6-12-09

JONES CH. 6

GROWTH AND DEVELOPMENT

IDEA: THERE ARE COUNTRIES THAT ARE NOT PUSHING THE "TECHNOLOGICAL FRONTIER"

- ARE NOT DEVOTING LABOR TO PUSH TECH. FRONTIER.
- ARE "NOT ABLE" TO "USE" THE MOST "ADVANCED" TECHNOLOGIES/CAPITAL GOODS BECAUSE OF "LACK OF SKILLS" OR "SKILL DEFICIENCIES"

THESE COUNTRIES STILL HAVE TO "DEVELOP"

$\boxed{\text{DEVELOPED COUNTRY :}}$ → ADVANCES TECH. FRONTIER :

$$\hat{A} = g$$

[ROMER MODEL SET UP]

$\boxed{\text{LDC}}$
"SKILL LEVEL "LOW" ⟹ ONLY SUBSET OF A CAN BE USED] h

$A(t)$ = LEVEL OF DC COUNTRY TECHNOLOGY ≈ # OF INT. GOODS / DIFF. K-GOODS BEING PRODUCED.

$h(t)$ = # OF INT. GOODS THAT CAN BE USED IN LDC. OR TECH. LEVEL IN LDC. THIS IS A FUNCTION OF AMOUNT OF TIME AGENTS DEVOTE TO SKILL ACCUMULATION (≈ YEARS OF SCHOOLING)

LDC

(0) $Y = L^{1-\alpha} \int_0^h x_j^{\alpha} \, dj$

$\rightarrow \alpha \ll 1$

EQUILIBRIUM/ FEASIBILITY:

$\rightarrow h_t < A_t$ ← GIVEN OUTSIDE MODEL

$$\boxed{K} = \int_0^h x_j \, dj = \boxed{hX}$$

$\rightarrow \hat{A} = g > 0$

BECAUSE OF SYMMETRY: $x_j = X$

$$\Rightarrow \boxed{X = \frac{K}{h}} \quad (\Delta)$$

USING (Δ) INTO (0)

$$Y = L^{1-\alpha} \int_0^h \underbrace{\left(\frac{K}{h}\right)^{\alpha}}_{\text{CONSTANT}} dj$$

$$= L^{1-\alpha} h \left(\frac{K}{h}\right)^{\alpha}$$

$$= L^{1-\alpha} K^{\alpha} h^{1-\alpha}$$

(1) $\boxed{Y} = \boxed{K^{\alpha} (hL)^{1-\alpha}}$

AGG. PROD. FUNCTION:

(1) $$Y = K^{\alpha}(hL)^{1-\alpha}$$

$$\hat{L} = n$$

CAPITAL STOCK:

(2) $$\dot{K} = s_K Y - dK$$

TECHNOLOGY IN LDC / # INT. GOODS THAT CAN BE USED IN LDC

(3) $$\dot{h} = \underbrace{\mu}_{\text{EASE OF ADOPTION}} . \underbrace{e^{\psi u}}_{\text{SCHOOLING}} \underbrace{A^{\gamma} . h^{1-\gamma}}_{\text{STOCKS OF } A, h}$$

u = TIME DEVOTED TO SKILL ACC ≈ YEARS OF SCHOOLING

$0 < \gamma < 1$

$\psi > 0$

$\mu > 0$: PARAMETER THAT REFLECT HOW EASY IS TO GET TECHNOLOGY FROM ABROAD.

$\Rightarrow$

(4) $$\hat{h} = \frac{\dot{h}}{h} = \mu e^{\psi u} \frac{A^{\gamma} h^{1-\gamma}}{h}$$

$$\boxed{\hat{h} = \mu e^{\psi u} . \left(\frac{A}{h}\right)^{\gamma}}$$

TASK: DESCRIBE SYSTEM AT BGP

AT BGP: $\hat{k}, \hat{Y}, \hat{h}$ CONSTANT

TAKE GROWTH RATES OF (1):

$$\hat{Y} = \alpha \hat{k} + (1-\alpha)[\hat{h} + \underbrace{\hat{L}}_{= n}]$$

(i) $$\hat{Y} = \alpha \hat{k} + (1-\alpha)[\hat{h} + n] \longrightarrow \text{ALWAYS}$$

