6-17-09

CH. 9 JONES

NAT. RESOURCES → LAND (FIXED)
→ NON RENEWABLE (STOCK ↓ AS WE USE "FLOW" FOR PRODUCTION)

OLD HYPOTHESIS:

→ MALTHUS: INCOME p.c → 0

WHY? POPULATION: GROWS GEOMETRICALLY 1, 2, 4, 8

RESOURCES: ARITHMETICALLY 1, 2, 3, 4, ...

→ LATE 60'S:

LIMITS TO GROWTH:

TOO MANY PEOPLE CONSUME TOO MANY RESOURCES ⇒ DISASTER IMMINENT

→ CURRENT VIEW:

"SUSTAINABILITY NEEDS TO BE TAKEN SERIOUSLY"

SOLOW WITH LAND

(1) $Y = B\ K^{\alpha}\ T^{\beta}\ L^{1-\alpha-\beta}$ CRS

$\hat{B} = g_B$ (TECH. CHANGE)

$\hat{L} = n$

$0 < \alpha < 1$

$0 < \beta < 1$

T = STOCK OF LAND

(2) $\dot{K} = sY - dK$

$0 < d < 1$

$\Rightarrow \hat{K} = s\frac{Y}{K} - d$ (3)

BGP: $\hat{Y}, \hat{K}$, CONSTANT

Eq. (3) $\Rightarrow \frac{Y}{K}$ CONSTANT $\Rightarrow$

(4) $\hat{Y} = \hat{K}$

TAKING GROWTH RATES OF (1)

TRUE EVERYWHERE:

$$\hat{Y} = \underbrace{\hat{B}}_{= g_B} + \alpha\hat{K} + \beta\underbrace{\hat{T}}_{= 0} + (1-\alpha-\beta)\underbrace{\hat{L}}_{= n}$$

ALWAYS $\boxed{\hat{Y} = g_B + \alpha\hat{K} + (1-\alpha-\beta)n}$ (5)

REMARK: $\hat{y} = \hat{Y} - \hat{L}$ EVERYWHERE?

$$\hat{y} = \hat{Y} - n = g_B + \alpha\hat{k} + \cancel{n} - \alpha n - \beta n - \cancel{n}$$

$$= g_B + \alpha\underbrace{(\hat{k} - n)}_{\hat{h}} - \beta n$$

ALWAYS $\Rightarrow$ $\boxed{\hat{y} = g_B + \alpha\hat{h} - \beta n}$ (6)

ALONG BGP: $\hat{Y} = \hat{k}$ OR $\hat{y} = \hat{h}$

$$(5) \Rightarrow \hat{Y}(1-\alpha) = g_B + (1-\alpha-\beta)n$$

$$\hat{Y} = \frac{g_B}{1-\alpha} + \left(\frac{1-\alpha-\beta}{1-\alpha}\right)n$$

$$\Rightarrow \hat{Y} = \frac{g_B}{1-\alpha} + \left[\frac{1-\alpha}{1-\alpha} - \underbrace{\frac{\beta}{1-\alpha}}_{\bar{\beta}}\right]n$$

$$\Rightarrow (7) \quad \boxed{\hat{Y}_{BGP} = \frac{g_B}{1-\alpha} + (1-\bar{\beta})n} \quad \text{ALONG BGP}$$

WHERE $\bar{\beta} = \frac{\beta}{1-\alpha}$

USING (6) OR (7)

$$(8) \quad \boxed{\hat{y}_{BGP} = \frac{g_B}{1-\alpha} - \bar{\beta} n} \quad \text{ALONG BGP}$$

REMARKS:

(1) IF $\beta = 0 \Rightarrow \bar{\beta} = 0$

$\Rightarrow$ BACK AT SOLOW WITH TECH. CHANGE AND ONLY L & K

(2) ASSUME $\beta > 0$.

