6-15-09

SUMMARY

- SOLOW WITH EX. TECH. CHANGE : $\hat{y}_{BGP} = g$ (EX. RATE OF TECH. CHANGE)

y_{BGP} = FUNCTION OF PARAMETERS $(s_k, \delta, \ldots)$

- ROMER (ENDOGENIZE RATE OF TECH. CHANGE) : $\hat{y}_{BGP} = \frac{\partial y}{1-\phi}$

y_{BGP} = FUNCTION OF PARAMETERS $(s_k, \delta, \ldots)$

REMARK: GOV. POLICIES

- HAVE LEVEL EFFECTS & DO NOT CHANGE GROWTH RATE
- THEY SHOW BY ADJUSTMENT TO BGP.

AK MODEL: REMOVES THE DECREASING RETURNS FEATURE - RESULT:

GOV. POLICY CAN INFLUENCE GROWTH RATE OF y.

AK MODEL

ASSUME NO TECH. PROGRESS $(\hat{A}=0)$ & NO POPULATION GROWTH $(n=0)$

SOLOW MODEL $\Rightarrow$

$\hat{y}_{BGP} = \hat{A} = g = 0 \qquad \hat{C} = \hat{k} = 0$

CONSIDER INSTEAD THE FOLLOWING PRODUCTION FUNCTION AND ASSUME $\alpha = 1$

$$Y = A \cdot k \quad (1)$$

$$\dot{k} = sY - \delta k \quad (2) \Rightarrow \boxed{\hat{k} = s\frac{Y}{k} - \delta}$$

$(1) \Rightarrow \frac{Y}{k} = A$ THEN: $\boxed{\hat{k} = sA - \delta}$

OR $\boxed{\dot{k} = sAk - \delta k}$

TAKING GROWTH RATES OF (1)

$$\boxed{\hat{Y}} = \hat{A} + \hat{k} = \boxed{\hat{k}} \Rightarrow \boxed{\hat{y}} = \hat{Y} - n = \hat{k}$$

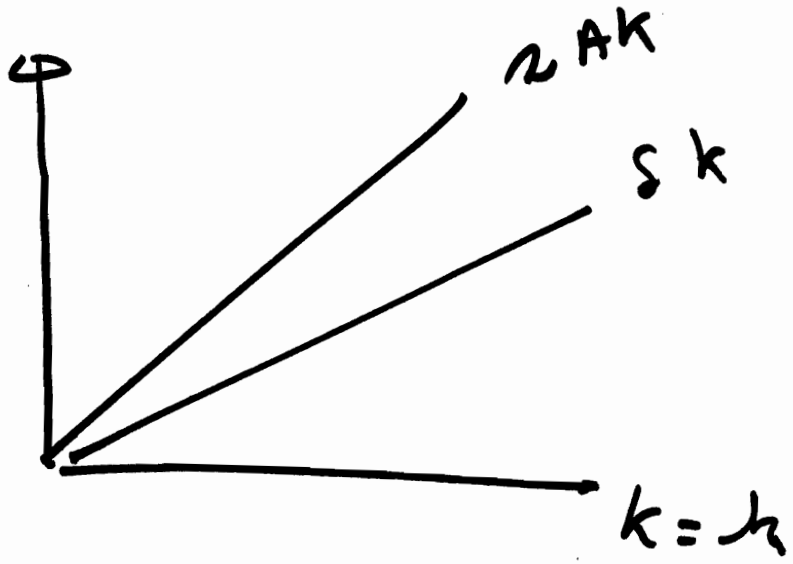
($\hat{A} = 0$; $n = 0$)

