6-1-09

→ NUMERICAL EXAMPLE

→ COMPARATIVE STATICS
→ DATA ON COUNTRIES
→ HOMEWORK ISSUES

→ GROWTH ACCOUNTING

NUMERICAL EXAMPLE

$$Y = K^{1/3} L^{2/3} \quad (\text{i.e. } \alpha = 1/3)$$

$$s = 0.30 \qquad \delta = 0.10 \qquad n = 0$$

(1) CALCULATE OUTPUT PER WORKER / CAPITAL PER WORKER AT THE BGP.

(2) SUPPOSE $k_0 = 2$ IS k INCREASING, DECREASING OR CONSTANT?

(3) CALCULATE GROWTH RATE OF k AND y AT $k_0 = 2$

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(1) THIS IS A SOLOW MODEL WITHOUT TECH. CHANGE ⟹ WE MODIFY THE SYSTEM BY DIVIDING EVERYTHING BY L TO ANALYZE THE BGP. THE MODIFIED SYSTEM HAS A S.S. THAT COINCIDES WITH

BGP OF ~~ORIGINAL~~ ORIGINAL ONE.

$$\frac{Y}{L} = \frac{K^{1/3}\, L^{2/3}}{L} = \left(\frac{K}{L}\right)^{1/3}$$

$$(+) \quad \boxed{y = k^{1/3}}$$

ORIGINAL SYSTEM:

$$\dot{K} = s \cdot Y - \delta K$$

$$\boxed{\hat{K}} = \frac{\dot{K}}{K} = s\frac{Y/L}{K/L} - \delta = s\frac{y}{k} - \delta$$

$$(++) \quad = \boxed{s\, k^{\alpha-1} - \delta}$$

USING (+)

WE WANT $\hat{k}$ AND $\dot{k}$ $\Rightarrow$

$$(+++) \quad \boxed{\hat{k}} = \widehat{\left(\frac{K}{L}\right)} = \hat{K} - \underset{\substack{\| \\ 0}}{\hat{L}} = \boxed{\hat{K}}$$

USING (++) & (+++)

$$\boxed{\hat{k}} = \hat{K} = \boxed{s\, k^{\alpha-1} - \delta}$$

AND $\boxed{\dot{k}} = \hat{k} \cdot k = \boxed{s\, k^{\alpha} - \delta k}$

AT S.S. $\hat{k} = 0$

$\Rightarrow \hat{k} = a k^{\alpha-1} - \delta = 0$

Here

$\Rightarrow a k^{\alpha-1} = \delta$

$\frac{a}{\delta} = k^{1-\alpha}$

$\Rightarrow \boxed{k^*} = \left[\frac{a}{\delta}\right]^{\frac{1}{1-\alpha}} = \left[\frac{0.3}{0.1}\right]^{3/2} = 5.196 \approx \boxed{5.2}$

$y^* = k^{*1/3} = (5.2)^{1/3}$

(2) SINCE $k_0 = 2 < k^* = 5.2$

$\Rightarrow k \uparrow$

(3) $\hat{k}_0 = 0.3\, k_0^{-2/3} - 0.10$

$= 0.3 \times 2^{-2/3} - 0.10$

$= \frac{0.3}{2^{2/3}} - 0.10 \cong 0.088$

$= 8.8\%$

$\hat{y}_0 = \alpha\, \hat{k}_0 = \frac{1}{3} \times 0.088 \cong 0.0296$

$\approx 2.96\%$

SOLOW WITH TECH. CHANGE (LABOR AUGMENTING)

$$\boxed{\begin{aligned} Y &= K^{\alpha}(AL)^{1-\alpha} \\ \dot{K} &= s \cdot Y - \delta K \end{aligned}}$$

$\hat{A} = g$

$\hat{L} = n$

MODIFIED SYSTEM IN NEW VARIABLES:

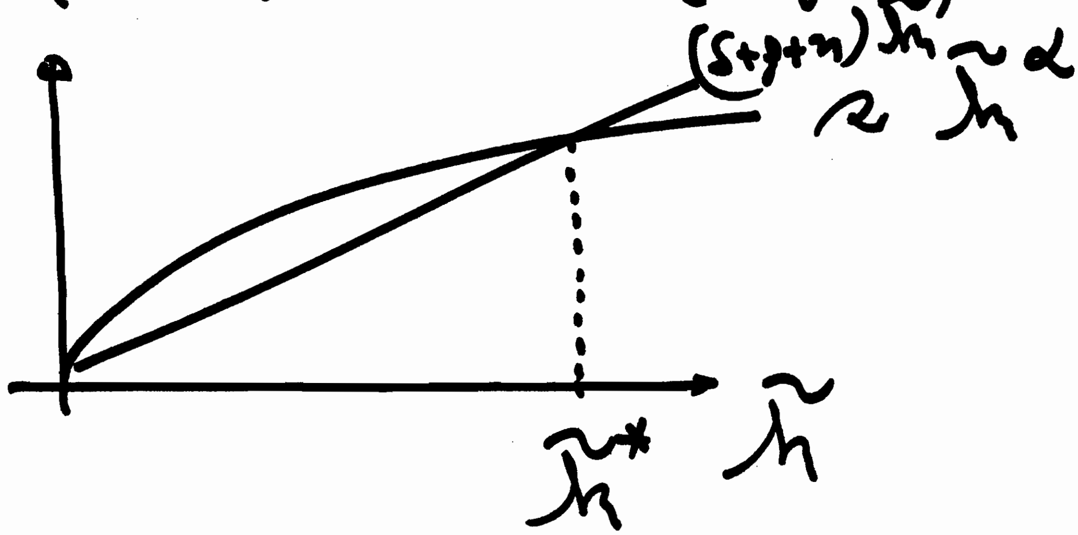
$$\boxed{\tilde{y}} = \frac{Y}{AL} = \boxed{\frac{y}{A}}, \quad \boxed{\tilde{k}} = \frac{K}{AL} = \boxed{\frac{k}{A}}$$