THE LARGER THE β, THE MORE IMPORTANT LAND IS IN THE PRODUCTION PROCESS $\Rightarrow$ THE "DRAG" THE FIXED RESOURCE INTRODUCES IN INCOME P.C. GROWTH IS LARGER

(3) POPULATION GROWTH ENTERS NEGATIVELY IN $\hat{y}_{BGP}$

THIS IS THE OPPOSITE OF WHAT WE FIND IN ROMER

(4) EQ. (8) SHOWS THAT THERE IS A "RACE" BETWEEN TECH. CHANGE & THE DIMINISHING RETURNS INTRODUCED BY THE FIXED FACTOR

IF $g_B = 0 \Rightarrow \hat{y}_{BGP} = -\bar{\beta} n$

$\Rightarrow y \rightarrow 0$ (EXTREME VERSION OF MALTHUS)

ANALYSIS OF ADJUSTMENT TO BGP

USE THE FACT THAT ALONG BGP $\frac{K}{Y}$ IS CONSTANT

$\frac{K}{Y} \rightarrow$ CAPITAL OUTPUT RATIO

WE CAN DEFINE A NEW VARIABLE:

$$z = \frac{K}{Y} \quad \text{CAPITAL OUTPUT RATIO}$$

AND LOOK AT $\hat{z}$ EVERYWHERE

$\hat{z}$ BGP

& LEARN ABOUT ADJUSTMENT TO BGP.

STEP 1: FIND $\hat{z}$ EVERYWHERE

DIVIDE (1) BY K

$$\frac{Y}{K} = B \frac{K^{\alpha}}{K} T^{\beta} L^{1-\alpha-\beta}$$

$$\boxed{\frac{Y}{K}} = \boxed{B \quad K^{\alpha-1} \quad T^{\beta} \quad L^{1-\alpha-\beta}}$$

$$\widehat{\left(\frac{Y}{K}\right)} = \hat{Y} - \hat{K} = \underset{\overset{\shortparallel}{g_B}}{\hat{B}} + (\alpha-1)\hat{K} + (1-\alpha-\beta)n$$

$$\hat{Y} - \hat{k} = g_B + (\alpha - 1)\hat{k} + (1 - \alpha - \beta) n$$

$$\hat{z} = \widehat{\left(\frac{k}{Y}\right)} = \hat{k} - \hat{Y} = -[\hat{Y} - \hat{k}]$$

$$(**)\quad \hat{z} = -g_B + (1-\alpha)\hat{k} - (1-\alpha-\beta) n$$

SINCE $\hat{k} = r \frac{Y}{k} - d = \boxed{r z^{-1} - d}$ $(*)$

WE PLUG $(*)$ INTO $(**)$ TO GET:

