1

6-5-09

SOLOW H-K

$$h = e^{\psi u}$$

h → INDICATION OF SKILL LEVEL

ψ CONSTANT

u = CONSTANT BETWEEN 0 & 1 = % TIME DEVOTED TO H-K ACC.

DEFINE:

$$H = h \cdot L$$

$\approx$ "AMOUNT OF SKILLED" LABOR USED IN PRODUCTION — EFFECTIVE AMOUNT OF LABOR

L → UNSKILLED LABOR

REMARK: IF $u = 0$ (NO TIME DEVOTED TO H-K ACCUMULATION) $\Rightarrow h = 1$

$\Rightarrow H = L \Rightarrow$ EFFECTIVE LABOR = RAW LABOR.

PROD. FUNCTION

$$(1) \quad Y = K^{\alpha} (A \cdot h \cdot L)^{1-\alpha}$$

OR

$$(1') \quad Y = K^{\alpha} (A \cdot H)^{1-\alpha}$$

WHERE

$\hat{A} = g$, h CONSTANT

$0 < \alpha < 1$, $\hat{L} = n$

$$(2) \quad \dot{K} = sY - \delta K \Rightarrow (2') \quad \hat{K} = s\frac{Y}{K} - \delta$$

<u>TASK</u>: ANALIZE SYSTEM AT BGP & ADJUSTMENT

<u>BGP</u>

AT BGP , $\hat{Y}$, $\hat{K}$, $\hat{A}$ CONSTANT

$\Longrightarrow$ BY (2)' $\Longrightarrow \boxed{\hat{Y} = \hat{K}}$

TAKING GROWTH RATES OF (1')

$$\hat{Y} = \alpha \underset{\underset{\hat{Y}}{\parallel}}{\hat{K}} + (1-\alpha) [\hat{A} + \hat{H}]$$

$$\hat{Y}(1-\cancel{\alpha}) = (1-\cancel{\alpha}) [\underset{\underset{g}{\parallel}}{\hat{A}} + \hat{H}]$$

SINCE $\hat{H} = \widehat{(hL)} = \underset{\substack{\| \\ 0}}{\hat{h}} + \underset{\substack{\| \\ n}}{\hat{L}}$

SINCE CONSTANT OVER TIME

$$\boxed{\hat{Y} = g + n} \quad \text{OR} \quad \boxed{\hat{Y} = [\hat{A} + \hat{H}]}$$

$$\Longrightarrow \widehat{\left(\frac{Y}{L}\right)} = \hat{y} = g$$

OR

$$\widehat{\left(\frac{Y}{H}\right)} = g$$

$$\| $$

$$\widehat{\left(\frac{Y}{h.L}\right)}$$

$$\Downarrow$$

$$\frac{Y}{AH} \quad \text{OR} \quad \frac{Y}{AhL}$$

CONSTANT AT BGP

MODIFIED SYSTEM

$$\frac{Y}{AH} = \frac{k^{\alpha} (AH)^{1-\alpha}}{AH}$$

$$\frac{Y}{AH} = \left(\frac{k}{AH}\right)^{\alpha} \quad \text{OR} \quad \frac{Y}{Ah.L} = \left(\frac{k}{AhL}\right)^{\alpha}$$

MODIFIED VARIABLES:

$$\frac{Y}{A.h.L} = \frac{y}{A.h} = \tilde{\tilde{y}}$$

$$\frac{K}{AhL} = \frac{k}{A.h} = \tilde{\tilde{k}}$$

PROD. FUNCTION

$$(10) \quad \boxed{\tilde{\tilde{y}} = \tilde{\tilde{k}}^{\alpha}}$$

WHAT is $\dot{\tilde{\tilde{k}}}$ or $\widehat{\tilde{\tilde{k}}}$?

$$\widehat{\tilde{\tilde{k}}} = \widehat{\left(\frac{k}{AhL}\right)} = \hat{k} - [\underset{g}{\underset{\|}{\hat{A}}} + \underset{0}{\underset{\|}{\hat{h}}} + \underset{n}{\underset{\|}{\hat{L}}}]$$

$$(12) \quad \boxed{\widehat{\tilde{\tilde{k}}}} = s\frac{\tilde{\tilde{y}}}{\tilde{\tilde{k}}} - \delta - (g+n) = \boxed{s\tilde{\tilde{k}}^{\alpha-1} + \delta - (g+n)}$$

