack{\| \\ n}}{\hat{L}_A} = (1-\phi)\hat{A}$$

$$\Rightarrow \boxed{\hat{A} = \frac{\lambda n}{1-\phi}} \quad (5)$$

<u>CONCLUSION</u>:

ALONG BGP:

$$\boxed{\hat{y} = \hat{k} = \hat{A} = \frac{\lambda n}{1-\phi}}$$

<u>QUESTION</u>:

WHAT IS THE EQUATION FOR OUTPUT PER WORKER AT BGP?

$$y^*(t) = \left[\frac{s_K}{d + \hat{A} + n}\right]^{\frac{\alpha}{1-\alpha}} \left(1 - s_R\right) A(t)$$

DERIVATION SIMILAR TO OTHERS

DONE: 1) CHANGE OF VARIABLES

2) FIND S.S. OF MODIFIED SYSTEM

<u>DONE</u> IN <u>APPENDIX A</u> OF HANDOUT IN WEB PAGE.

REMARK: NEW VARIABLE: $\tilde{y} = \frac{Y}{AL}$

(II) COMPARATIVE STATICS

THIS SYSTEM HAS 2 STOCKS MOVING : k , A
↓
ENDOGENOUSLY

(i) $\hat{A}$ (GROWTH RATE OF STOCK OF A)

ALWAYS TRUE : (4) $\hat{A} = \frac{\delta L_A^{\lambda}}{A^{1-\phi}}$

BGP : (5) $\hat{A} = \frac{\lambda n}{1-\phi}$

WE CAN HANDLE CHANGES IN : $\rightarrow \lambda, n, \phi, \lambda, \delta$] PARAMETERS

$\rightarrow A, L_A$

WE WILL SHOW DIAGRAMS

(ii) $\hat{k}$ (GROWTH RATE OF STOCK OF k)

ALWAYS TRUE : $\hat{k} = s_Y \frac{Y}{K} - d$

BGP : $\hat{k}$ CONSTANT : $\hat{k} = \hat{Y} = \hat{A} + n$
$= [\lambda n / 1-\phi] + n$

BETTER TO WORK WITH MODIFIED SYSTEM AT TIMES.

WE CAN HANDLE:
CHANGES IN :

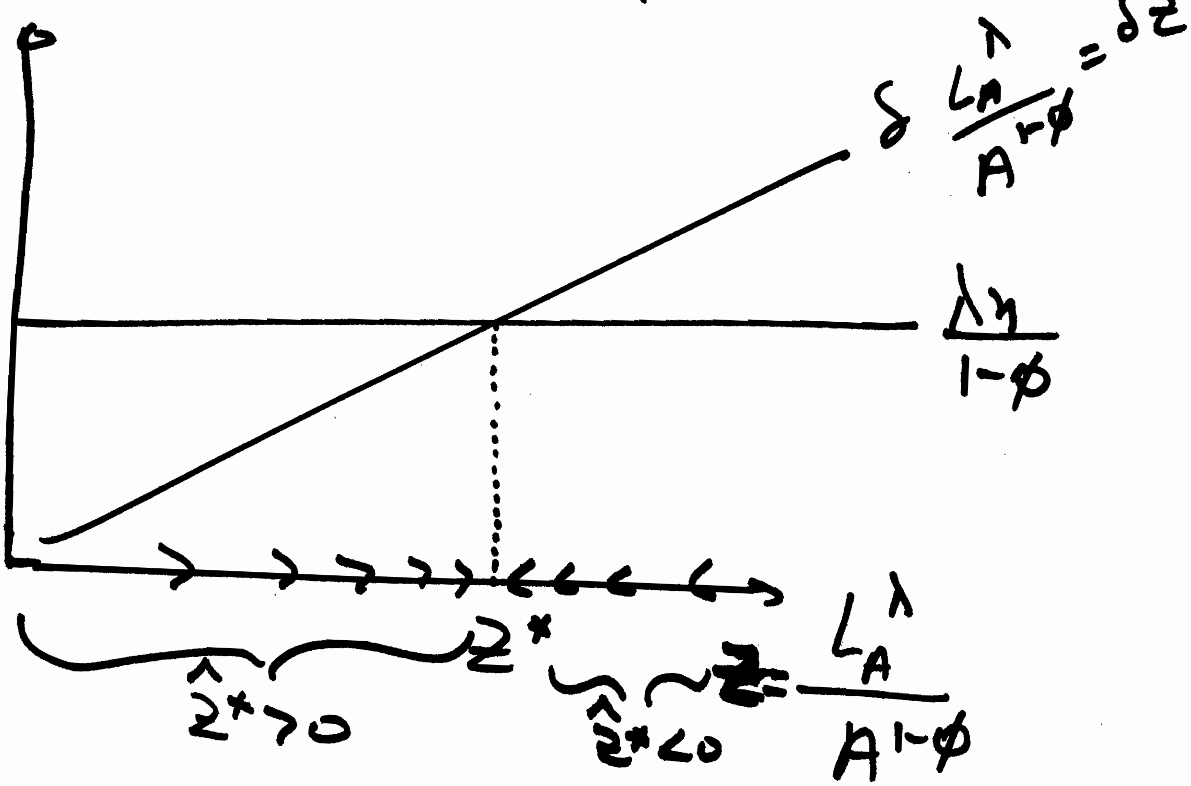
→ ρ_k, d, α, ...] PARAME

→ k, OTHER.