$$\hat{z} = -g_B + (1-\alpha) r z^{-1} + (1-\alpha)(-d) - (1-\alpha-\beta) n$$

ALWAYS

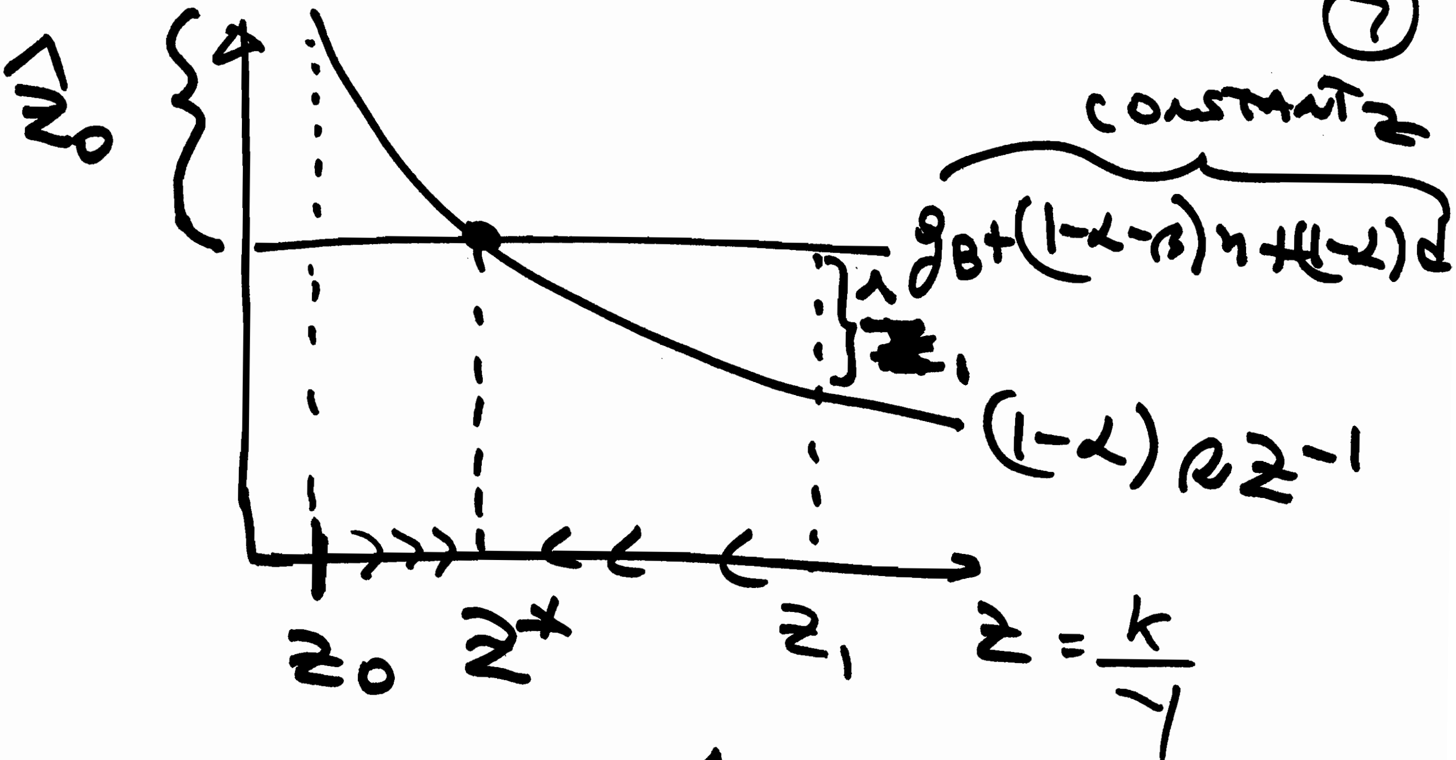
$$\hat{z} = (1-\alpha) r z^{-1} - \underbrace{[g_B + (1-\alpha-\beta) n + (1-\alpha) d]}_{\text{CONSTANT}_z}$$

