Econ 475 M. Muniagurria

Answer Key Problem Set 1

(I)

a - This production function has constant returns to scale because:

$$F(\lambda K, \lambda L) = (\lambda K)^{1/2} (\lambda L)^{1/2} = \lambda^{1/2} K^{1/2} \lambda^{1/2} L^{1/2} = \lambda K^{1/2} L^{1/2} = \lambda F(K,L),$$

for any λ different from zero. In other words, we have shown that $\% \Delta K = \% \Delta L = \% \Delta Y$. Alternative less formal explanations are acceptable.

b- The per-worker production function is: $f(k) = k^{1/2}$ since,

$$y = F(K, L) / L = (K^{1/2} L^{1/2}) / L = (K/L)^{1/2} (L/L)^{1/2} = k^{1/2}\, 1^{1/2} = k^{1/2}$$

c- We assume that :

$\delta = 0.05$

Savings rate in country A = $s^A = 0.10$

Savings rate in country B = $s^B = 0.20$

At BGP: $s\, f(k^*) = \delta\, k^*$ (see explanation in I above before equation (3)) so we can use this equation , the relevant savings rate and our particular functional form for f(k) to calculate k* for each country.

Country A:

$s^A f(k) = \delta\, k$ can be written as : $0.10\ k^{1/2} = 0.05\ k$

We solve for k:

$0.10 / 0.05 = k\, k^{-1/2}$

$2 = k^{1/2}$

$4 = k$

Then at the BGP :

Capital per worker : $k^{A*} = 4$

Income per worker: $y^{A*} = f(k^{A*}) = 4^{1/2} = 2$

Consumption per worker: $c^{A*} = (1-s^A)\, y^{A*} = 0.90 \cdot 2 = 1.8$

Country B:

$s^B f(k) = \delta\, k$ can be written as : $0.20\ k^{1/2} = 0.05\ k$

We solve for k:

$0.20 / 0.05 = k\, k^{-1/2}$

$4 = k^{1/2}$

$16 = k$

Then at the BGP :

Capital per worker : $k^{B*} = 16$

Income per worker: $y^{B*} = f(k^{B*}) = 16^{1/2} = 4$

Consumption per worker: $c^{B*} = (1-s^B)\, y^{B*} = 0.80 \cdot 4 = 3.2$

d - Suppose at time zero (t=0) both countries have 2 units of capital per worker:

$$k_0^A = k_0^B = 2$$

time	k^A	y^A	c^A	k^B	y^B	c^B
0	2.00	1.414	1.2730	2.00	1.414	1.1314
1	2.04	1.428	1.2850	2.18	1.477	1.1819
2	2.08	1.442	1.2980	2.37	1.539	1.2313
3	2.12	1.456	1.3104	2.56	1.599	1.2800
4	2.16	1.469	1.3227	2.75	1.658	1.3267

It will take four years for consumption in country B to exceed that of country A.

We calculated the capital stock per worker in different periods as:
$k_{t+1} = k_t + \Delta k = k_t + s \,.\, f(k_t) - \delta\, k_t = k_t + s \,.\, k_t^{1/2} - 0.05\, k_t$, so at time 1 in Country A for instance,
$k^A_1 = 2.00 + 0.10 \,.\, 2^{1/2} - 0.05 \,.\, 2 = 2.04$
Income per worker (y) is just $f(k) = k^{1/2}$, so : $y^A_1 = 2.04^{1/2} = 1.428$ Consumption per worker c is
$c = (1\text{-}s) \,.f(k)$, so: $c^A_1 = 0.90 \,.\, 1.428 = 1.285$

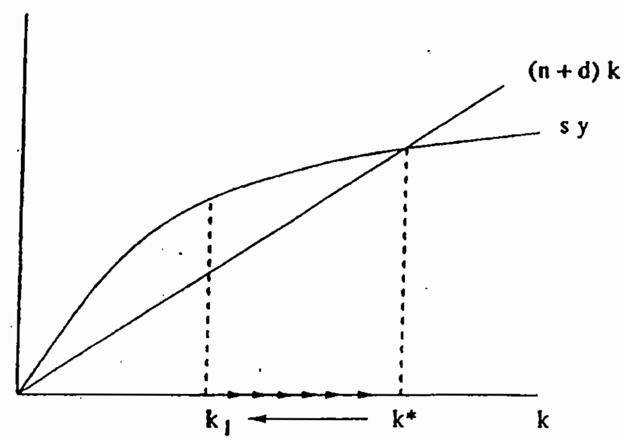
(II) Problems 1, 2, 3 in Jones pages 51 and 52

