5-29-09

TECHNOLOGICAL CHANGE REMARKS

ONE NOTION: MORE OUTPUT WITH SAME AMOUNT OF INPUTS

NEUTRAL OR INPUT AUGMENTING OR BIASED

EX: $Y = \overbrace{K^{\alpha} L^{1-\alpha}}^{F(K,L)}$ ORIGINAL

NEUTRAL

- WE ASSUME A ONCE AND FOR ALL TECH. IMPROVEMENT:

$$F^{NEW}(K,L) = A \cdot K^{\alpha} L^{1-\alpha}$$

WHERE $A > 1$

- ASSUME MANY PERIODS AND THAT: $\hat{L} = n$, $\underbrace{\hat{A} = g}_{\text{TECH. CHANGE SUSTAINED}}$

OTHER ASSUMPTIONS SAME AS IN ORIGINAL SOLOW

LABOR AUGMENTING

$$F^{NEW}(K,L) = K^{\alpha} (AL)^{1-\alpha}$$

$\hat{A} = g$, $\hat{L} = n$, ...OTHER ASSUMPTIONS

SOLOW WITH LABOR AUGMENTING TECH. CHANGE

PROD. FUNCTION

(1) $Y = K^{\alpha}(AL)^{1-\alpha}$ $\qquad 0<\alpha<1$, $A>0$

"AL" ≈ EFFECTIVE AMOUNT OF LABOR USED IN PRODUCTION

(2) $\hat{A} = g > 0$ RATE OF TECH. CHANGE (EXOGENOUS)

(3) $S = s \cdot Y$ $\qquad 0<s<1$

(4) $\hat{L} = n$ $\qquad n>0$ RATE OF POP. GROWTH

(5) $\dot{K} = I - \delta K$ $\qquad 0<\delta<1$ DEPRECIATION RATE

FEASIBILITY / EQUILIBRIUM

$$\left.\begin{array}{l} I = S \\ Y = C + I \end{array}\right] \Longrightarrow C = (1-s)\cdot Y$$

(6) $\dot{K} = sY - \delta K$

(7) $\Rightarrow \boxed{\hat{K}} = \frac{\dot{K}}{K} = \boxed{s\frac{Y}{K} - \delta}$

ASSUME # WORKERS = POPULATION

I DESCRIBE/ CHARACTERIZE BGP

ALONG BGP, VARIABLES GROW AT CONSTANT RATES: i.e. $\hat{k}, \hat{Y}, \ldots$ CONSTANT

USING:

$$(7) \quad \hat{k} = s \cdot \frac{Y}{k} - \delta$$

WE GET THAT: $\frac{Y}{k}$ CONSTANT

$$\Rightarrow \boxed{\hat{Y} = \hat{k}} \quad (*)$$

WE TAKE GROWTH RATES OF ~~THE~~ PROD. FUNCTION:

$$(1) \quad Y = k^{\alpha} \cdot (AL)^{1-\alpha}$$

SO

$$\hat{Y} = \widehat{k^{\alpha}} + \widehat{(AL)^{1-\alpha}}$$

$$= \alpha \hat{k} + (1-\alpha)\widehat{(AL)}$$

$$= \alpha \hat{k} + (1-\alpha)(\underbrace{\hat{A}}_{=g} + \underbrace{\hat{L}}_{=n})$$

$$(8) \quad \boxed{\hat{Y} = \alpha \hat{k} + (1-\alpha)[g+n]}$$

USING (*) AND (8): $\hat{Y} = \alpha \hat{Y} + (1-\alpha)(g+n)$

$$\cancel{(1-\alpha)}\,\hat{Y} = \cancel{(1-\alpha)}(g+n)$$

$$(9) \quad \boxed{\hat{Y} = g+n}$$

AT BGP TOTAL GROWS AT A CONSTANT RATE EQUAL TO $g+n$

EQUATION (9) IMPLIES THAT ALONG BGP.