$$= \boxed{sA - \delta}$$

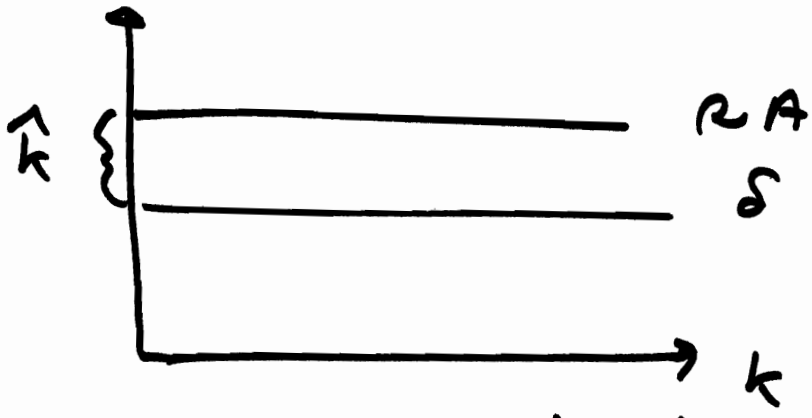
IF $sA > \delta \Rightarrow \hat{k} = \hat{Y} = \hat{y}$ POSITIVE

IF $sA = \delta \Rightarrow \hat{k} = \hat{Y} = \hat{y}$ ZERO

IF $sA < \delta \Rightarrow \hat{k} = \hat{Y} = \hat{y}$ NEGATIVE



IF $\Omega A > \delta$



IF $\Omega \uparrow \Rightarrow \hat{k} = \hat{Y} = \hat{y} \uparrow$

SO GOV. POLICIES INFLUENCE GROWTH OF OUTPUT PER WORKER.

<u>GMAC CASE</u>

$$Y = F(k, L)$$

FOR GOV. POLICIES TO INFLUENCE GROWTH RATE WE NEED THAT MPK DOES NOT GO TO ZERO AS $K \rightarrow$ INFINITY.

EXAMPLE:

$$Y = Ak + Bk^{\alpha} L^{1-\alpha} \qquad 0 < \alpha < 1,\ A > 0$$

NOTICE:

$$MPK = \frac{\partial Y}{\partial k} = A + \alpha k^{\alpha-1} L^{1-\alpha} B$$

$$= A + \alpha B \left(\frac{L}{k}\right)^{1-\alpha}$$

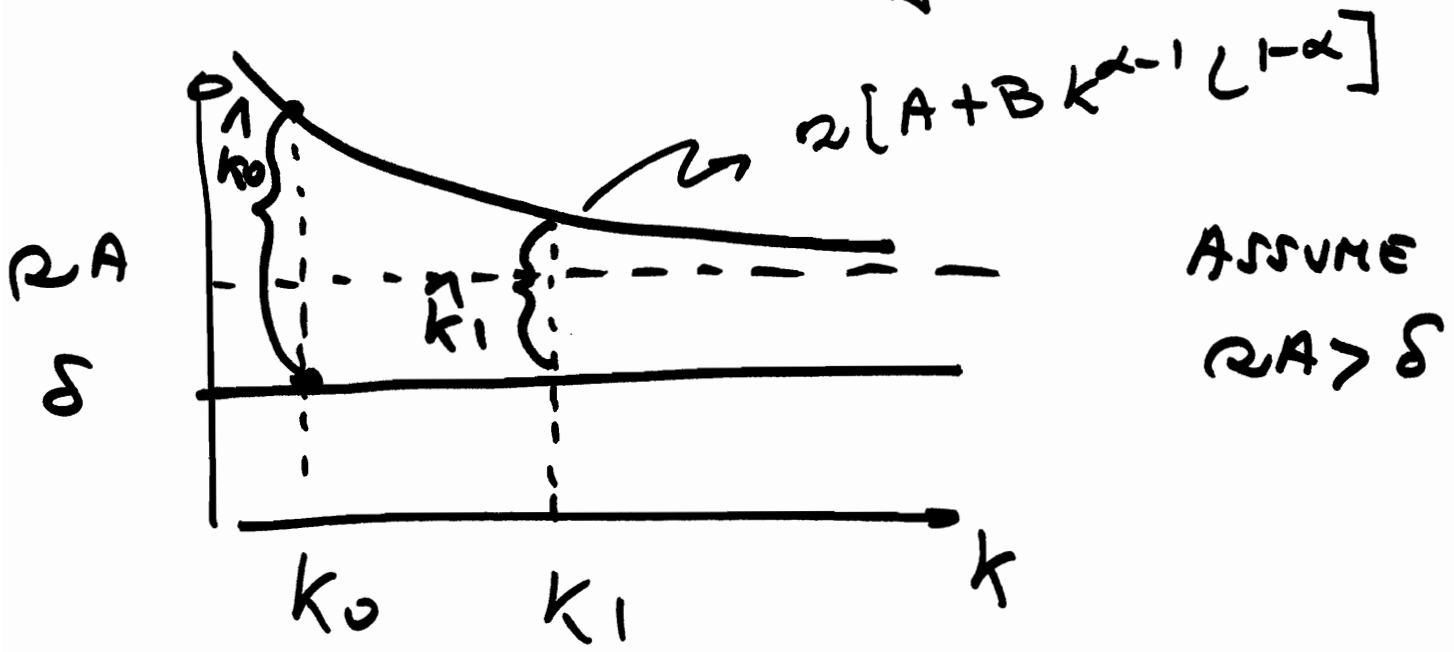
AS $k \uparrow$, MPK $\downarrow$ BUT HAS LOWER BOUND POSITIVE