AT BGP, $\hat{k} = s_Y \frac{Y}{k} - d$ CONSTANT

$$\Longrightarrow \boxed{\hat{Y} = \hat{k}} \quad \text{(ii)}$$

AT BGP, $\hat{h} = \underbrace{\mu e^{\psi \mu} \left(\frac{A}{h}\right)^{\gamma}}_{\text{CONSTANT}}$

$$\Longrightarrow \frac{A}{h} \text{ CONSTANT} \Longrightarrow \hat{A} = \hat{h}$$

SINCE $\hat{A} = g \Longrightarrow \boxed{\hat{h} = g}$ (iii)

USING (i)-(iii):

$$\hat{Y} = \alpha \hat{Y} + (1-\alpha)[g+n]$$

$$\cancel{(1-\alpha)}\hat{Y} = \cancel{(1-\alpha)}[g+n]$$

BGP $\Rightarrow \hat{Y} = g + n = \hat{K}$ BGP

$$\boxed{\hat{y}} = \widehat{\left(\frac{Y}{L}\right)} = \hat{Y} - \underset{\underset{n}{\parallel}}{\hat{L}} = g + n - n = \boxed{g}$$

$$\boxed{\hat{k}} = \widehat{\left(\frac{K}{L}\right)} = \hat{K} - \hat{L} = \boxed{g}$$

ALWAYS:

$$\hat{Y} = \alpha \hat{K} + (1-\alpha)\left[\hat{h} + n\right]$$

NOTICE $\hat{K} = s_K \frac{Y/L}{K/L} - d$

$$= s_Y \frac{Y/hL}{K/hL} - d$$

$$= s_Y \frac{\tilde{y}}{\tilde{k}} - d = \boxed{s_K \tilde{k}^{\alpha-1} - d}$$

WHERE $\tilde{y} = \frac{Y}{hL} = \tilde{k}^{\alpha} = \left(\frac{K}{hL}\right)^{\alpha}$

$$\underset{\text{ALWAYS}}{\hat{y}} = \alpha\underbrace{\left[s\,\cancel{k}\,\tilde{h}^{\alpha-1} - \delta \right]}_{\hat{k}} + (1-\alpha)\left[\hat{h} + n\right]$$

REMARK:

THIS SYSTEM HAS A S.S. IN THE MODIFIED VARIABLES:

$$\frac{Y}{hL}, \quad \frac{K}{hL}$$

SINCE WE SHOWED THAT AT BGP: $\hat{Y} = \underset{\hat{h}}{\overset{\|}{g}} + n$

$$\boxed{\text{WHAT IS THE OUTPUT PER WORKER } \sim\sim\text{AT}\sim\sim \text{ AT BGP?}}$$

$y^*(t)$

WE CAN USE:

$$\hat{h}_{\text{AT BGP}} = g \quad \text{OR} \quad \hat{\tilde{h}}_{\text{BGP}} = 0$$

$$\hat{h}_{BGP} = \hat{k} - n = \underbrace{r_k \tilde{h}^{\alpha-1} - d}_{\text{FROM PAGE (6)}} - n = g$$

$$\hat{h}_{BGP} = r_k \left(\frac{\tilde{h}}{h}\right)^{\alpha-1} - d - n = g$$

$$r_k \tilde{h}^{\alpha-1} h^{1-\alpha} = g + d + n$$

$$\frac{r_k}{g+d+n} h^{1-\alpha} = \tilde{h}^{1-\alpha}$$

$$\boxed{\tilde{h}_{BGP} = \left[\frac{r_k}{g+d+n}\right]^{\frac{1}{1-\alpha}} h}$$

OUTPUT PER WORKER:

$$\boxed{y} = \frac{Y}{L} = \frac{K^{\alpha}}{L^{\alpha}} \frac{(hL)^{1-\alpha}}{L^{1-\alpha}} = \boxed{\tilde{h}^{\alpha} h^{1-\alpha}}$$

$$y_{BGP} = \tilde{h}_{BGP}^{\alpha} h^{1-\alpha} = \left[\frac{r_k}{g+d+n}\right]^{\frac{\alpha}{1-\alpha}} h^{\alpha} h^{1-\alpha}$$

$$\boxed{y^{*}_{BGP}(t) = \left[\frac{s_K}{g+d+n}\right]^{\frac{\alpha}{1-\alpha}} \cdot h}$$ $\triangle$

WHAT IS h AT BGP?