$\frac{Y}{AL}$ IS CONSTANT SINCE

$$\hat{Y} = \underbrace{\widehat{AL}}_{\hat{A}+\hat{L}\,=\,g+n}$$

OUTPUT IN EFFECTIVE LABOR UNITS

AND $\frac{K}{AL}$ ALSO CONSTANT AT BGP

THEN WE CAN DIVIDE THE INITIAL PROD. FUNCTION BY "AL" AND TRANSFORM THE SYSTEM.

THE SYSTEM IN THE NEW VARIABLES WILL HAVE A STEADY STATE.

REMARK: S.S. OF MODIFIED SYSTEM WILL COINCIDE WITH BGP OF ORIGINAL SYSTEM.

II CHANGE OF VARIABLE

ANALYSIS OF NEW SYSTEM S.S

NOTATION: $\tilde{y} = \frac{Y}{A.L} = \frac{y}{A}$ OUTPUT PER EFFECTIVE UNIT OF LABOR

$$\tilde{k} = \frac{K}{A.L} = \frac{k}{A}$$

$$\tilde{c} = \frac{C}{AL} = \frac{c}{A}$$

→ DIVIDE PROD. FUNCTION BY AL

$$\frac{Y}{AL} = \frac{K^{\alpha}(AL)^{1-\alpha}}{AL} = K^{\alpha}(AL)^{-\alpha} = \left(\frac{K}{AL}\right)^{\alpha}$$

$$(10) \quad \boxed{\tilde{y} = \tilde{k}^{\alpha}}$$

PROD. FUNCTION IN MODIFIED VAR.

NEED TO CALCULATE $\widehat{\tilde{k}}$

$\rightarrow$ (△) $$\widehat{\tilde{k}} = \widehat{\left(\frac{k}{AL}\right)} = \hat{k} - (\underset{\overset{\|}{g}}{\hat{A}} + \underset{\overset{\|}{n}}{\hat{L}}) = \hat{k} - (g+n)$$

NEED TO WRITE $\hat{k}$ IN TERMS OF NEW VARIABLE $\tilde{k}$.

USING EQ. (7):

$$\hat{k} = r \cdot \frac{Y}{K} - \delta = r \frac{Y/AL}{K/AL} - \delta =$$

$$= r \frac{\tilde{y}}{\tilde{k}} - \delta$$

USING (10): $\tilde{y} = \tilde{k}^{\alpha}$

WE GET

(△△) $$\boxed{\hat{k}} = r \frac{\tilde{k}^{\alpha-1}}{\tilde{k}} - \delta = \boxed{r \tilde{k}^{\alpha-1} - \delta}$$

USING (△) & (△△) WE GET LAW OF MOTION OF $\tilde{k}$:

(11) $$\boxed{\widehat{\tilde{k}} = r \tilde{k}^{\alpha-1} - (\delta + g + n)}$$

SINCE

$$\dot{\tilde{k}} = \hat{\tilde{k}} \cdot \tilde{k}$$

$$(12) \quad \boxed{\dot{\tilde{k}} = s\,\tilde{k}^{\alpha} - (\delta+g+n)\,\tilde{k}}$$

