6-18-09

→ LAST REMARKS SOLOW

EX. NAT. RESOURCES

→ EX. NAT. RESOURCES IN AK MODEL

→ OLD EXAMS REVIEW

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## SOLOW NON RENEWABLE NAT. RESOURCES

$$Y = Bk^{\alpha} E^{\gamma} L^{1-\gamma-\alpha}$$

WHERE $\dot{R}_t = -E_t \Rightarrow \hat{R} = -\frac{E_t}{R_t} = \overline{-R_E}$

↓ CONSTANT

$$\Rightarrow \boxed{Y = Bk^{\alpha}(R_E R)^{\gamma} L^{1-\alpha-\gamma}}$$

$\Rightarrow$ CONSTANT ENERGY SHARE IN TOTAL INCOME.

$$\frac{P_E \cdot E}{P_Y \cdot Y} = \frac{P_E \cdot \bar{E}}{Y}$$

QUESTION:

DO WE OBSERVE THIS IN THE REAL WORLD? US?

NO

RESPONSE: CHANGE SPECIFIC FORM OF PROD. FUNCTION

CES PROD. FUNCTION:

$$Y = \left[k^{\rho} + (BE)^{\rho}\right]^{\frac{1}{\rho}}$$

$\rho < 1$

FORGET ABOUT LABOR

HERE $\sigma = \frac{1}{1-\rho}$ ELASTICITY OF SUBSTITUTION BETWEEN K & E

$\hat{B} = g_B$ ENERGY SPECIFIC TECH. CHANGE

WHEN $\rho \to 0$ [$\sigma \to 1$]: COBB DOUGLAS

THIS GENERAL CES ALLOWS THE MODEL TO FIT OBSERVED BEHAVIOR OF FACTOR SHARES IN US ECONOMY -

# Ex 3 S07

## Solow with Land & no Tech Change

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$$Y = B K^{\alpha} T^{\beta} L^{1-\alpha-\beta}$$

$\hat{L} = n$

$\hat{B} = 0$

$$\dot{k} = sY - \delta k \Rightarrow \boxed{\hat{k} = \frac{sY}{k} - s}$$

?

i) $\hat{Y}_{BGP}$ , $\hat{y}_{BGP}$ ?

$$\hat{Y} = \underbrace{\hat{B}}_{0} + \alpha \hat{k} + \beta \underbrace{\hat{T}}_{0} + (1-\alpha-\beta)\underbrace{\hat{L}}_{n}$$

$$\boxed{\hat{Y} = \alpha \hat{k} + [1-\alpha-\beta]\, n}$$ EVERY WHERE

AT BGP: $\hat{k}$ , $\hat{Y}$ CONST.

$\Rightarrow \frac{Y}{k}$ CONSTANT

$\Rightarrow \hat{Y} = \hat{k}$

$$\hat{Y}(1-\alpha) = (1-\alpha-\beta)\, n$$

$$\boxed{\hat{Y}_{BGP}} = \left[\frac{1-\alpha-\beta}{1-\alpha}\right] n = \underbrace{\left[1 - \frac{\beta}{1-\alpha}\right]}_{<1} n$$

$$\hat{g}_{BGP} = \hat{Y}_{BGP} - n = -\left[\frac{\beta}{1-\beta}\right] n < 0 \;!!$$

(4) $\Rightarrow g_{BGP} \to 0$

(3) $\hat{Y}_{BGP} < \underbrace{\hat{L}}_{=n}$ — SOLOW WITH LAND

$\Rightarrow \hat{g}_{BGP} < 0 \Rightarrow g_{BGP} \to 0$

SOLOW WITHOUT LAND $\Rightarrow$

$\hat{Y}_{BGP} = \underbrace{\hat{L}}_{=n} \Rightarrow \hat{g}_{BGP} = 0$

$\Rightarrow g_{BGP} =$ SOMETHING POSITIVE

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(II) AK

$Y = AK \underbrace{L^{1-\alpha}}_{=1} = Ak$

$\dot{k} = sY - dk \Rightarrow \hat{k} = s\frac{Y}{k} - d$

$\hat{L} = 0$

$\hat{k} = sA - d$

(1) $\hat{y}$ ?