$$\tilde{y} = \tilde{k}^{\alpha}$$

$$\dot{\tilde{k}} = s \cdot \tilde{k}^{\alpha} - (\delta + g + n)\tilde{k}$$

or

$$\hat{\tilde{k}} = s\,\tilde{k}^{\alpha-1} - (\delta + g + n)$$



AT BGP

$$\hat{\tilde{y}} = 0 \Rightarrow \hat{y} = \hat{A} = g$$

OUTPUT PER WORKER
GROWS AT RATE
OF TECH. CHANGE

$$\hat{\tilde{k}} = 0 \Rightarrow \hat{k} = \hat{A} = g$$

CAPITAL PER WORKER
GROWS AT RATE
OF TECH. CHANGE

<u>OUTPUT PER WORKER</u>
<u>AT BGP</u> <u>IN SOLOW</u>
<u>MODEL</u> <u>WITH</u> <u>TECH.</u>
<u>CHANGE</u>.

AT BGP: $\hat{\tilde{k}} = 0$

$$\Rightarrow \hat{\tilde{k}} = s\tilde{k}^{\alpha-1} - (\delta+n+g) = 0$$

$$\Rightarrow \tilde{k} = \left[\frac{s}{\delta+n+g}\right]^{\frac{1}{1-\alpha}}$$

SINCE $\tilde{y} = \tilde{k}^{\alpha} = \left[\frac{s}{\delta+n+g}\right]^{\frac{\alpha}{1-\alpha}}$ ⑥

BUT $\tilde{y} = \frac{y}{A} \Rightarrow \boxed{y = \tilde{y} \cdot A}$

NOTICE $\tilde{y}$ = CONSTANT

$A(t)$ MOVING

$\Rightarrow$

$$y(t) = \tilde{y} \cdot A(t) = \left[\frac{s}{\delta+n+g}\right]^{\frac{\alpha}{1-\alpha}} A(t)$$

GDP PER WORKER VERSUS THE INVESTMENT RATE

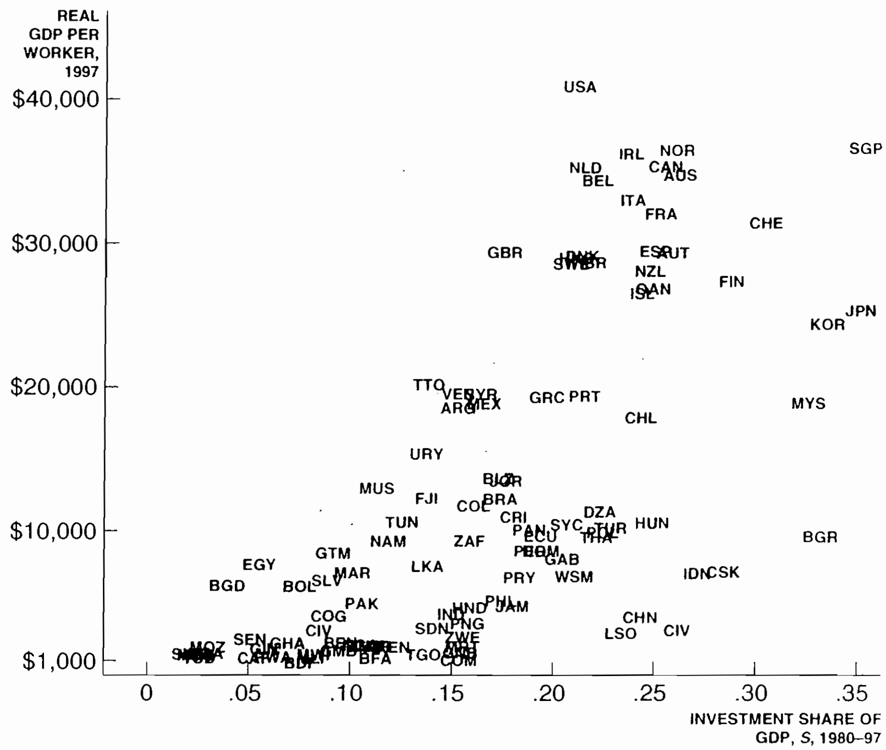
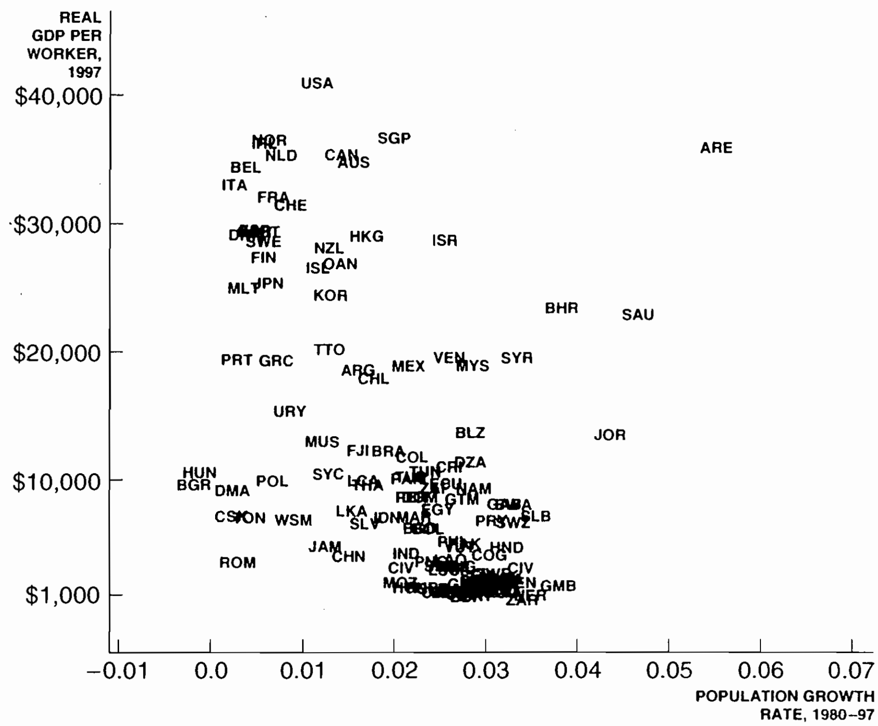