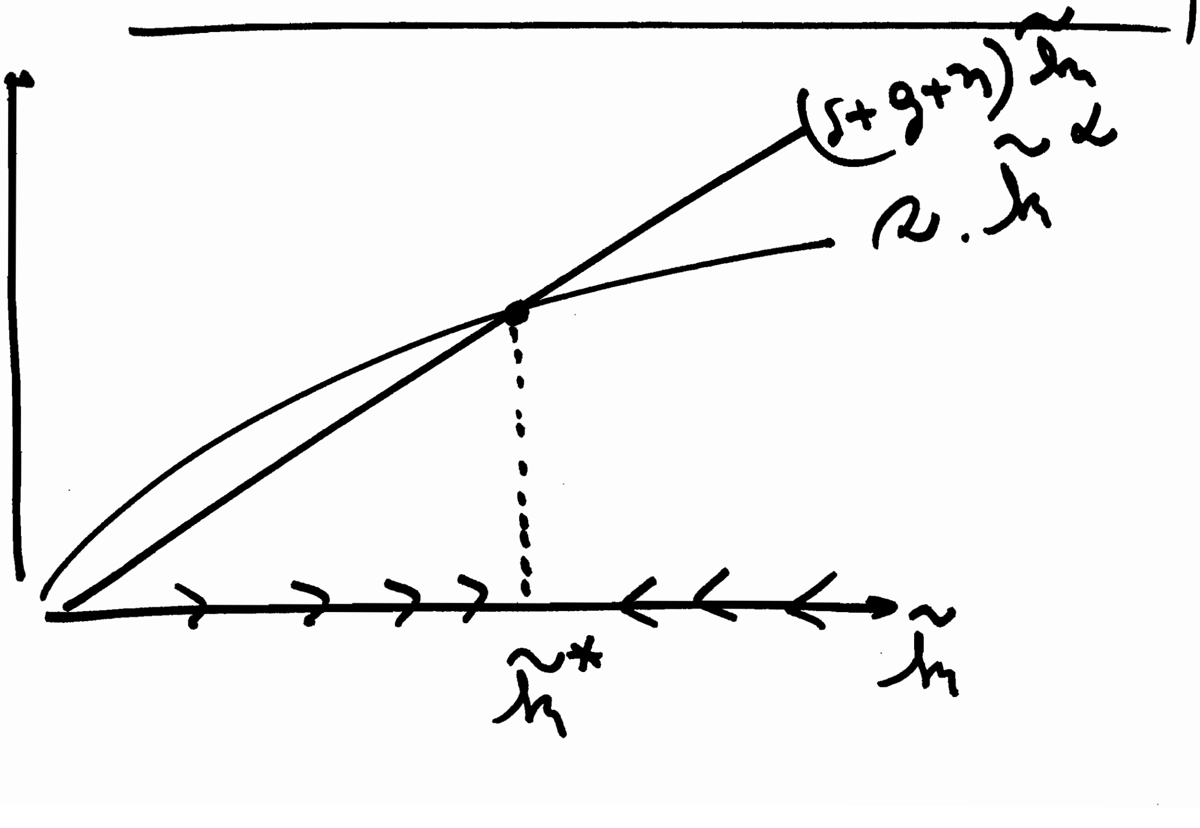
SUMMARY OF MODIFIED SYSTEM

$$(10) \quad \boxed{\tilde{y} = \tilde{k}^{\alpha}}$$

$$(11) \quad \hat{\tilde{k}} = s \cdot \tilde{k}^{\alpha-1} - (\delta+g+n)$$

OR

$$(12) \quad \boxed{\dot{\tilde{k}} = s \cdot \tilde{k}^{\alpha} - (\delta+g+n)\,\tilde{k}}$$



$\tilde{k}^*$ IS THE S.S. LEVEL OF $\tilde{k}$

IF $\tilde{k} < \tilde{k}^* \Rightarrow \dot{\tilde{k}} > 0 \Rightarrow \tilde{k} \uparrow$

$\tilde{k} > \tilde{k}^* \Rightarrow \dot{\tilde{k}} < 0 \Rightarrow \tilde{k} \downarrow$

THE S.S. OF MODIFIED SYSTEM COINCIDES WITH BGP OF ORIGINAL SYSTEM –

AT S.S. (OR BGP OF ORIGINAL)

$$\widehat{\tilde{k}^*} = 0 \Rightarrow \widehat{\tilde{k}} = \widehat{\left(\frac{K}{AL}\right)} = \widehat{\left(\frac{k}{A}\right)} = 0$$

$$\Rightarrow \hat{k} = \hat{A}$$

OR $\boxed{\hat{k} = g}$ CAPITAL PER WORKER GROWS AT RATE OF TECH. CHANGE

SINCE ~~$y = k^\alpha$~~

$\tilde{y} = \tilde{k}^\alpha$ THEN AT S.S.

$$\boxed{\hat{\tilde{y}}} = \alpha \underbrace{\hat{\tilde{k}}}_{=0} = \boxed{0}$$

