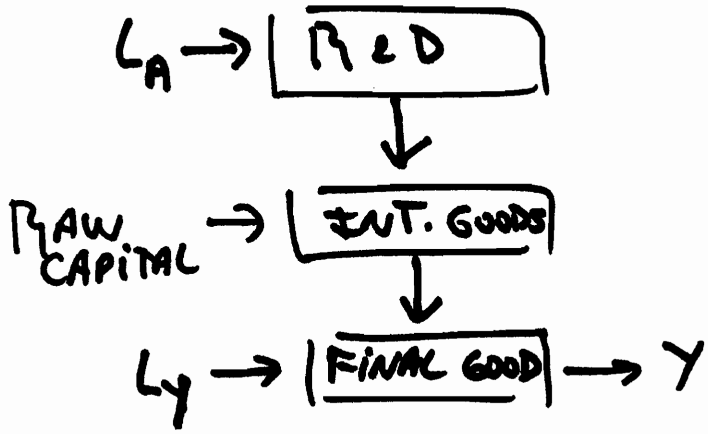
6-9-09

→ MICRO FOUNDATIONS
ROMER



TO SIMPLIFY ASSUME NO DEPRECIATION

$d = 0$

FINAL GOOD SECTOR

(1) $Y = L_Y^{1-\alpha} \int_0^A X_j^{\alpha} \, dj$

WHERE $0 < \alpha < 1$
X_j'S = DIFF. CAPITAL GOODS.
= INTERMEDIATE DIFF. GOODS.

PERFECT COMPETITION
OUTPUT: $Y \rightarrow$ PRICE OF $Y = 1$
INPUTS: LABOR : $L_y \rightarrow$ PRICE: W
INT : X_j'S $\rightarrow$ PRICE : p_j

PROFIT MAXIMIZATION IN P. COMPETITION
$\Longrightarrow$ INPUTS PAID VALUE OF MARGINAL PRODUCT -

ASIDE

x_j's : SPECIALIZED OR DIFFERENTIATED CAPITAL GOODS

CONSIDER:

$$F(x_1, \ldots x_A) = x_1^{\alpha} + x_2^{\alpha} + x_3^{\alpha} + \ldots + x_A^{\alpha}$$

WHERE $0 < \alpha < 1$

$A \approx$ # OF DIFF. CAPITAL GOODS.

EFFICIENCY REQUIRES THAT FIRM BUY SAME QUANTITY OF ALL AVAILABLE CAPITAL GOODS (x_i's) ASSUMING THEY COST THE SAME $\Rightarrow$

$$x_1 = x_2 = \ldots\ldots = x_A = x$$

$$\Rightarrow F(x_1, x_2 \ldots, x_A) = \underbrace{x^{\alpha} + x^{\alpha} + \ldots + x^{\alpha}}_{A}$$

$$= \boxed{A\, x^{\alpha}}$$

LABOR:

$$MPL_Y = \frac{\partial Y}{\partial L_Y} = (1-\alpha) L_Y^{1-\alpha-1} \int_0^A x_j^\alpha \, dj$$

$$= \frac{(1-\alpha)}{L_Y} \underbrace{L^{1-\alpha} \int_0^A x_j^\alpha \, dj}_{Y}$$

$$= \boxed{(1-\alpha) \frac{Y}{L_Y}}$$

PROFIT MAX $\Longrightarrow$

$$\boxed{w = \underbrace{(1-\alpha) \frac{Y}{L_Y}}_{\text{JUST MPL SINCE } P_Y = 1}} \quad (12)$$

INTERMEDIATES

$$MPX_j = L_Y^{1-\alpha} \cdot \alpha x_j^{\alpha-1}$$

$$= \boxed{\alpha L_Y^{1-\alpha} x_j^{\alpha-1}}$$

PROFIT MAX $\Longrightarrow$

$$\boxed{p_j = \underbrace{\alpha L_Y^{1-\alpha} x_j^{\alpha-1}}_{\text{JUST MPX}_j \text{ SINCE } p_Y = 1}} \quad (13)$$

QUESTION:

WHAT IS THE DEMAND ELASTICITY FOR x_j?