AT BGP: $\frac{K}{Y} = z$ IS CONSTANT

$$\Rightarrow \hat{z} = 0$$

$$(1-\alpha) r z^{-1} = \text{CONSTANT}_z$$

$$\boxed{z_{BGP}} = \frac{(1-\alpha) r}{\text{CONSTANT}_z}$$



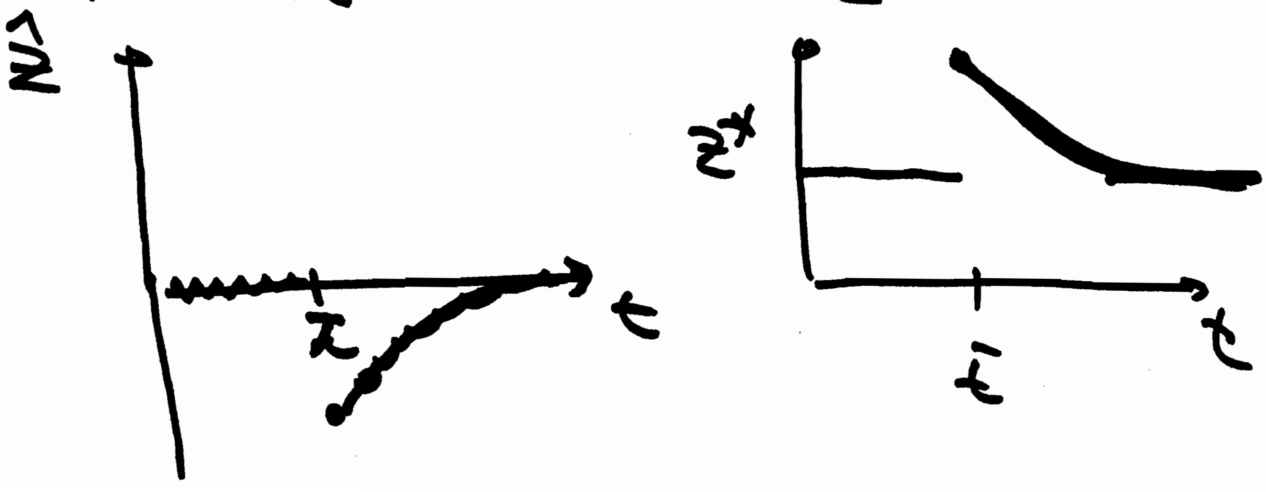
IF $z < z^*$, $\hat{z} > 0 \Rightarrow z \uparrow$

IF $z > z^*$, $\hat{z} < 0 \Rightarrow z \downarrow$

EX: $\uparrow k_{\text{GIFT}} \Rightarrow z \uparrow$ TO z_1

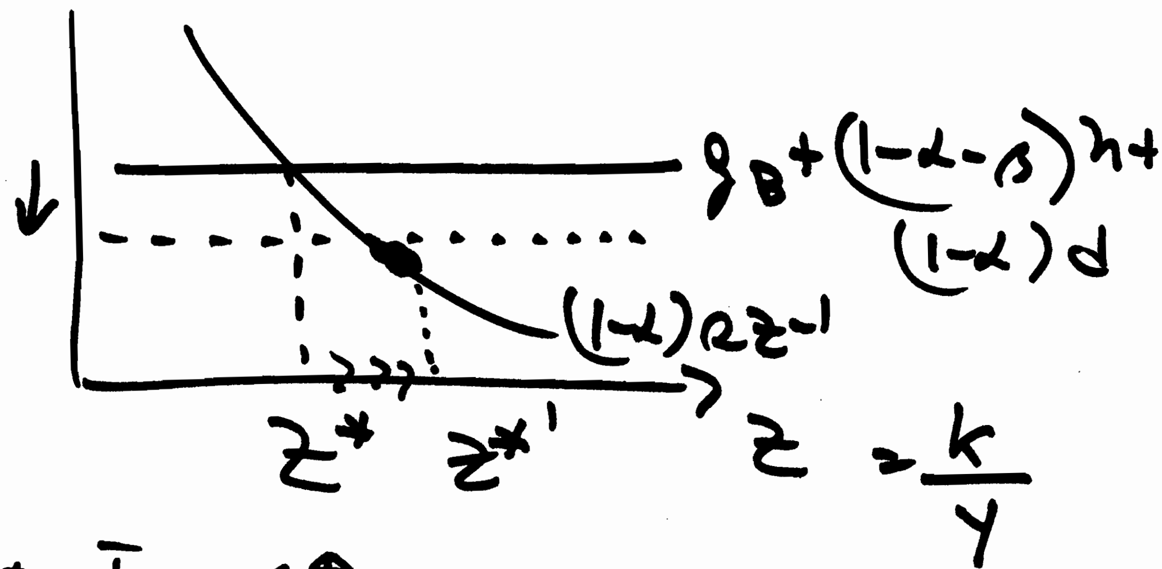
(BEFORE WE WERE AT BGP)