SINCE AT S.S.

$$\hat{\tilde{y}} = 0 \implies \widehat{\left(\frac{Y}{AL}\right)} = \widehat{\left(\frac{y}{A}\right)} = 0$$

$$\implies \hat{y} = \hat{A}$$

OR $\boxed{\hat{y} = g}$

OUTPUT PER WORKER GROWS AT RATE OF TECH. CHANGE

NEXT CLASS

NUMERICAL EXAMPLE

$\alpha = 1/3$ $\quad s = 0.30$ $\quad \delta = 0.1$ $\quad n = 0$

QUESTIONS:

1) y^*, k^* ?

2) SOME GROWTH PATH IF WE START BELOW S.S.

$k_0 = 2$

DISCRETE / CONT. TIME

$$\dot{k}(t) = \frac{d\,k(t)}{dt}, \quad \hat{k}(t) = \frac{\dot{k}}{k}$$

CONT.

$$\dot{k} = s \cdot Y - \delta k$$

$$\hat{k} = s\frac{Y}{k} - \delta$$

$$\Delta k(t) = k(t) - k(t-1)$$

$$\hat{k} = \frac{\Delta k}{k}$$

DISCRETE

$$\Delta k = sY - \delta k$$

$$\hat{k} = s\frac{Y}{k} - \delta$$

Economic Growth Maria Muniagurria

Notes on Excel Simulations of Solow Model

Basic Equations:

(1) $Y = K^{\alpha} (AL)^{1-\alpha}$

(2) $\hat{L} = n \quad , \quad \hat{A} = g$

(3) $S = s\,Y$

(4) $\hat{K}_t = s\,(Y_t/K_t) - \delta$

(5) $K_{t+1} = (1+\hat{K}_t)\,K_t$

(6) $L_{t+1} = (1+\hat{L}_t)\,L_t = (1+n)\,L_t$

(7) $A_{t+1} = (1+\hat{A}_t)\,A_t$

(8) $\hat{y}_t = (y_{t+1} - y_t) / y_t$

(9) $y = Y/L, \quad k = K/L$

We assume that population (N) is equal to the labor force (L)

There are 3 countries (USA, Foreign 1, Foreign 2)
The program shows the following variables for each country for 100 periods:

$\hat{y}$ y Y/AL K/AL Y K AL L A $\hat{K}$

SIMULATIONS

For all countries and in all cases:

$\alpha = 0.35 \qquad \delta = 0.02$

$K_0 = 1 \qquad L_0 = 10$

(1) **Solow Model with population growth and without technological change**: countries differ in their **rate of population growth** .

All countries $\hat{A}=0 \;\; A_0 = 1, \;\; s= 0.10$

USA: $\hat{L} = 0.10$ Foreign 1: $\hat{L} = 0.05$ Foreign 2: $\hat{L} = 0.15$

(2) **Solow Model with population growth and without technological change** :countries differ in their **savings rates**.

All countries: $\hat{A}=0$, $\hat{L}= n= 0.10$, $A_0 = 1$

USA: $s = 0.10$ Foreign 1: $s = 0.15$ Foreign 1: $s = 0.05$

(3) **Solow Model with population growth and technological change** :countries differ in their **initial technology level** .