FIGURE 2.7 GDP PER WORKER VERSUS POPULATION GROWTH RATES



DATA?

SAVINGS RATE? AND OUTPUT PER WORKER : (CLOSED ECONOMY $\Rightarrow$ SAVINGS RATE = INV. RATE

POPULATION GROWTH & OUTPUT PER WORKER ?

COMPARATIVE STATICS

TWO TYPES OF CHANGES

(I) SHOCKS THAT CHANGE STEADY STATE OR DISTURB SYSTEM

BGP DOES NOT CHANGE

- $\downarrow$ K DUE TO WAR
- $\uparrow$ K DUE TO GIFT
- $\downarrow$ L DUE TO VIRUS
- $\uparrow$ L DUE TO IMMIGRATION

(II) PARAMETER CHANGES

CHANGES IN: α, δ, n, g

$\Rightarrow$ BGP CHANGES $\Rightarrow$ EVALUATE ADJUSTMENT TO BGP

(I) EXAMPLE:

$\uparrow K$ DUE TO GIFT

IN SOLOW WITHOUT TECH. CHANGE ASSUMING ECONOMY WAS AT THE BGP BEFORE THE GIFT

$n > 0$

<u>QUESTIONS</u>:

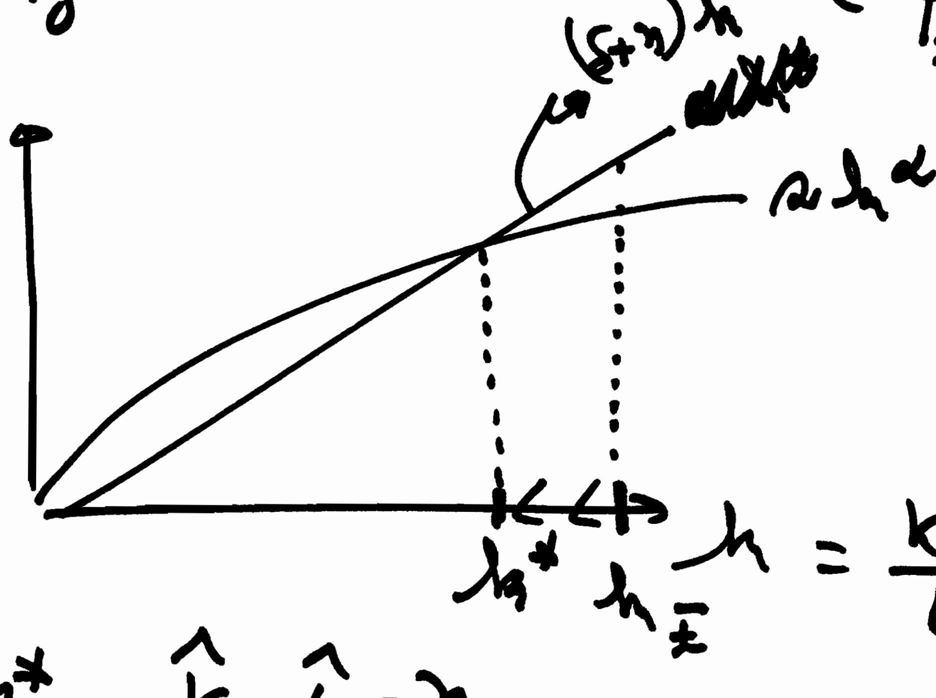
(i) $\hat{y}^*$? y^* ?