$\hat{Y} = \hat{A} + \hat{k} = \hat{k}$

$\hat{y} = \hat{Y} - \hat{L} = \hat{Y} = \hat{k}$

WHAT IS $\hat{k} = g_A - d$ ?

Result $\boxed{\hat{Y} = \hat{y} = g_A - d}$

(2) MPK CONSTANT = (A)

(AS OPPOSED TO STANDARD SOLOW WHERE MPK $\to 0$ AS $K \to \infty$)

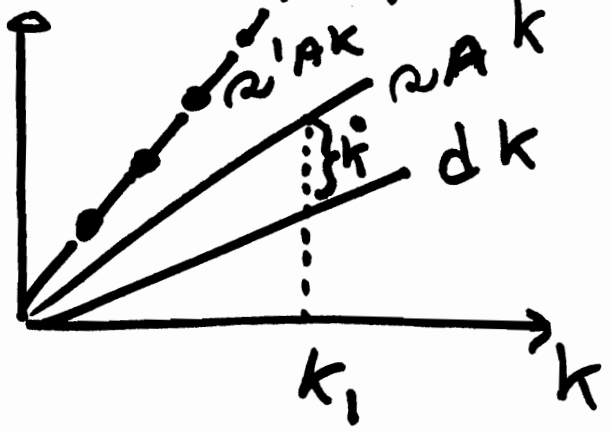
$Y = B K^{\alpha} L^{1-\alpha}$

$\boxed{MPK} = B \alpha K^{\alpha-1} L^{1-\alpha}$

$= \boxed{B \alpha \left(\frac{L}{K}\right)^{1-\alpha}}$

(3) ASSUME ↑ R AT $\bar{t}$

$\hat{y}$ ? $\hat{m}$ ?




$\dot{k} = RAk - dk$

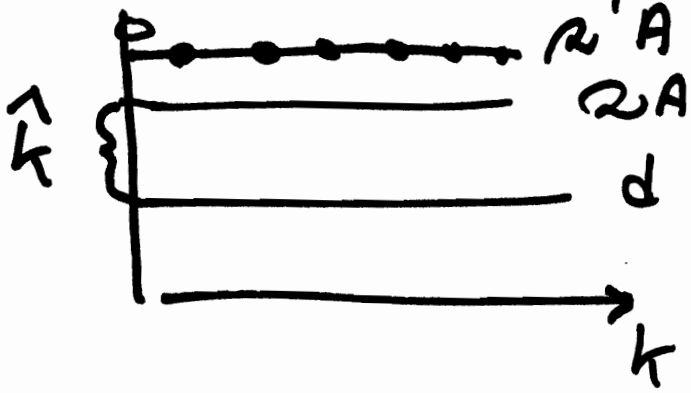
$\hat{k} = RA - d$

REMEMBER:

$\hat{y} = \hat{k} = \hat{m} = RA - d$

FROM QUESTION (1)