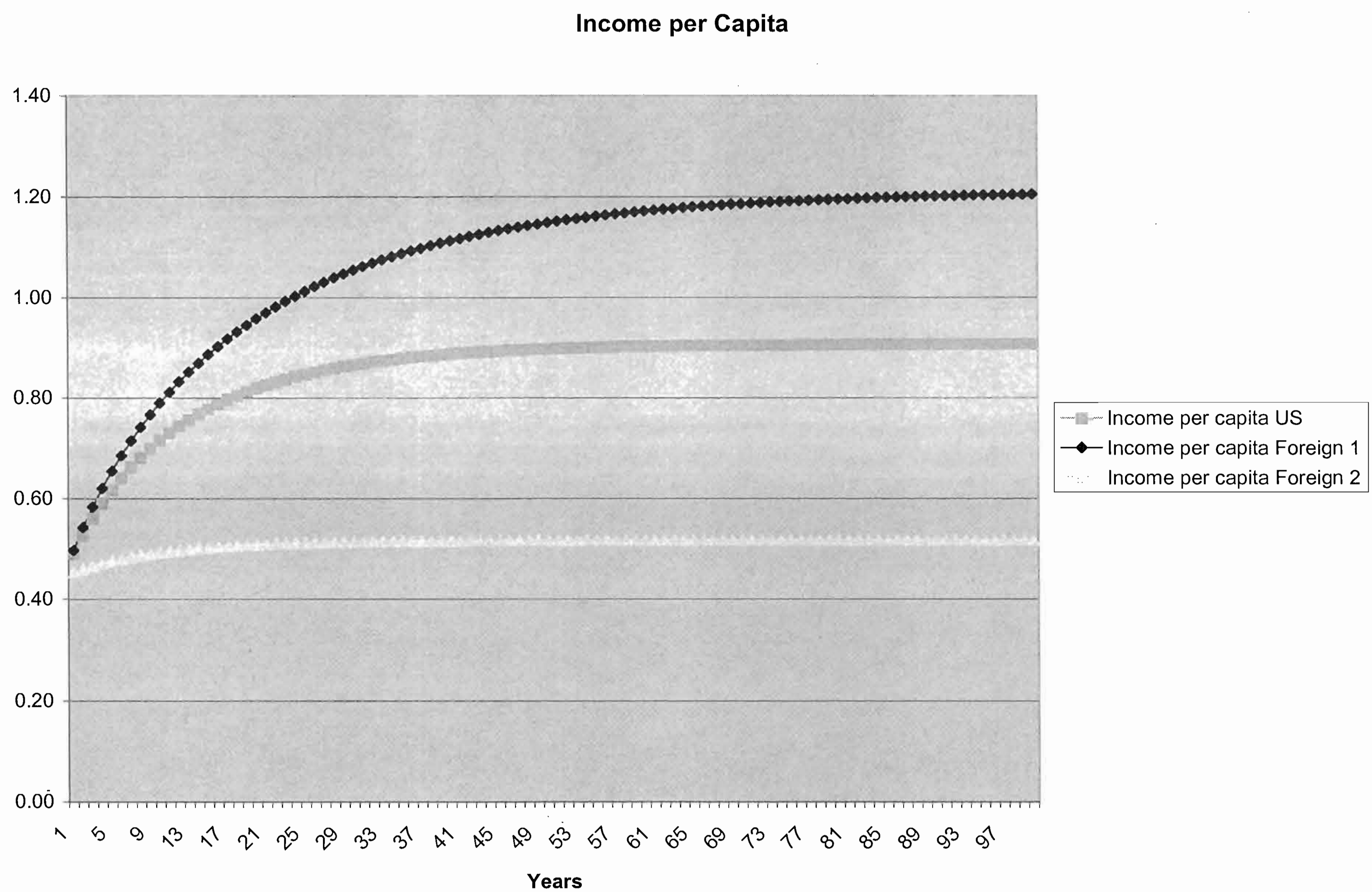
All countries: $\hat{A}=0.02$, $\hat{L}= n= 0.10$, $s = 0.10$