WE ADJUST BACK TO z^*

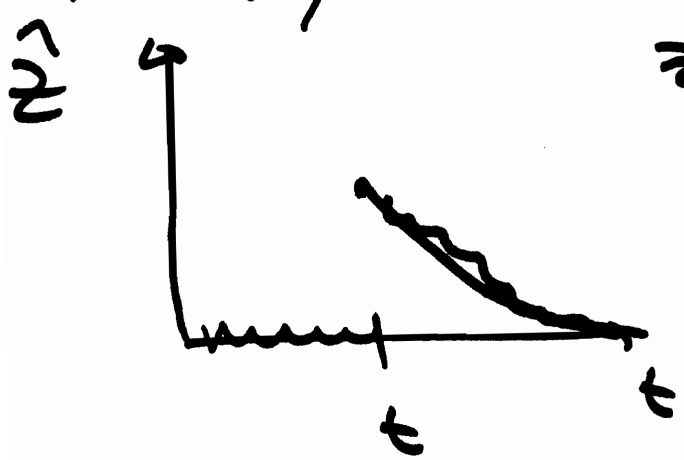


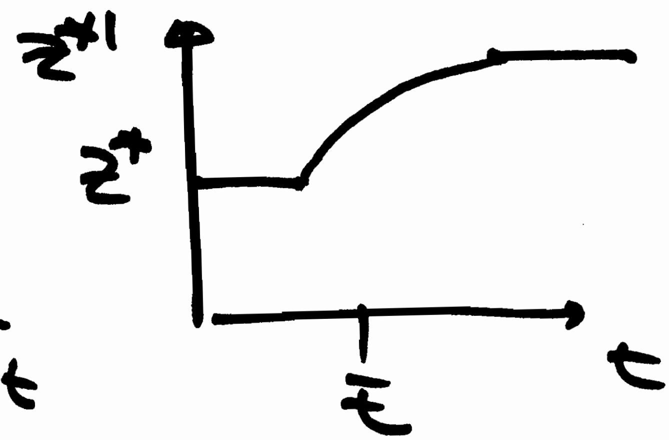
EX: $\uparrow \beta$ (LAND BECOMES MORE IMPORTANT)

$\uparrow \beta \Rightarrow (1-\alpha-\beta) \downarrow$



At $\bar{t}$, $\beta \uparrow$





$$\hat{m}_{BGP} = \frac{g_B}{1-\alpha} - \bar{\beta} n \quad \downarrow$$

SINCE $\uparrow \beta \Rightarrow \uparrow \bar{\beta}$

EX: $\downarrow g_B$

SOLOW WITH NON RENEWABLE NAT. RESOURCE

R_t = STOCK OF NAT. RESOURCE AT TIME t

WE ASSUME THAT IT IS EXTRACTED AS FOLLOWS:

$$\boxed{E_t = \kappa_E \cdot R_t} \quad (1) \quad 0 < \kappa_E < 1$$

AND $\dot{R}_t = -E_t$ (2)

$$\Rightarrow \boxed{\hat{R}_t} = \frac{\dot{R}_t}{R_t} = \frac{-E_t}{R_t} = \boxed{-\kappa_E} \quad (3)$$

MANY REASONABLE MODELS SHOW THIS RESULT.