Exercise 2 *An increase in the labor force.*

The key to this question is to recognize that the initial effect of a sudden increase in the labor force is to reduce the capital-labor ratio since $k \equiv K/L$ and K is fixed at a moment in time. Assuming the economy was in steady state prior to the increase in labor force, k falls from k^* to some new level k_1. Notice that this is a movement *along* the sy and $(n+d)k$ curves rather than a *shift* of either schedule: both curves are plotted as functions of k, so that a change in k is a movement along the curves. (For this reason, it is somewhat tricky to put this question first!)

At k_1, $sy > (n+d)k_1$, so that $\dot{k} > 0$, and the economy evolves according to the usual Solow dynamics, as shown in Figure 1.

Figure 1: An Increase in the Labor Force



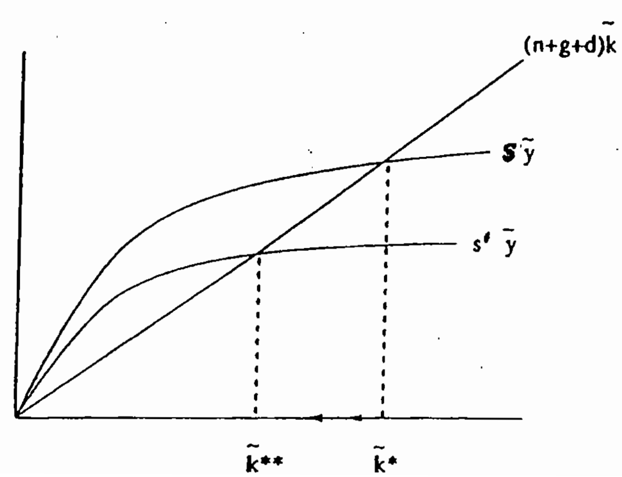
In the short run, per capita output and capital drop in response to a inlarge flow of workers. Then these two variables start to grow (at a decreasing rate), until in the long run per capita capital returns to tne original level, k^*. In the long run, nothing has changed!

Exercise 1. *A decrease in the investment rate.*

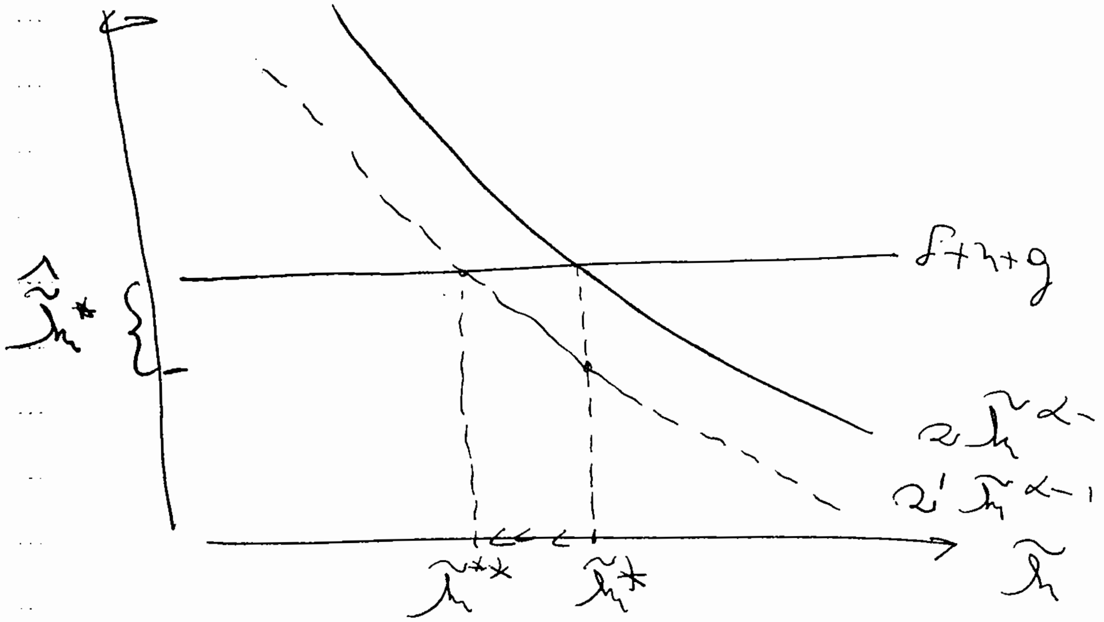
A decrease in the investment rate causes the $s\tilde{y}$ curve to shift down: at any given level of $\tilde{k}$, the investment-technology ratio is lower at the new rate of saving/investment.