DEMAND ELASTICITY
FOR x_j :

$$\frac{\partial x_j}{\partial p_j} \cdot \frac{p_j}{x_j} = \frac{\frac{\partial x_j}{x_j}}{\frac{\partial p_j}{p_j}} = \frac{\hat{x}_j}{\hat{p}_j}$$

FROM EQUATION (13), TAKING GROWTH RATES :

$$\hat{p}_j = \underbrace{\widehat{\left[\alpha L_Y^{1-\alpha}\right]}}_{=0} + \widehat{\left(x_j^{\alpha-1}\right)}$$

$$\hat{p}_j = (\alpha - 1)\,\hat{x}_j \implies \boxed{\frac{\hat{x}_j}{\hat{p}_j} = \boxed{\frac{1}{\alpha-1}}}$$

ELASTICITY

OR $\boxed{\frac{\hat{p}_j}{\hat{x}_j} = \alpha - 1}$ INVERSE OF ELASTICITY

INTERMEDIATE GOODS SECTOR

$\underbrace{\text{1 UNIT OF RAW CAPITAL}}_{\text{RENTED AT } r} + \underbrace{\text{1 BLUEPRINT/DESIGN}}_{\text{BOUGHT AT PRICE } P_A} = \underbrace{\text{1 UNIT OF INT. GOOD/DIFF. CAPITAL } X_j}_{\text{SOLD AT } P_j}$

REMARK: A SINGLE BLUEPRINT CAN BE USED TO PRODUCE ANY NUMBER OF UNITS OF X_j

COST FUNCTION FOR FIRM j (PRODUCING X_j) $= P_A + r \cdot X_j = C_j(X_j)$

$P_A \downarrow$ FIXED COST

1 UNIT OF RAW CAPITAL $\longrightarrow$ $\boxed{\text{BLUEPRINT FOR } X_j}$ $\longrightarrow$ 1 UNIT OF X_j

ASSUME MONOPOLIST COMPETITION

$\Longrightarrow$ $MRx_j = MC_j$

$\Longrightarrow$ ZERO PROFITS OR

$$\text{PROD. PROFITS} = \underbrace{\text{PRICE OF BLUEPRINT/DESIGN}}_{P_A}$$

MARGINAL REVENUE:

$$MRx_j = p_j\left(1+\frac{1}{\varepsilon_j}\right) = p_j\left[1+\frac{\hat{p}_j}{\hat{x}_j}\right]$$

ELAST. OF DEMAND

WHERE $\varepsilon_j = \dfrac{\hat{x}_j}{\hat{p}_j}$

USING WORK FROM PAGE 3:

$$\frac{\hat{p}_j}{\hat{x}_j} = \alpha - 1 \Longrightarrow$$

$$\boxed{MRx_j} = p_j\left[1+\alpha-1\right] = \boxed{\alpha\, p_j}$$

WE SET $MRx_j = \underbrace{MC_j}_{\Omega}$

$\Longrightarrow$

$$\underbrace{\alpha p_j}_{MAX_j} = \underbrace{r}_{MC_j}$$

$$\Rightarrow \boxed{p_j = \underbrace{\frac{1}{\alpha}}_{\text{BIGGER THAN ONE i.e. MARK-UP}} r} \quad (14)$$

<u>IMPLICATION</u> :

SINCE r IS THE SAME FOR EVERY FIRM $\Rightarrow$

$p_j = p$ FOR EVERY X_j

ALL INT. HAVE ~~THE~~ SAME PRICE : $\boxed{\frac{1}{\alpha} r = p}$ (14)

OR $\boxed{r = \alpha p}$ (14')

$\Rightarrow$ DEMAND FOR EACH INTERMEDIATE IS THE SAME $\Rightarrow X_j = X$

REMARK:

FIRM RENT GENERIC/RAW CAPITAL FOR 1 PERIOD AT A RENTAL PRICE "r".