OUTPUT:

$$(4) \quad Y = B\, K^{\alpha} E^{\gamma} L^{1-\alpha-\gamma}$$

USING (1)

$$(5) \quad \boxed{Y = B\, K^{\alpha} (\kappa_E R)^{\gamma} L^{1-\alpha-\gamma}} \quad \begin{array}{l} \alpha+\gamma < 1 \\ 0<\alpha<1 \\ 0<\gamma<1 \end{array}$$

$$(6) \quad \dot{K} = sY - dK \Rightarrow \boxed{\hat{K} = \frac{sY}{K} - d}$$

$\hat{B} = g_B$, $\hat{L} = n$

GOAL: LOOK AT BGP & ADJ. PATH

TAKING GROWTH RATES OF (5)

$$\hat{Y} = \hat{B} + \alpha\hat{K} + \gamma\big[\widehat{\underbrace{r_E}_{\text{CONSTANT}} \cdot R}\big] + (1-\alpha-\gamma)\hat{L}$$

$$= g_B + \alpha\hat{K} + \gamma \underbrace{\hat{R}}_{= -r_E} + (1-\alpha-\gamma)n$$

$$(7)\quad \boxed{\hat{Y} = g_B + \alpha\hat{K} - \gamma r_E + (1-\alpha-\gamma)n} \quad \text{ALWAYS}$$

IN PER WORKER TERMS

$$\boxed{\hat{y}} = (\widehat{Y/L}) = \hat{Y} - n = g_B + \alpha\hat{K} - \gamma r_E \cancel{+n} - \alpha n - \gamma n \cancel{-n}$$

$$(8)\quad \boxed{\hat{y}} = \boxed{g_B + \alpha\hat{k} - \gamma[r_E + n]} \quad \underline{\text{ALWAYS}}$$

AT BGP: $\hat{Y}, \hat{K}, \ldots$ CONSTANT

$(6) \Rightarrow \frac{Y}{K}$ CONSTANT $\Rightarrow \hat{Y} = \hat{K}$

AND $\hat{k} = \hat{y}$

$$(7) \Rightarrow \hat{Y} = g_B + \alpha\hat{Y} - \gamma r_E + (1-\alpha-\gamma)n$$

$$\Longrightarrow \hat{Y} = \frac{g_B}{1-\alpha} - \frac{\gamma}{1-\alpha} r_E + \left(\frac{1-\alpha-\gamma}{1-\alpha}\right)n$$

$$\hat{Y} = \underbrace{\frac{g_B}{1-\alpha}}_{g} - \underbrace{\frac{\gamma}{1-\gamma}}_{\bar{\gamma}} r_E + \underbrace{\left(1 - \frac{\gamma}{1-\alpha}\right)}_{1-\bar{\gamma}} n$$

$\Rightarrow$

$$\boxed{\hat{Y}_{BGP} = g + n - \bar{\gamma}\left[r_E + n\right]} \qquad (9)$$

USING (8) OR (9)

$$\boxed{\hat{y}_{BGP} = g - \underbrace{\bar{\gamma}\left[r_E + n\right]}_{\text{"DRAG ON GROWTH"}}} \qquad 10$$

BGP ISSUES

REMARKS

1) $\uparrow r_E \Rightarrow \uparrow$ TODAY'S Y

BUT $\hat{y}_{BGP} \downarrow$

USEFUL TOOLS TO ANALIZE MODEL

DIVIDE (5) BY K

$$\frac{Y}{K} = B\,\frac{K^{\alpha}}{K}\left(r_E R\right)^{\gamma} L^{1-\alpha-\gamma}$$

$$\frac{Y}{K} = B\,K^{\alpha-1}\left(r_E R\right)^{\gamma} L^{1-\alpha-\gamma}$$

TAKING GROWTH RATES

$$\widehat{\left(\frac{Y}{K}\right)} = \hat{Y} - \hat{K} = \underset{\overset{\|}{g_B}}{\hat{B}} + (\alpha-1)\hat{K} + \gamma\hat{R} + (1-\alpha-\gamma)n$$