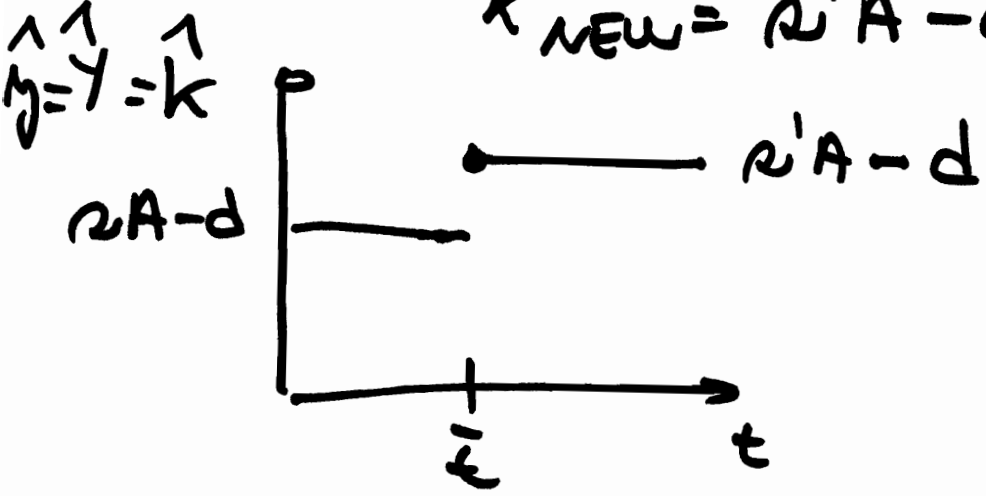
ASSUME $RA > d$

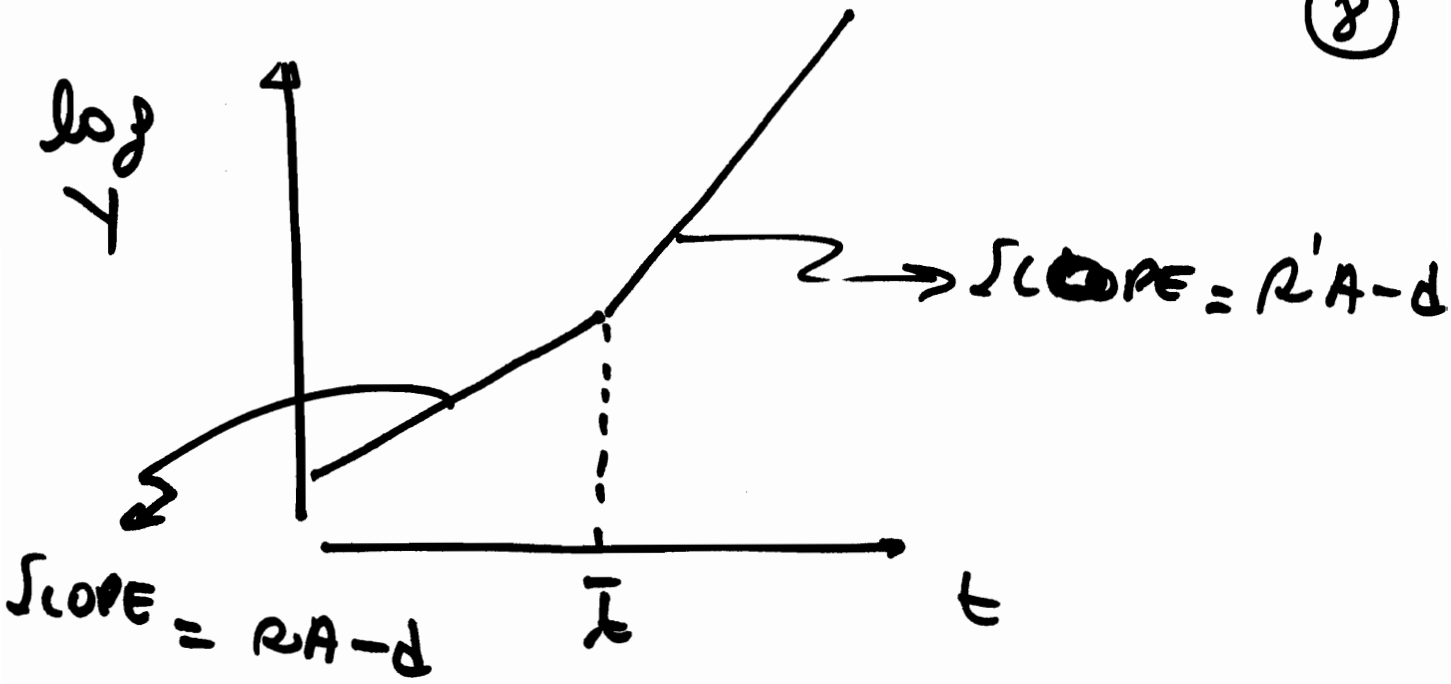



$\hat{k} = \frac{\dot{k}}{k}$

$\uparrow R \Rightarrow \uparrow \hat{k} = \hat{y} = \hat{m}$

$\hat{k}_{NEW} = R'A - d$




log Y
SLOPE = R'A - d
SLOPE = RA - d
t

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(III)

$$\boxed{Y = L^{1-\alpha} \int_0^h x_j^{\alpha} dj} \overset{\text{MAGIC}}{=} \boxed{k^{\alpha} (hL)^{1-\alpha}}$$