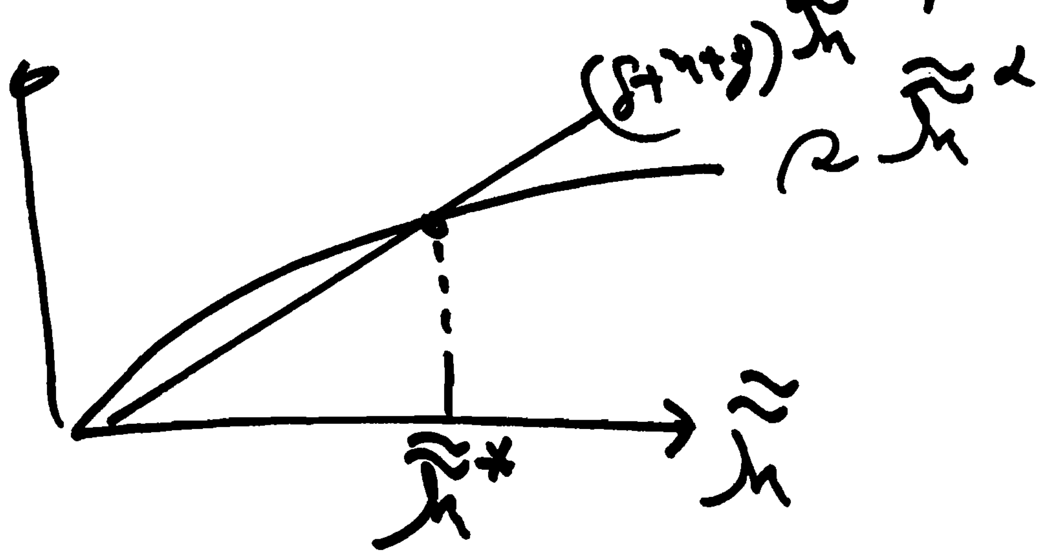
USING (10)

$$\Rightarrow (11) \quad \boxed{\dot{\tilde{\tilde{k}}} = s\tilde{\tilde{k}}^{\alpha} - (g+n+\delta)\tilde{\tilde{k}}}$$

MODIFIED SYSTEM

$$(10) \quad \tilde{\tilde{y}} = \tilde{\tilde{h}}^{\alpha}$$

$$(11) \quad \dot{\tilde{\tilde{h}}} = \Omega\, \tilde{\tilde{h}}^{\alpha} - (\delta + g + n)\, \tilde{\tilde{h}}$$



WHAT IS $y(t)$ AT BGP?

AT S.S. MODIFIED = BGP ORIGINAL

$\dot{\tilde{\tilde{h}}} = 0$ USING (11)

$$\Omega\, \tilde{\tilde{h}}^{\alpha - 1} = \delta + g + n$$

$$(13) \quad \boxed{\tilde{\tilde{h}} = \left[\frac{\Omega}{\delta + g + n}\right]^{\frac{1}{1-\alpha}}}$$

USING (10)

(14) $$\boxed{\tilde{y} = \left[\frac{s}{\delta+g+n}\right]^{\frac{\alpha}{1-\alpha}}}$$

$$\frac{y(t)}{A(t)\,h} = \tilde{y} \Rightarrow y(t) = \tilde{y} \cdot h \cdot A(t)$$

(15) $$\boxed{\underbrace{y(t) =}_{\text{USING (14)}} \left[\frac{s}{\delta+g+n}\right]^{\frac{\alpha}{1-\alpha}} h \cdot A(t)}$$

CONCLUSION:

→ OUTPUT LEVELS DEPEND ON SKILL OF LABOR FORCE / HUMAN CAPITAL

→ GROWTH RATES OF OUTPUT PER WORKER CONTINUE TO BE EQUAL TO RATE OF TECH. CHANGE

→ RELATIVE INCOMES AT BGP

PER CAPITA

$$y^{r*} = \frac{y^*}{y^{*\,US}}$$

$$\boxed{y^{r*} = \frac{s^{r\,\frac{\alpha}{1-\alpha}}}{x^{r\,\frac{\alpha}{1-\alpha}}}\; h^{r}\; A^{r}(t)}$$

WHERE $x^{r} = \dfrac{n+g+\delta}{(n+g+\delta)^{US}}$

__REMARK__: UNLESS BOTH A'S GROW AT THE SAME RATE

$$y^{r*} \to \infty \text{ OR ZERO}$$

THEN FOR RELATIVE INCOME PER CAPITA TO BE CONSTANT AT BGP WE NEED TO ASSUME THAT $\hat{A}$'S ARE THE SAME AT __BGP__

PREDICTIONS

① ASSUME COUNTRIES HAVE SAME S.S / BGP ⟹

POORER COUNTRIES (i.e. FURTHER BELOW S.S.) WILL GROW FASTER THAN RICH COUNTRIES (CLOSER TO S.S.)