(ii) PATH OF k, $\hat{k}$, y, $\hat{y}$

(i) y^* DOES NOT CHANGE

$\hat{y}^*$ " " " (EQUALS ZERO)

(ii)

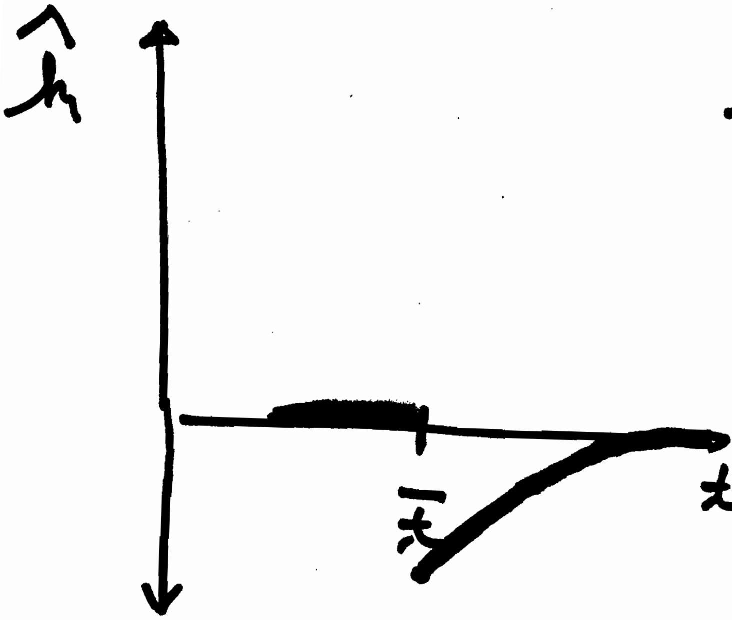


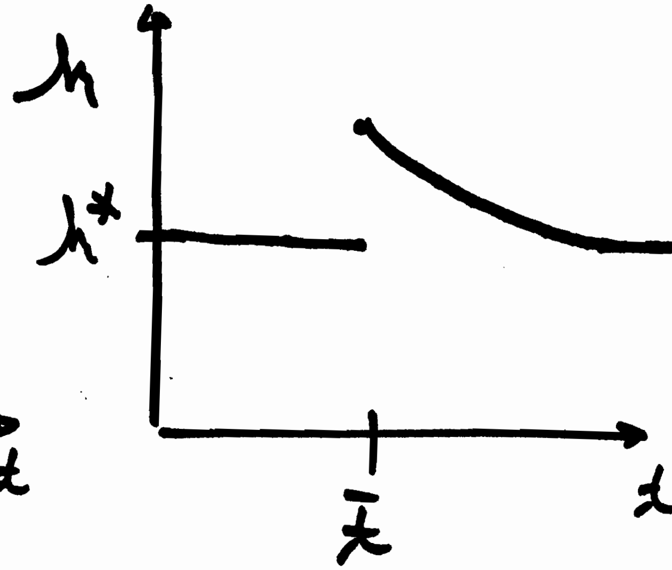
$k = \frac{K}{L}$

AT k^*, $\hat{K} = \hat{L} = n$

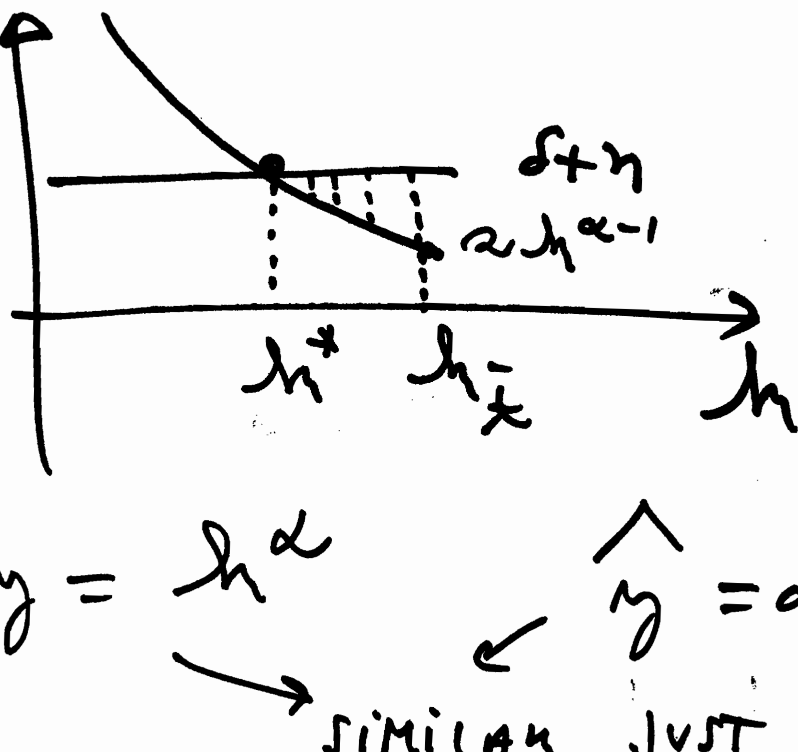
ASSUME K JUMPS UP AT $\bar{t}$

PATHS





$$\boxed{\hat{h} = s h^{\alpha-1} - (\delta+n)} = \frac{s}{h^{1-\alpha}} - (\delta+n)$$