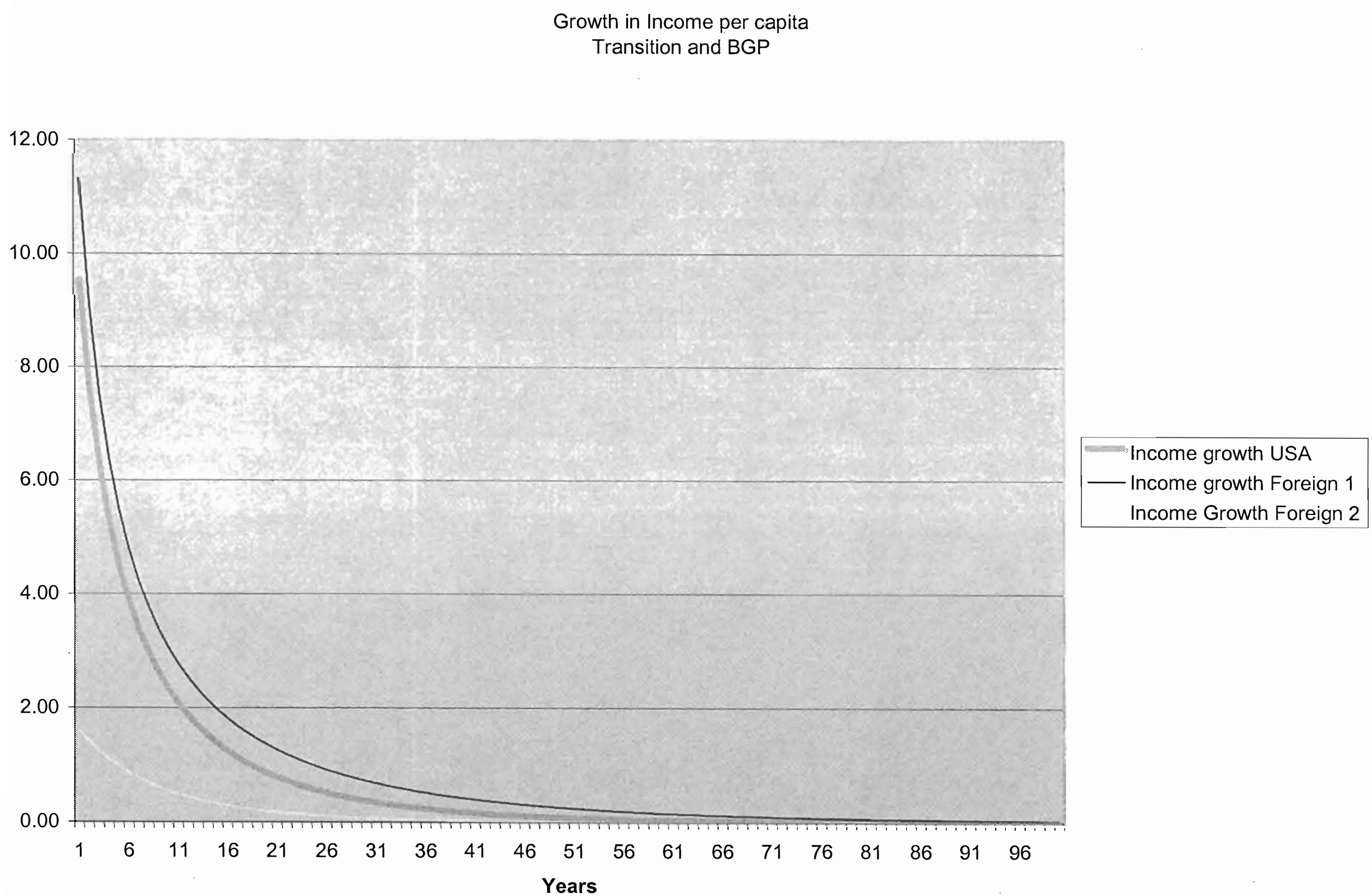
USA: $A_0 = 8$ Foreign 1: $A_0 = 4$ Foreign 1: $A_0 = 2$

(4) **Solow Model with population growth and technological change** :countries differ in their **savings rates**.