AT BGP: $\hat{h}$ = CONSTANT = g

USE EQUATION (4)

$$g = \mu e^{\psi u} A^{\gamma} h^{-\gamma}$$

$$\Rightarrow$$

$$h^{\gamma}_{BGP} = \frac{\mu e^{\psi u} A^{\gamma}}{g}$$

$$\boxed{h_{BGP} = \left[\frac{\mu e^{\psi u}}{g}\right]^{\frac{1}{\gamma}} A(t)}$$ $\triangle\!\triangle$

$$\boxed{y^*_{BGP}(t) = \left[\frac{s_K}{g+\delta+n}\right]^{\frac{\alpha}{1-\alpha}} \cdot \left[\frac{\mu e^{\psi\mu}}{g}\right]^{\frac{1}{\gamma}} \cdot A(t)}$$

SIMILAR TO SOLOW H-K
CH. 3

$\uparrow \mu$ (SCHOOLING) $\Rightarrow \uparrow y^*_{BGP}(t)$

BUT NO GROWTH EFFECTS

$\uparrow \mu$: EASIER TO ADOPT FOREIGN TECHNOLOGY

$\uparrow y^*_{BGP}(t)$

BUT NO GROWTH EFFECT.

SUMMARY OF MODEL

(1) $Y = K^{\alpha}(hL)^{1-\alpha}$, $\hat{L} = n$, $\hat{A} = g$

$$\hat{K} = e_K \frac{Y}{K} - d$$

$0 < \gamma < 1$

$0 < \alpha < 1$

$$\hat{h} = \mu \, e^{\psi u} \left(\frac{A}{h}\right)^{\gamma}$$

$\mu, u > 0$

TAKING GROWTH RATES OF (1)

$$\hat{Y} = \alpha \hat{K} + (1-\alpha)\big[\hat{h} + \underbrace{\hat{L}}_{= n}\big] = \alpha \hat{K} + (1-\alpha)\big[\hat{h} + n\big]$$

$$= \alpha \hat{K} + (1-\alpha)\hat{h} + (1-\alpha) n$$

$$\Rightarrow \boxed{\hat{y}} = \widehat{\left(\frac{Y}{L}\right)} = \hat{Y} - n = \alpha \hat{K} + (1-\alpha)\hat{h} + (1-\alpha) n - n$$

$$= \alpha \underbrace{(\hat{K} - n)}_{= \widehat{\left(\frac{K}{L}\right)} = \hat{k}} + (1-\alpha)\hat{h} = \boxed{\alpha \hat{k} + (1-\alpha)\hat{h}}$$

AT BGP:

$$\hat{Y} = \hat{K} = g + n$$

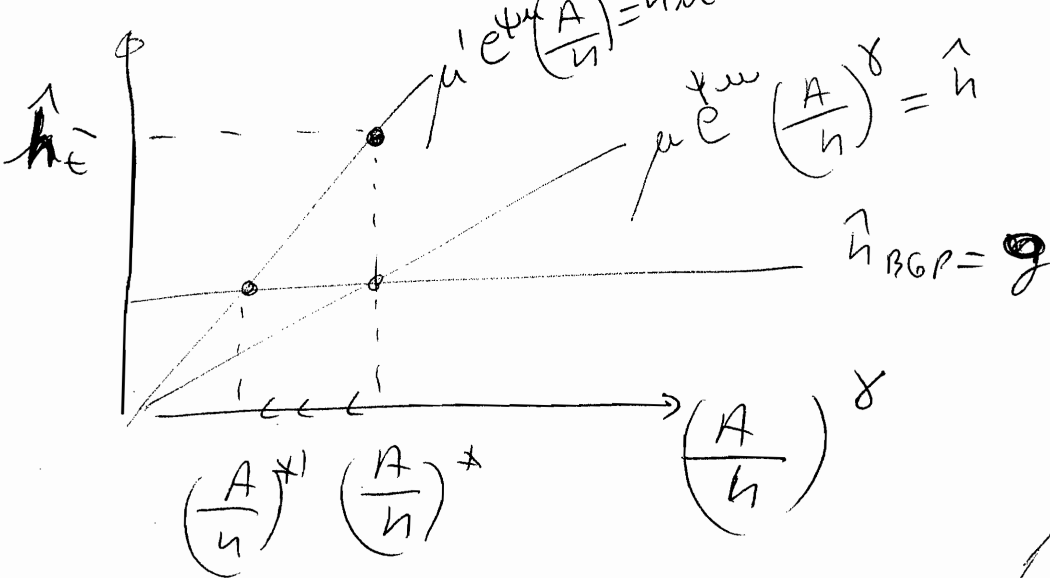
$$\hat{h} = g$$

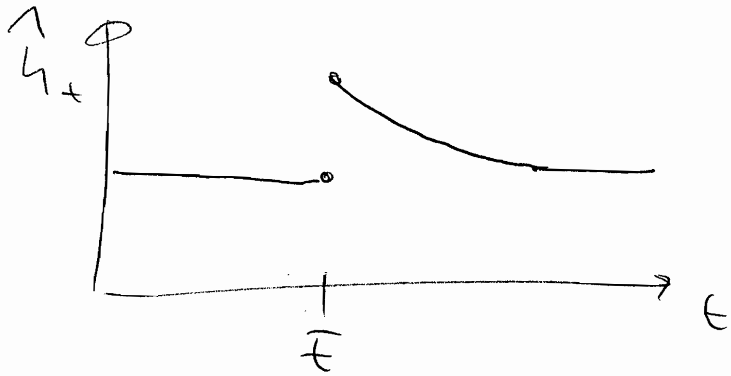
$$\hat{y} = \hat{k} = g$$

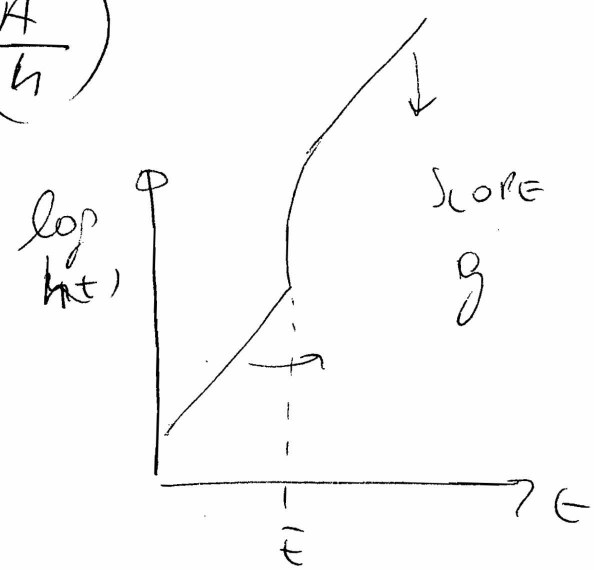
$$\boxed{y^{*}(t) = \left[\frac{e_K}{d+n+g}\right]^{\frac{\alpha}{1-\alpha}} \left[\frac{\mu e^{\psi u}}{g}\right]^{\frac{1}{\gamma}} A(t)}$$

COMPARATIVE STATICS: $\mu \uparrow$

ASSUME COUNTRY WAS AT BGP & AT $\bar{t}$: $\uparrow \mu$ (i.e. it is EASIER TO ADOPT TECHNOLOGY)







SINCE $\hat{y} = \alpha \underbrace{\hat{k}}_{\text{NO CHANGE}} + (1-\alpha)\hat{h} \Rightarrow$