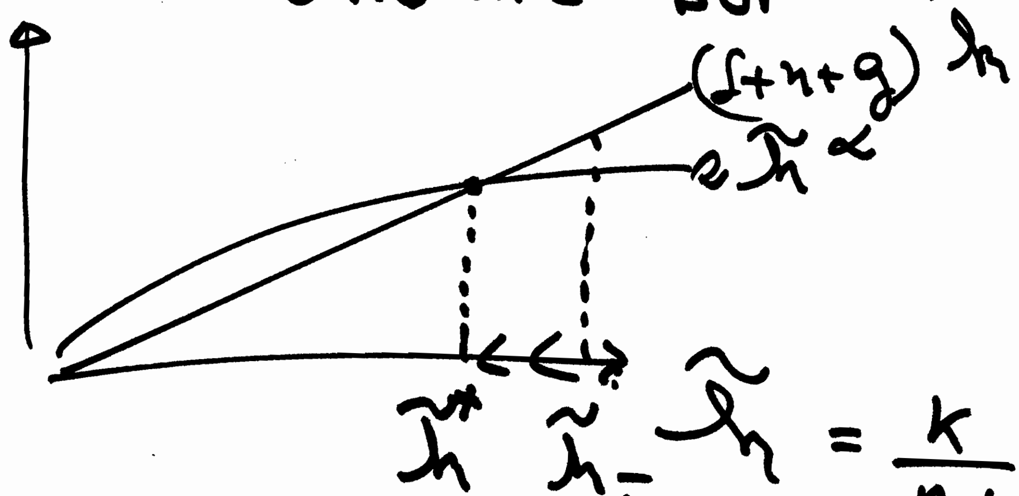
$y = h^{\alpha}$ $\quad$ $\hat{y} = \alpha \hat{h}$

SIMILAR JUST A CONSTANT CHANGES

REMARK.

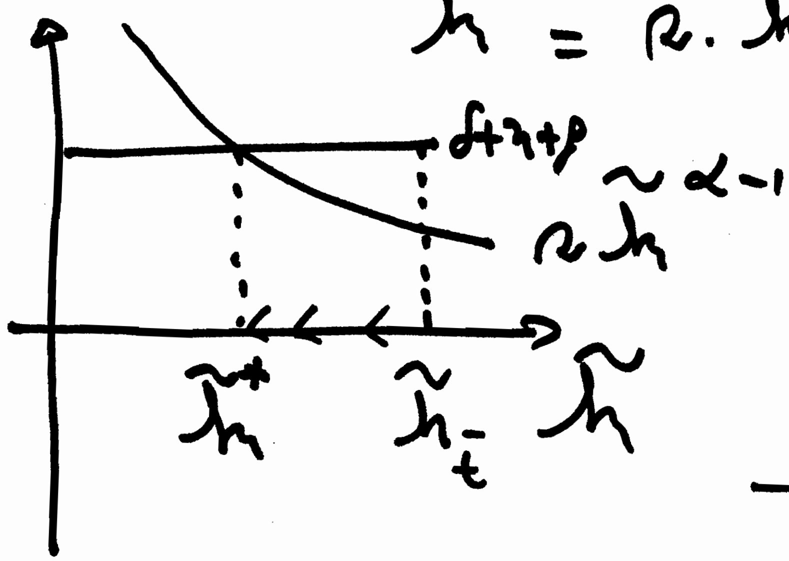
WHAT IF WE HAVE ↑K WITH THE SOLOW MODEL WITH TECH. CHANGE?

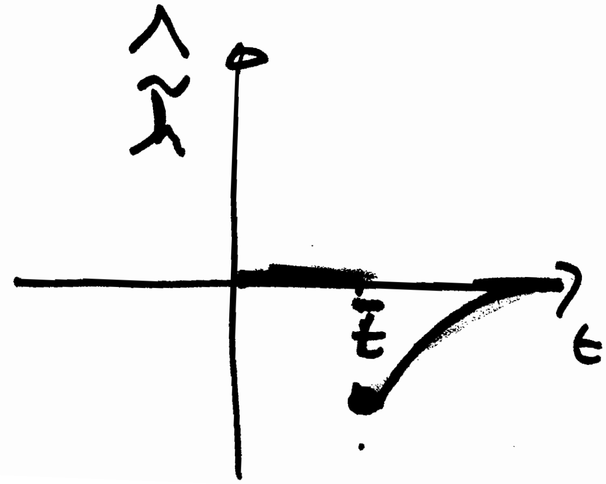
AGAIN: SHORT RUN ADJUSTMENT TO GET BACK TO "ORIGINAL BGP"