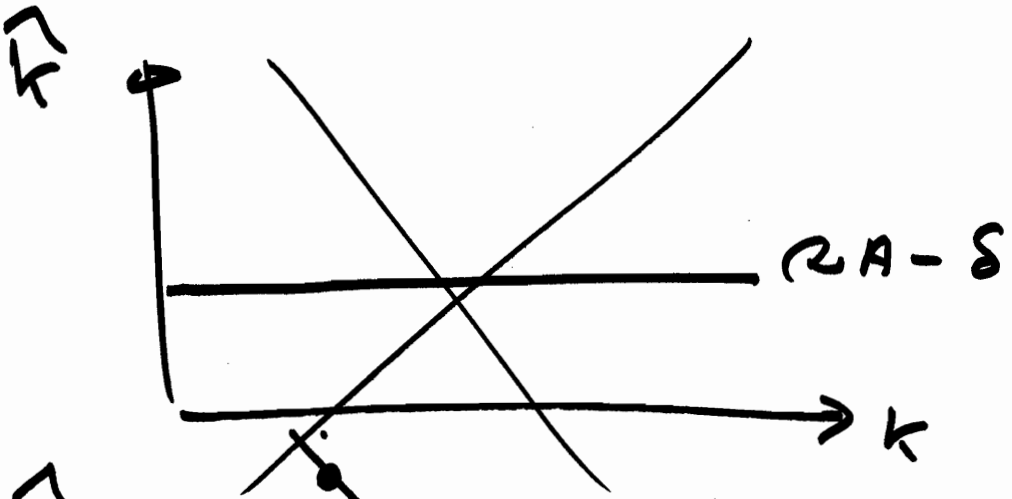
MIXED MODEL: TRANSITIONAL DYNAMICS + BGP LIKE AK