WHAT IS THE TOTAL DEMAND FOR RAW CAPITAL? HOW MANY UNITS?

$$\underbrace{\int_0^A X_j}_{\approx \sum_0^A X_j} = A \cdot X$$

BECAUSE FIRMS TRANSFORM 1 UNIT OF RAW CAPITAL INTO 1 UNIT OF X_j

IF FIRMS IDENTICAL $X_j = X$

TOTAL SUPPLY OF RAW CAPITAL IS K

$\Longrightarrow$ FEASIBILITY/EQUILIBRIUM: $\boxed{AX = K}$ OR $\boxed{X = \frac{K}{A}}$

WILL SHOW THAT (1) [PROD. FUNCTION OF FINAL GOODS SECTOR] COINCIDES WITH: $\cancel{Y = L_Y^{1-\alpha} A}$

$$Y = K^{\alpha} (A L_Y)^{1-\alpha}$$

PROOF:

$$\boxed{Y} = L_Y^{1-\alpha} \int_0^A x_j^{\alpha}\, dj$$

$$\underset{\text{SINCE } x_j = X}{=} \boxed{L_Y^{1-\alpha} A \cdot X^{\alpha}} \quad (15)$$

$$\underset{\text{SINCE } X = \frac{K}{A}}{=} L_Y^{1-\alpha} A \left(\frac{K}{A}\right)^{\alpha}$$

$$= L_Y^{1-\alpha} K^{\alpha} A^{1-\alpha}$$

$$\boxed{Y} = \boxed{K^{\alpha} (L_Y A)^{1-\alpha}}$$

WE CAN WRITE (13) IN A DIFFERENT FORM:

$$(13)\ p_j = \underbrace{\alpha\, X_j^{\alpha-1} L_Y^{1-\alpha}}_{\frac{\partial Y}{\partial X_j} = MPX_j} =$$

$$\underset{\substack{\text{SINCE}\\ X_j = X = \frac{K}{A}}}{=} \alpha\, L_Y^{1-\alpha} \left(\frac{K}{A}\right)^{\alpha-1}$$

$$= \frac{\alpha}{K} L_Y^{1-\alpha} A^{1-\alpha} K^{\alpha}$$

$$= \frac{\alpha}{K} \underbrace{K^{\alpha} (L_Y A)^{1-\alpha}}$$

$$\boxed{p_j = p} = \boxed{\alpha \frac{Y}{K}}$$

PRODUCTION PROFIT FOR A FIRM IN THE INTERMEDIATE SECTOR $= \pi$ (i.e. EXCLUDES DESIGN/BLUEPRINT COST)

REMARK:

"ZERO PROFIT CONDITION" IS NOT THE SAME AS SETTING $\pi = 0$, WE NEED TO PAY FOR THE BLUEPRINT -

THE "FREE ENTRY / ZERO PROFIT CONDITION" IS MORE COMPLICATED -

PROD. PROFITS FOR ONE PERIOD $= \boxed{\pi = \alpha(1-\alpha)\frac{Y}{A}}$ (16)

SHOW USING (15), (13), (14')

BIG QUESTION: P_A? BLUEPRINT PRICE?

R&D SECTOR

$$L_Y \longrightarrow \underbrace{\text{DESIGN}}_{P_A}$$

P_A?

ARBITRAGE CONDITION

MONEY IN THE BANK

INVEST IN BUYING A BLUEPRINT/DESIGN

RETURNS HAVE TO BE THE SAME ~~IF~~

BOTH INVESTMENTS ARE TAKING PLACE

COST OF BLUEPRINT : P_A

$$\underbrace{r P_A}_{\text{PUT } P_A \text{ IN A "BANK" FOR 1 PERIOD.}} = \underbrace{\pi + \dot{P}_A}_{\text{RETURN OF BUYING A BLUEPRINT \& RESELLING IT AFTER 1 PERIOD}}$$

$\Rightarrow$

$$r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A}$$

$$\boxed{r = \frac{\pi}{p_A} + \hat{p}_A} \quad (17)$$

ALONG BGP: $\hat{Y}$, $\hat{K}$, $\hat{L} = n$

$\hat{p}_A$ CONSTANT

$\Rightarrow$ WILL GET IMPLICATIONS NEXT TIME:

$$\boxed{p_A = \frac{\pi}{r-n}}$$