DIAGRAMS ASSOCIATED WITH EQUATIONS (4) & (5)

ALWAYS TRUE (4) $\hat{A} = \frac{\delta L_A^{\lambda}}{A^{1-\phi}} = \delta \underbrace{\frac{L_A^{\lambda}}{A^{1-\phi}}}_{z}$

BGP ONLY (5) $\hat{A} = \frac{\lambda n}{1-\phi}$



RENAME AS z WHAT WE HAVE: $\frac{L_A^{\lambda}}{A^{1-\phi}}$

WE WANT TO SHOW THAT z^* IS A VALUE SUCH THAT THE SYSTEM CONVERGES TO IT.

WE SHOW IN APPENDIX B OF HANDOUT IN WEB PAGE THAT:

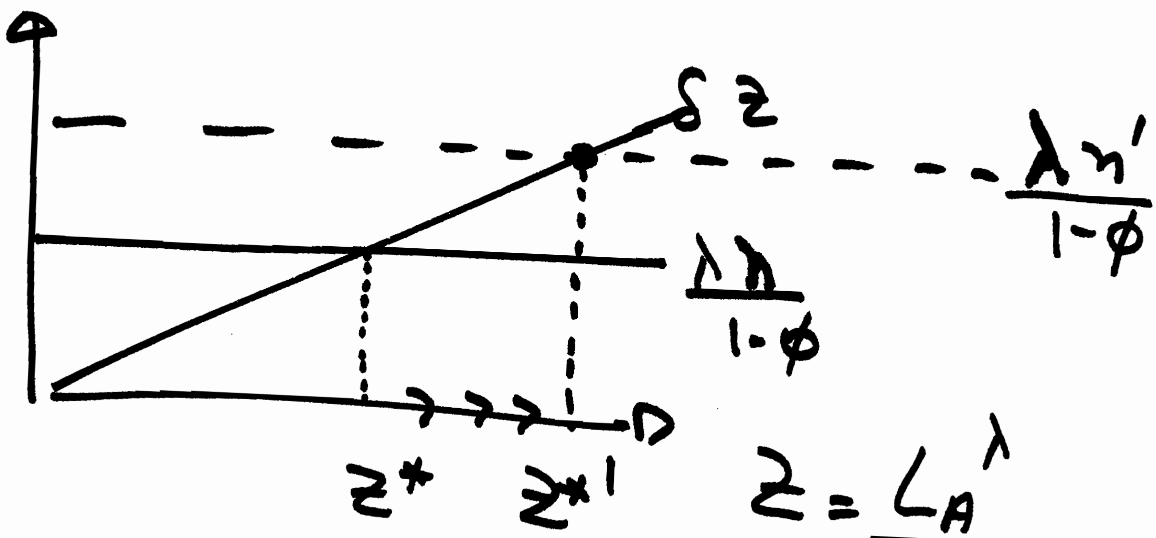
IF $z < z^*$, $\hat{z} > 0$ (i.e. $z \uparrow$)

IF $z > z^*$, $\hat{z} < 0$ (i.e. $z \downarrow$)

COMPARATIVE STATICS

(I) $\uparrow n$ IN MODEL IN GENERAL FORM ($0 < \lambda < 1$, $0 < \phi < 1$)

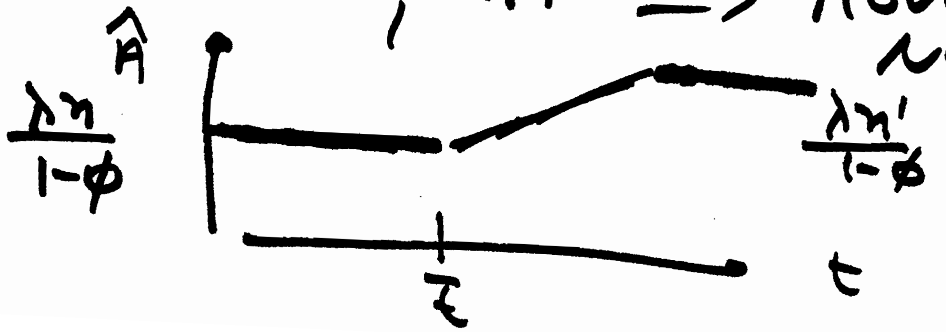
SYSTEM WAS AT BGP AND AT $\bar{t}$, $n \uparrow$ TO n'