LET $Z = K/Y \Rightarrow \hat{Z} = \hat{K} - \hat{Y}$

$$= -(\hat{Y} - \hat{K})$$

$$\hat{Z} = -g_B + (1-\alpha)\underset{\overset{\|}{rZ^{-1}-d}}{\hat{K}} - \gamma\underset{\overset{\|}{-r_E}}{\hat{R}} - (1-\alpha-\gamma)n$$

$$\hat{Z} = -g_B + (1-\alpha)rZ^{-1} + \gamma r_E + (1-\alpha)(-d)$$

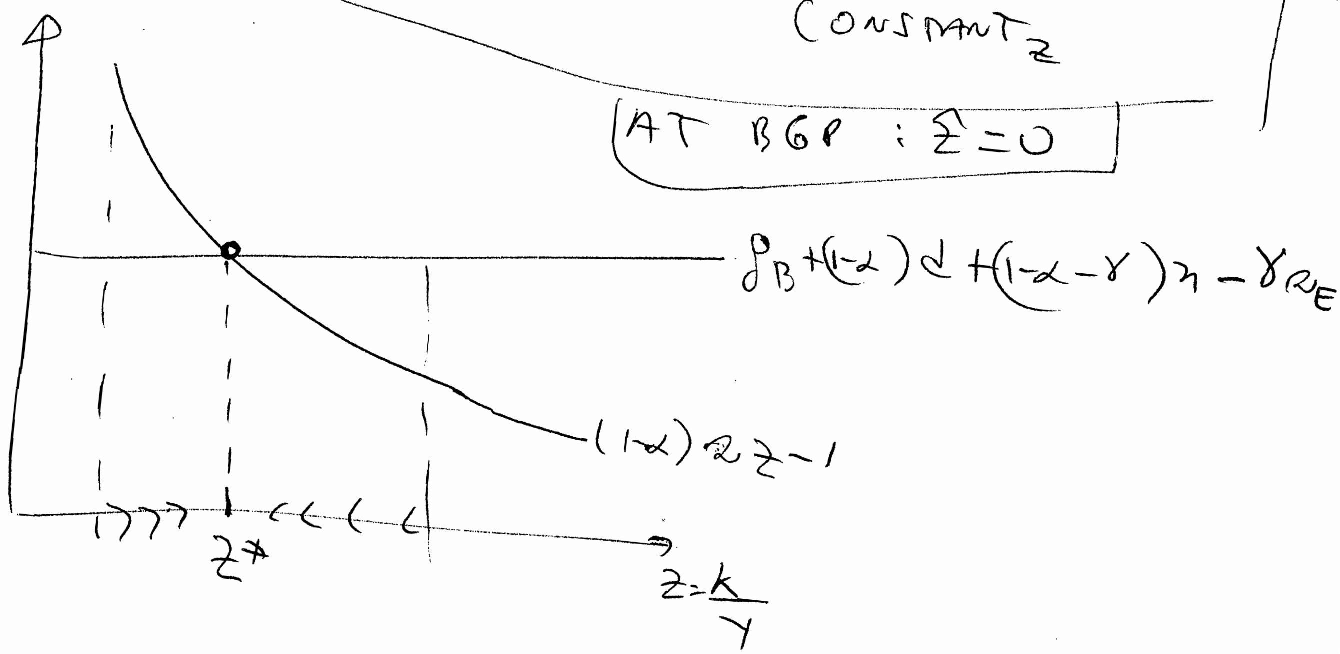
$$- (1-\alpha-\gamma)n$$

$$\hat{z} = -g_B + (1-\alpha)\left[r z^{-1} - d\right] - \gamma\hat{R} - (1-\alpha-\gamma)n$$

$$= (1-\alpha) r z^{-1} - \left[g_B + d(1-\alpha) + \gamma\underbrace{\hat{R}}_{= -r_E} + (1-\alpha-\gamma)n\right]$$

$$\boxed{\hat{z} = (1-\alpha) r z^{-1} - \left[\underbrace{g_B + (1-\alpha)d + (1-\alpha-\gamma)n - \gamma r_E}_{\text{CONSTANT}_z}\right]}$$ ALWAYS

AT BGP : $\hat{z} = 0$



COMP. STATICS:

$\uparrow r_E \Longrightarrow$