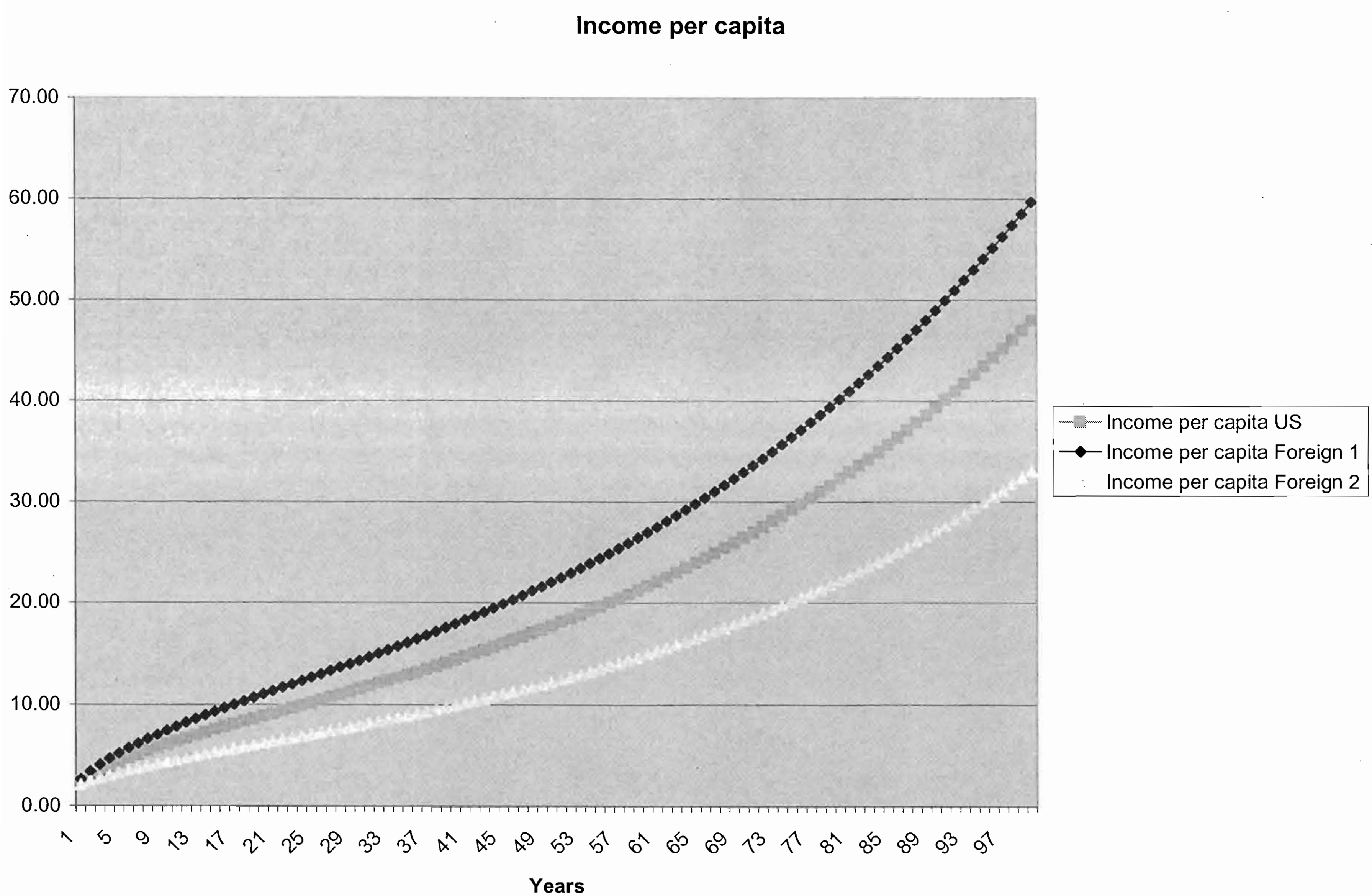
All countries: $\hat{A}=0.02$, $\hat{L}= n= 0.10$, $A_0 = 8$

USA: $s = 0.10$ Foreign 1: $s = 0.15$ Foreign 1: $s = 0.05$


Income per Capita
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
Years
Income per capita US
Income per capita Foreign 1
Income per capita Foreign 2



Growth in Income per capita
Transition and BGP
12.00
10.00
8.00
6.00
4.00
2.00
0.00
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
Years
Income growth USA
Income growth Foreign 1
Income Growth Foreign 2


Income per capita
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73 77 81 85 89 93 97
Years
Income per capita US
Income per capita Foreign 1
Income per capita Foreign 2