(4) $\dot{h} = \mu e^{\psi w} A$

$\mu$ = EASE OF TECH. ADOPTION

$\hat{A} = g$

$\psi > 0$

$\hat{L} = n$

$\mu$ = SKILL LEVEL/SCHOOLING

$\dot{k} = s_k \cdot Y - d k$

A = # INT. AVAILABLE IN ADVANCED COUNTRY

1)

2) $\hat{Y}_{BGP}, \hat{y}_{BGP}, \hat{h}_{BGP}$

$$\hat{Y} = \alpha \hat{K} + (1-\alpha)\left[\hat{h} + \hat{L}\right]$$

$$\hat{L} = n$$

(**) $\boxed{\hat{Y} = \alpha \hat{K} + (1-\alpha)(\hat{h} + n)}$ ALWAYS

(*) $\hat{k} = s_k \cdot \frac{Y}{k} - d$ ALWAYS

AT BGP: $\hat{Y}, \hat{k}, \hat{\eta}$ CONSTANT

(*) $\Rightarrow$ $\frac{Y}{k}$ CONSTANT $\Rightarrow \hat{Y} = \hat{k}$

USING (**)

$\hat{Y}(1-\alpha) = (1-\alpha)(\hat{h} + n)$

$\Rightarrow$ (***) $\hat{Y}_{BGP} = \hat{h} + n$

$\hat{h} = \mu e^{\psi m} \frac{A}{h}$ ALWAYS

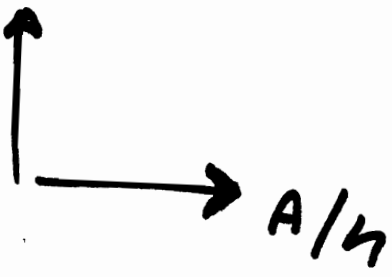
$\Rightarrow$ AT BGP $\Rightarrow$

$\left(\frac{A}{h}\right)$ CONSTANT $\Rightarrow$ $\hat{A} = \hat{h} = g_{BGP}$

$\hat{Y}_{BGP} = g + n$ $\Rightarrow$ $\hat{\eta}_{BGP} = \hat{Y} - n = g$

(3) $\downarrow \mu$ AT time $\bar{t}$

USE

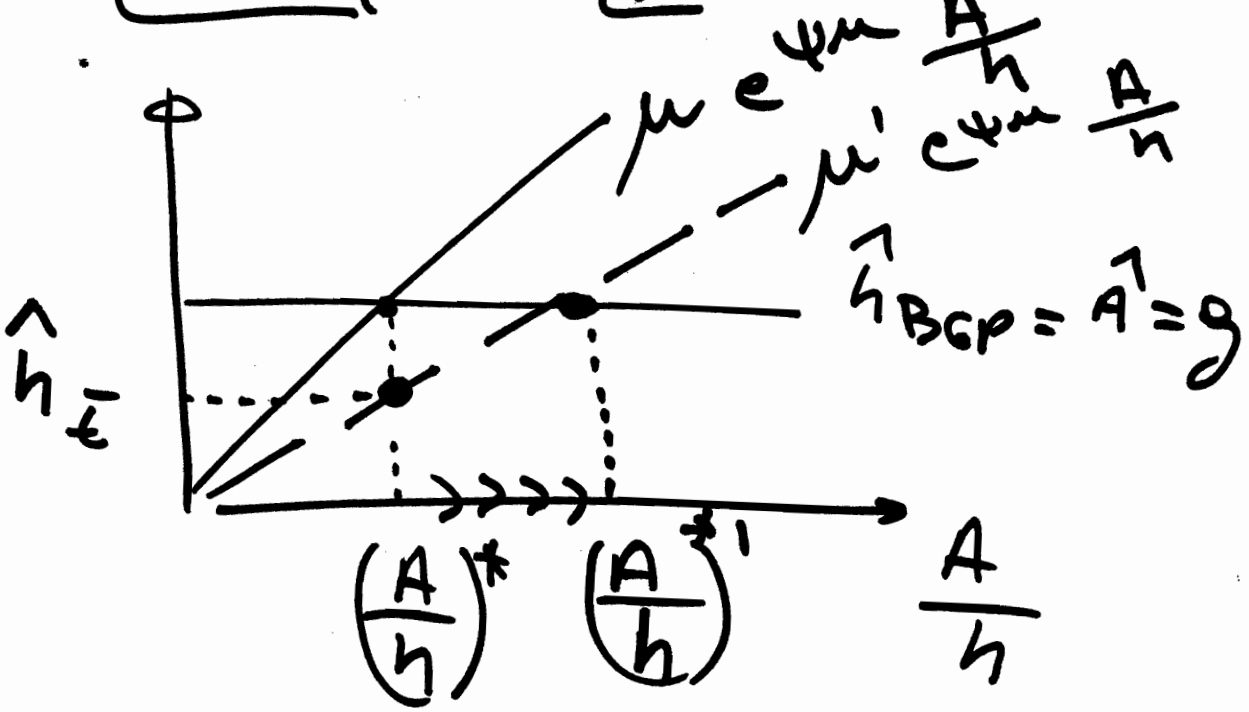


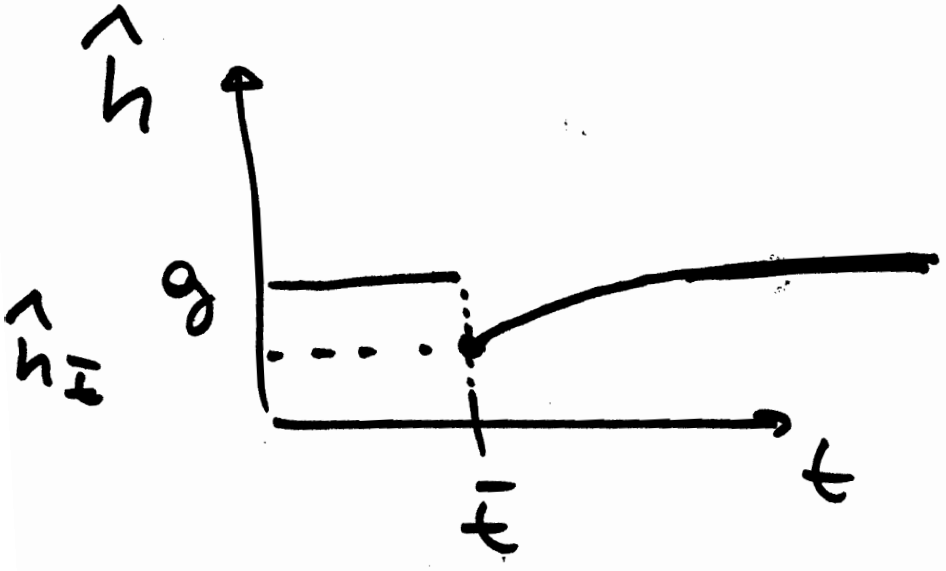

(a) $\frac{A}{h}$ PATH

(b) $\hat{h}$ PATH

$$\hat{h} = \mu e^{\psi \mu} \cdot \frac{A}{h}$$ ALWAYS

$$\hat{h}_{BGP} = \hat{A} = g$$






$$\hat{h}_{\bar{t}} = \mu' \cdot e^{\psi \mu \frac{A}{h}}$$