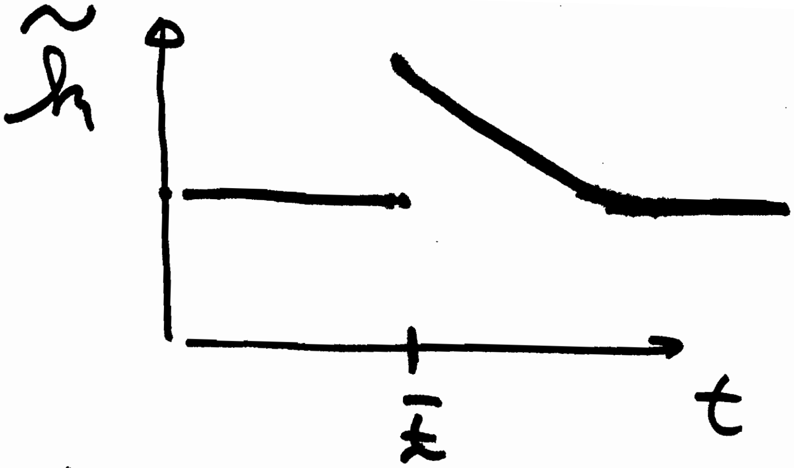


$$\tilde{k} = \frac{K}{AL}$$

$$\uparrow K \Rightarrow \tilde{k} \uparrow \quad \hat{\tilde{k}} = s \cdot \tilde{k}^{\alpha-1} - (\delta+n+g)$$





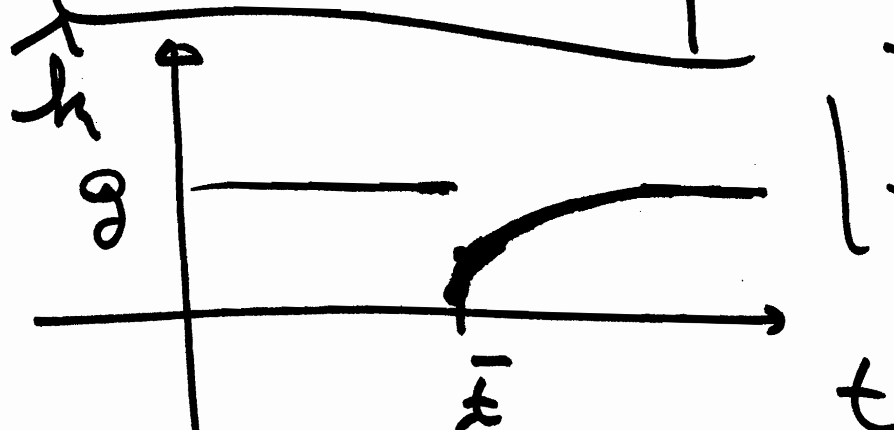


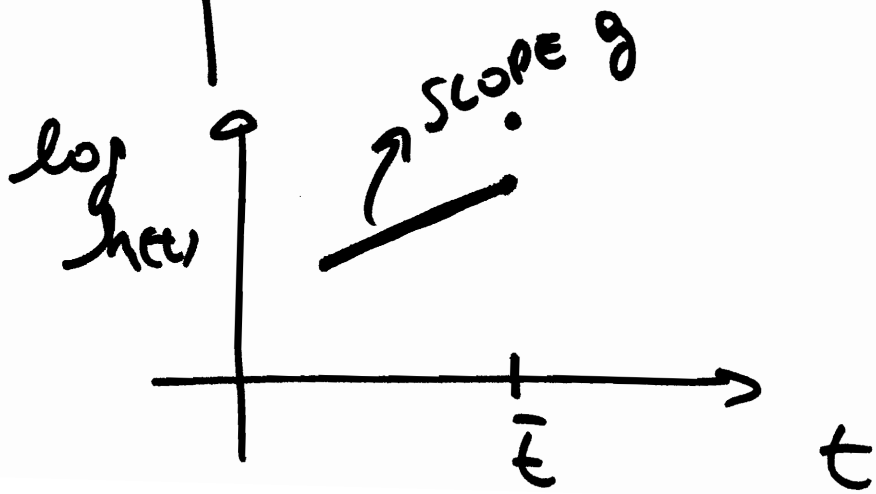
HOW ABOUT y? h?
WE WILL FOCUS ON $\hat{h}$?