$$z = \frac{L_A^{\lambda}}{A^{1-\phi}}$$

NEW BGP AT $z^{*\prime}$

AT $\bar{t}$, $n \uparrow \Rightarrow$ MOVE TOWARDS NEW BGP



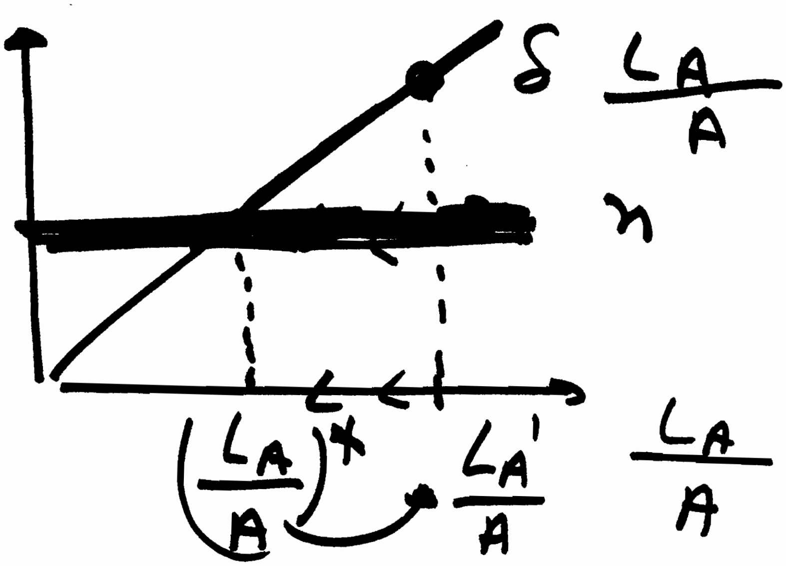
(II) ↑ λ_R (% OF WORKERS IN R&D SECTOR)

USING A SIMPLIFIED VERSION OF ~~THE~~ MODEL: $\lambda = 1$, $\phi = 0$

IN THIS CASE EQ. (4) & (5) BECOME:

$$(4') \quad \hat{A} = \delta \frac{L_A}{A} \qquad \text{ALWAYS TRUE}$$

$$(5') \quad \hat{A} = n \qquad \text{AT BGP}$$

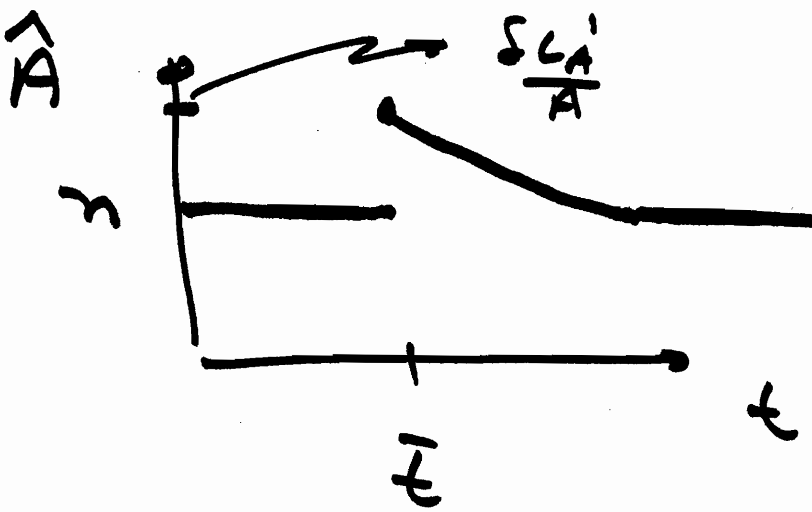


SYSTEM IS AT BGP.

AT $\bar{t}$, λ_R ↑ (<u>ONLY ONCE</u>)

SINCE: $\lambda_R = \frac{L_A}{L}$ THIS $\Rightarrow$ $\boxed{\uparrow L_A}$

THEN AT $\bar{t}$: $\hat{A} = \dfrac{\delta L_A^{NEW}}{A} = \dfrac{\delta L_A'}{A}$



OTHER CHANGES?

$$Y = K^\alpha (A L_Y)^{1-\alpha}$$

DIVIDE BY L

$$\underbrace{\frac{Y}{L}} = \frac{K^\alpha}{L^\alpha} \frac{(A L_Y)^{1-\alpha}}{L^{1-\alpha}}$$

$$y = A^{1-\alpha} k^\alpha \left(\frac{L_Y}{L}\right)^{1-\alpha}$$

$$\boxed{y = A^{1-\alpha} k^\alpha (1-s_R)^{1-\alpha}}$$

OR DIVIDING BY A.L.