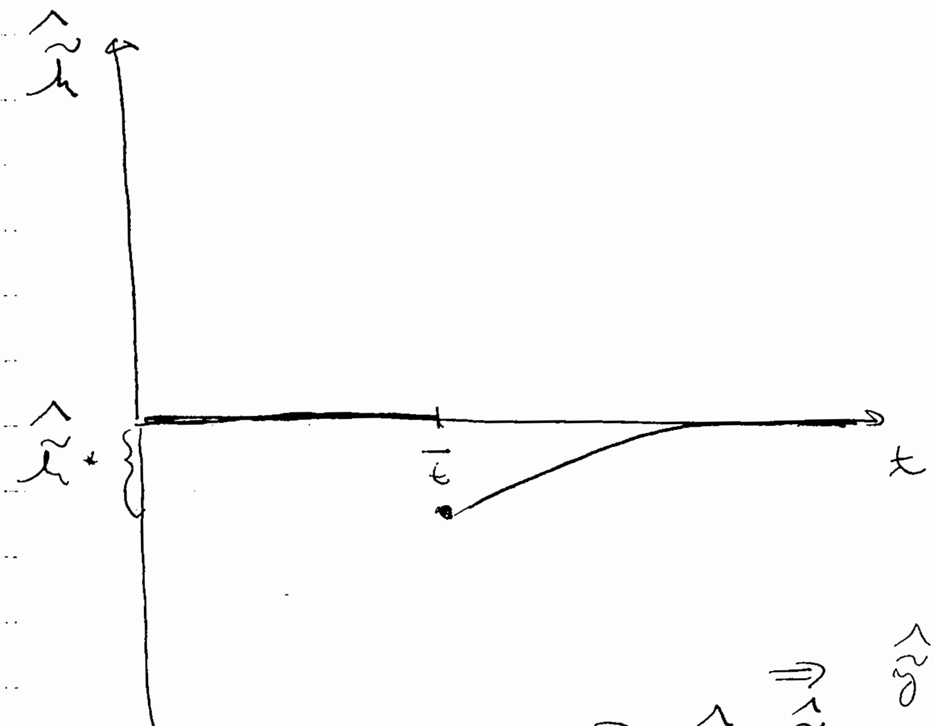
Assuming the economy began in steady state, the capital-technology ratio is now higher than is consistent with the reduced saving rate, so it declines gradually, as shown in Figure 2.

Figure 2: A Decrease in the Investment Rate



SINCE $\hat{\tilde{k}} = \frac{\dot{\tilde{k}}}{\tilde{k}} = s\tilde{k}^{\alpha-1} (n+g+d)$





SUPPOSE $s \downarrow$ AT TIME $\bar{t}$

& SINCE $\tilde{y} = \tilde{k}^{\alpha}$

$\Rightarrow$ $\boxed{\hat{\tilde{y}} = \alpha \hat{\tilde{k}}}$

BECAUSE $\tilde{y} = \frac{y}{A}$

$\Rightarrow \hat{\tilde{y}} = \hat{y} - \underset{= g}{\hat{A}}$

$\Rightarrow \hat{y} = \hat{\tilde{y}} + g$

$= \alpha \hat{\tilde{k}} + g$

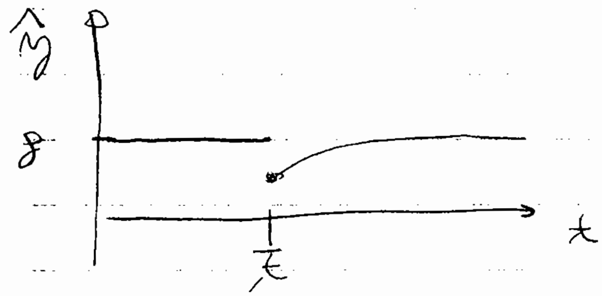
AT $t = \bar{t}$, $\hat{k} = \hat{k}^*$, $\hat{y}^* = \alpha \underbrace{\hat{\tilde{k}}^*}_{\text{NEGATIVE}} + g$