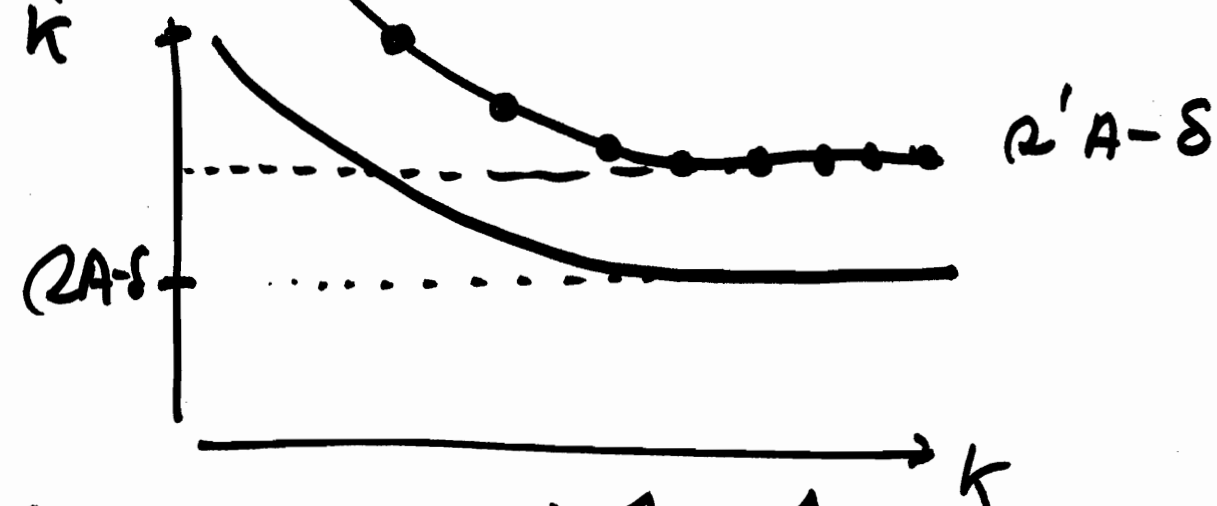
$$\dot{k} = sY - \delta k$$

$$\boxed{\frac{\dot{k}}{k}} = s\frac{Y}{k} - \delta \quad \text{WHERE} \quad \frac{Y}{k} = \underline{\underline{A + Bk^{\alpha-1}L^{1-\alpha}}}$$

$$\hat{K} = \Omega\left[A + BK^{\alpha-1}L^{1-\alpha}\right] - \delta$$







$\uparrow \Omega \Rightarrow$ Eq $\uparrow \hat{K}_{BGP} = \hat{Y}_{BGP}$

ANOTHER MODEL
WHERE GOV. POLICY CAN $\uparrow$

$\hat{y}$ AT BGP.

LUCAS 1988

$$Y = k^{\alpha}(hL)^{1-\alpha} \qquad 0 < \alpha < 1$$

h → H-K PER PERSON

$$\hat{h} = \underbrace{(1-u)}_{\text{CONSTANT}}$$

u = % OF TIME SPENT WORKING

TAKING GROWTH RATES:

$$\hat{y} = \alpha\hat{k} + (1-\alpha)\hat{h} + (1-\alpha)n$$

$\Rightarrow$ AT BGP: $\hat{y} = \hat{k} = \boxed{\hat{h}} = \boxed{1-u}$

(SAME PROOF AS BEFORE)

IF GOVERNMENT SUBSIDIZES SCHOOLING $\Rightarrow \uparrow 1-u \Rightarrow \uparrow \hat{y}_{BGP}$

GROWTH MODEL WITH EXTERNALITIES DERIVED FROM k-ACCUMULATION

IDEA: BY BUILDING THE k-STOCK, WE MAKE ↑ THE TECHNOLOGY LEVEL.

REMARK: THIS IS AN ALTERNATIVE MODEL WITH TECH. CHANGE GENERATE INSIDE THE MODEL (EXTERNALITY, BY PRODUCT OF k-ACCUMULATION)

(1) $$Y = B\,k^{\alpha}\,L^{1-\alpha}$$

INDIVIDUAL AGENTS TAKE B(t) AS GIVEN (THEY SEE IT AS EXOGENOUSLY CHANGING)

BUT

(2) $$B = A\,k^{1-\alpha}$$ ← CONSTANT

i.e.:

$$\hat{B} = \hat{A} + (1-\alpha)\hat{k}$$

$$\hat{B} = (1-\alpha)\hat{k}$$

AGGREGATE

PROD. FUNCTION & MOVEMENT.

$$\boxed{Y} = A K^{1-\alpha} k^{\alpha} L^{1-\alpha} = \boxed{A k L^{1-\alpha}}$$

USING (3)

$$\hat{Y} = \hat{A} + \hat{K} + (1-\alpha)\hat{L}$$

$$\hat{Y} = \hat{K} + (1-\alpha)n = \hat{K} + n - \alpha n$$

$$\boxed{\hat{y}} = \hat{Y} - n = \boxed{\hat{K} - \alpha n}$$

ASSUME $n = 0$ $\quad L = 1$

$$\hat{K} = \frac{RY}{K} - \delta = \boxed{RA - \delta}$$

$$\boxed{\hat{y}} = \hat{K} = \hat{Y} = \boxed{RA - \delta}$$

REMARK: EXTERNALITIES + P.C

$\Longrightarrow$ ENDOGENOUS GROWTH