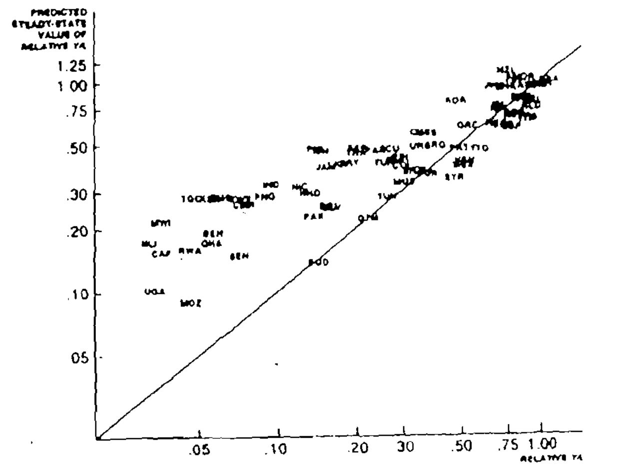
"CONVERGENCY HYPOTHESIS".

② ASSUME COUNTRIES HAVE ≠ S.S. / BGP'S -

COUNTRIES WILL GROW FASTER THE FURTHER BELOW THEY ARE FROM THEIR S.S.

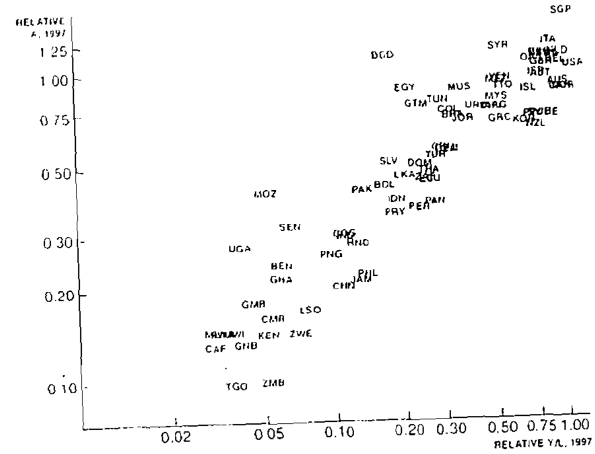
"CONDITIONAL CONVERGENCY"

FIGURE 3.1 THE "FIT" OF THE NEOCLASSICAL GROWTH MODEL, 1990



Note: A log scale is used for each axis

FIGURE ... PRODUCTIVITY LEVELS, 1997



Note. A log scale is used for each axis, and U.S. values are normalized to 1.

$$A = \left(\frac{y}{k}\right)^{\alpha/1-\alpha}\frac{y}{h}.$$

Figure 4
Income in the United States and United Kingdom
(log scale)

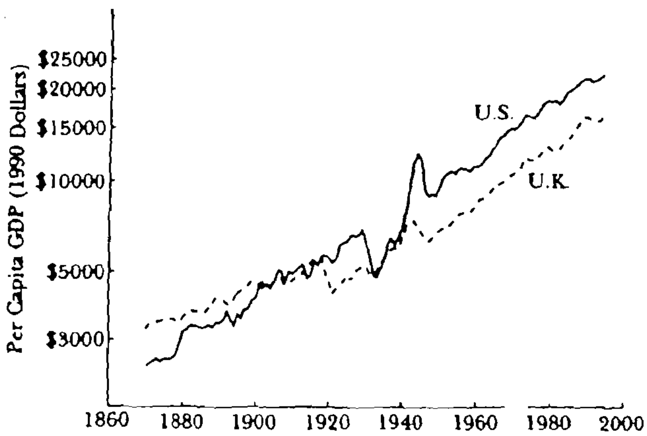
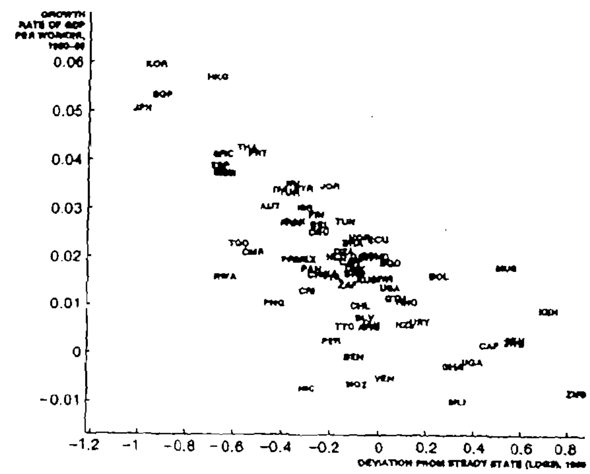


FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–90



Note: The U.S. deviation (in logs) from steady state in 1960 is normalized to zero. Estimates of A for 1970 instead of 1990 are used to compute the steady state.

Figure 1
World Income Distribution, 1960 and 1988