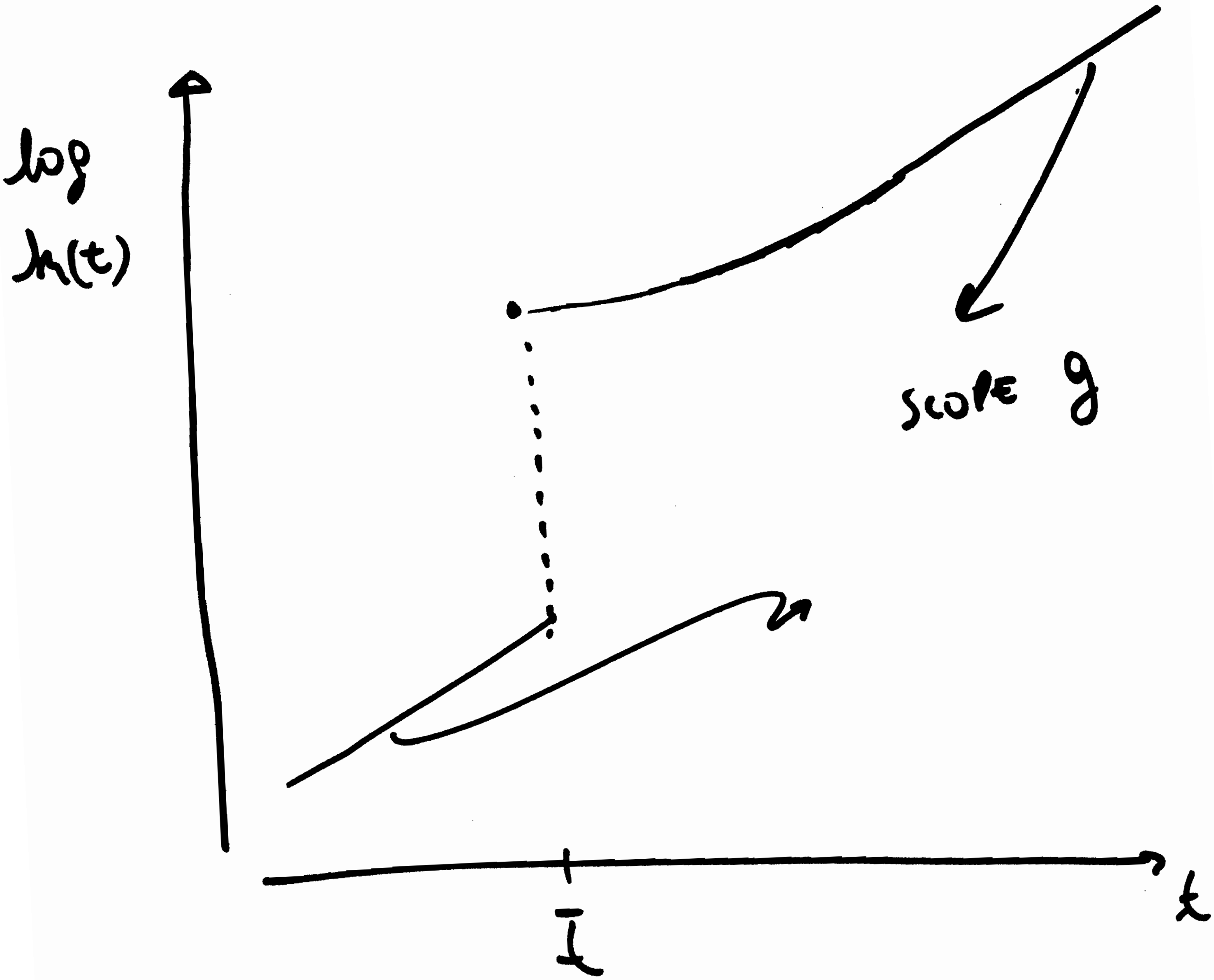
$$\tilde{h} = \frac{h}{A} \Longrightarrow \hat{\tilde{h}} = \hat{h} - g$$

$$\Longrightarrow \boxed{\hat{h} = \hat{\tilde{h}} + g}$$

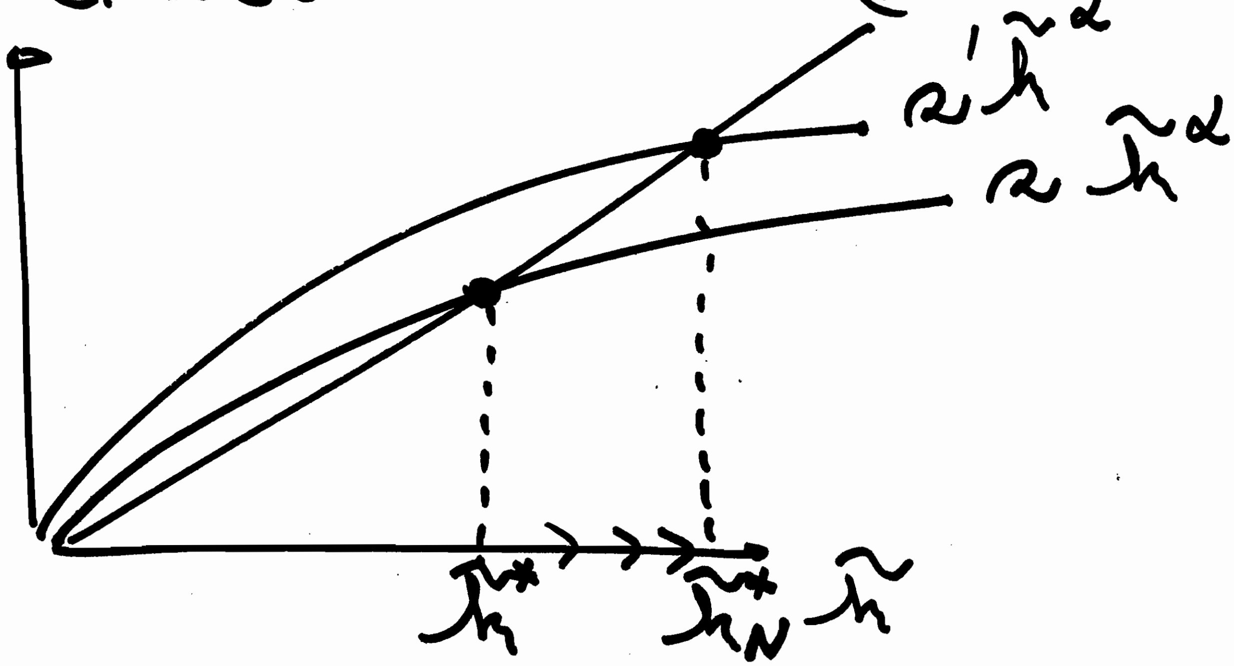
ASSUME THAT $|\hat{\tilde{h}}_{\bar{t}}| < g$






log
h(t)
scope g
t
I

(II) EXAMPLE: $\uparrow \tilde{A}$
IN SOLOW WITH TECH CHANGE



ASSUME WE WERE AT INITIAL S.S. (OR BGP)

LONG RUN EFFECTS (BGP)

$\hat{\tilde{k}}^*$? $\hat{\tilde{y}}^*$?