$$\frac{Y}{AL} = \frac{K^\alpha}{(AL)^\alpha} \left(\frac{A L_Y}{A L}\right)^{1-\alpha}$$

$$\boxed{\tilde{y} = \tilde{k}^\alpha (1-s_R)^{1-\alpha}}$$

ALSO LAW OF MOTION OF k WILL CHANGE

SINCE

$$\boxed{\hat{k}} = R_K \frac{Y}{K} - d$$

$$= R_K \frac{Y/AL}{K/AL} - d = \boxed{R_K \tilde{h}^{\alpha-1} (1-R_R)^{1-\alpha} - d}$$

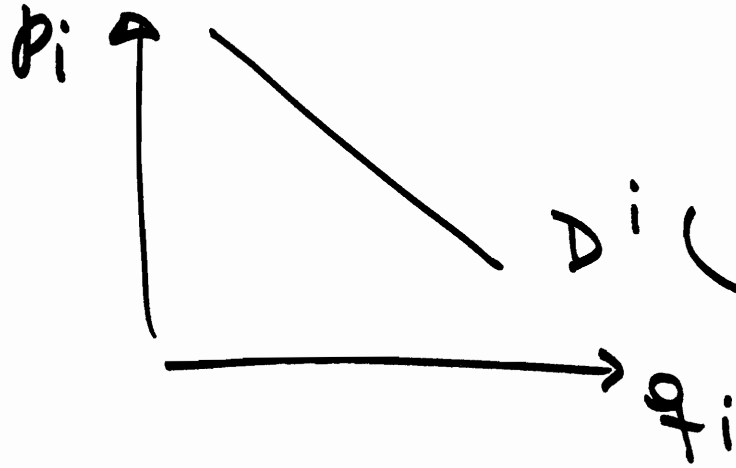
USING

$$\tilde{y} = \tilde{h}^{\alpha} (1-R_R)^{1-\alpha}$$

$$\boxed{\hat{\tilde{h}}} = \hat{k} - \hat{A} - n =$$

$$= \boxed{R_K \tilde{h}^{\alpha-1} (1-R_R)^{1-\alpha} - d - \hat{A} - n}$$

MONOPOLISTIC COMPETITION



(prices of others, set of substitutes)

- As # of substitutes ↑ : D^i ↓
- As p_j (price of subst.) ↓, D^i ↓

Equilibrium conditions:

① Profit max ⇒ $MR_i = MC_i$

$$\max_{q_i} \; p_i(\,)\cdot q_i - C_i(q_i)$$

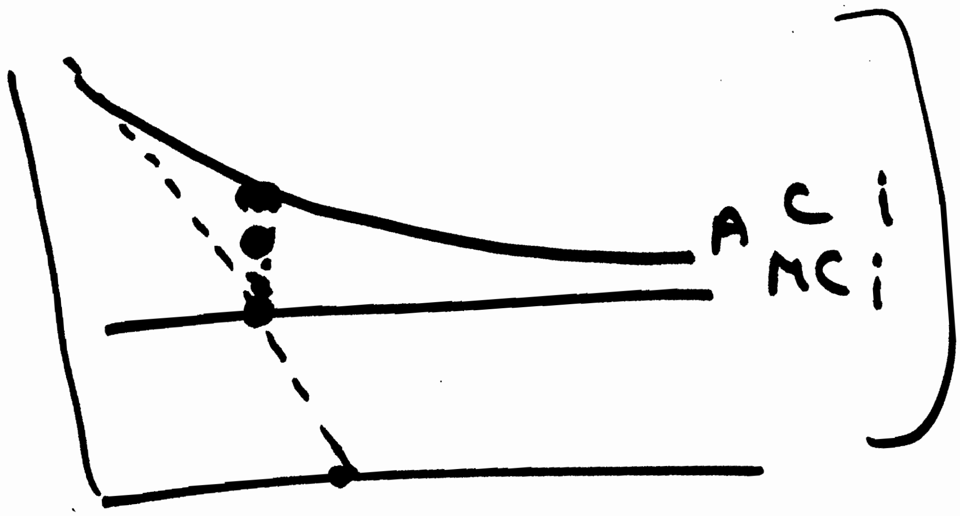
Taking derivatives:

$$\underbrace{\frac{\partial p_i q_i}{\partial q_i}}_{MR_i} - \underbrace{\frac{\partial C_i}{\partial q_i}}_{MC_i} = 0$$

(2) FREE ENTRY $\Longrightarrow$ ZERO PROFITS

$\Pi_i = 0$

IF FIRMS HAVE CONST. MC & A POSITIVE FIXED COST



NOT GOOD