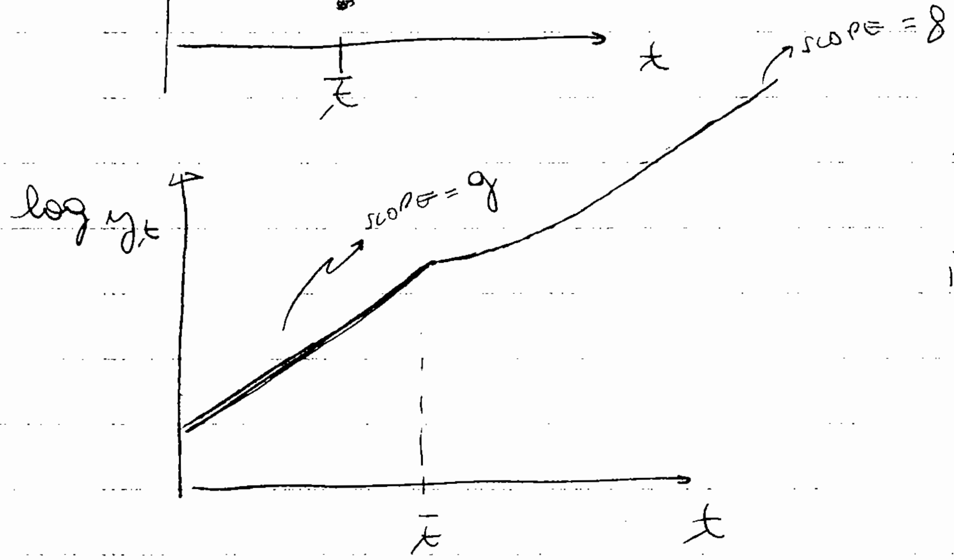
SO WE DON'T KNOW IN PRINCIPLE IF

$\hat{y}^* \gtrless 0$

ASSUME AS SPECIFIED IN THE HOMEWORK THAT

$\hat{y}^* > 0$ (i.e. $|\alpha \hat{x}^*| < g$)





NOTICE SLOPE OF $\log y_t$ IS THE GROWTH RATE OF y_t

i.e.

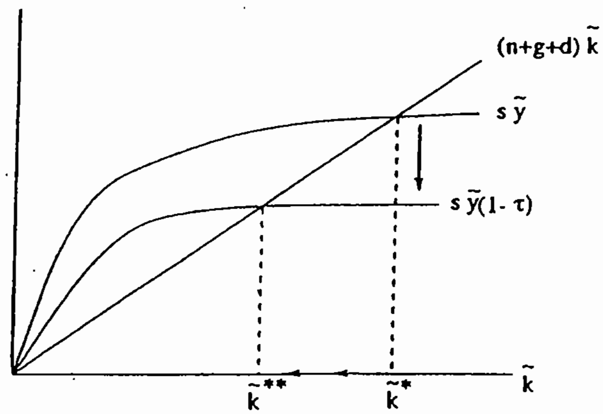
$$\hat{y} = \frac{d \log y_t}{dt}$$

AT $\bar{t}$, THE SLOPE CHANGES (IT DROPS TO A POSITIVE VALUE) & STARTS TO INCREASE UNTIL IT REACHES A VALUE OF g AGAIN –

Exercise 3. *An income tax.*

Assume that the government throws away the resources it receives in taxes. Then an income tax reduces the total amount available for investing and shifts the investment curve down as shown in Figure 5.

Figure 5: An Income Tax



The tax policy permanently reduces the level of output per worker, but the growth rate per worker is only temporarily lowered. Notice that this experiment has basically the same results as that in Exercise 2.