DO NOT CHANGE

$\hat{y}_{OLD} = g = \hat{y}_{NEW}$ AT BGP

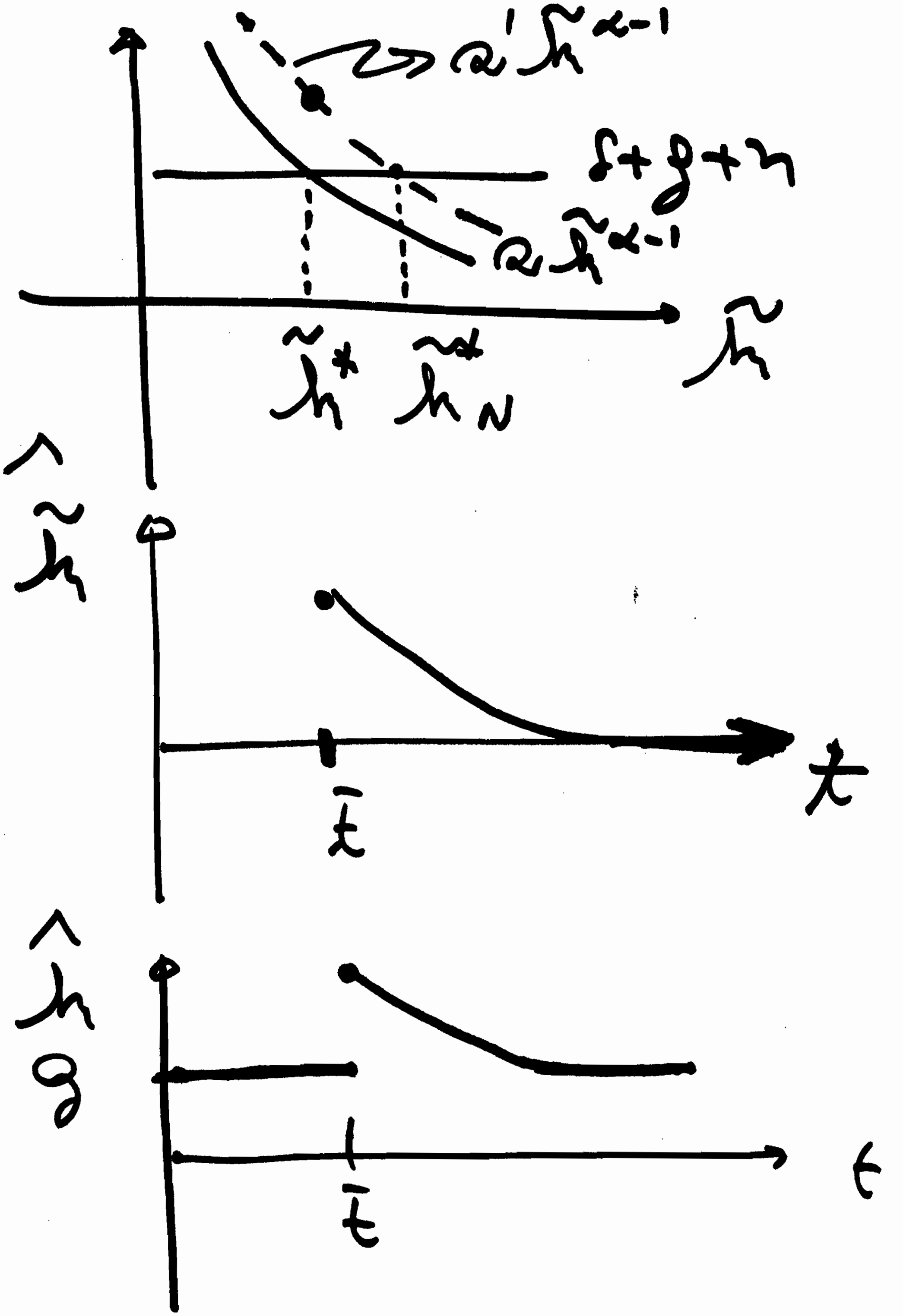
PATH k AT BGP ? PATH y AT BGP?

HIGHER $\tilde{A}$ ⇒ HIGHER PATH

AT $\bar{t}$, a JUMPS UP

$$\Rightarrow a' \tilde{h}^{\alpha-1} - (n+g+\delta) > 0$$

$$\Rightarrow \tilde{h} \uparrow \quad (\text{AT A} \downarrow \text{RATE})$$



PATH