ECON 475
5509

EXAM 2

(I) $Y = K^{\alpha}(AhL)^{1-\alpha}$  $\hat{A} = g$  $\hat{L} = n$

$\dot{K} = s_K Y - \delta K$

$\tilde{y} = \frac{Y}{AL h}$  $\tilde{k} = \frac{K}{AL h}$

$\dot{\tilde{k}} = \dot{\left(\frac{k}{Ah}\right)} = s_K \tilde{k}^{\alpha} - (\delta + g + n)\tilde{k}$

or

$\hat{\tilde{k}} = s_K \tilde{k}^{\alpha - 1} - (\delta + g + n)$

(1) (ΔΔ) $\hat{Y} = \alpha \hat{K} + (1-\alpha)[\underbrace{\hat{A}}_{g} + \underbrace{\hat{h}}_{0} + \underbrace{\hat{L}}_{n}] = \alpha \hat{K} + (1-\alpha)[g+n]$

ALSO

$\hat{K} = s_K \frac{Y}{K} - \delta$  (Δ)

BGP ⟹ $\hat{Y}, \hat{K}, \ldots$ CONSTANT

THEN (Δ) ⟹ $\hat{Y} = \hat{K}$

USING (ΔΔ) ⟹

$\hat{Y} = (1-\alpha)\hat{Y} + (1-\alpha)[g+n]$

$\hat{Y}(\cancel{1-\alpha}) = [g+n](\cancel{1-\alpha})$

THEN

ALONG BGP : $\hat{Y} = g + n = \hat{K}$

$\boxed{\hat{y}} = \widehat{\left(\frac{Y}{L}\right)} = \hat{Y} - \underbrace{\hat{L}}_{n} = g + n - n = \boxed{g}$

$\boxed{\hat{k}} = \underbrace{\hat{K}}_{\hat{Y}} - \hat{L} = g + n - n = \boxed{g}$

(2) $\tilde{h}$ AT BGP

AT BGP $\boxed{\hat{\tilde{h}}} = \widehat{\left(\frac{k}{ALh}\right)} = \hat{k} - \hat{A} - \hat{L} =$

$= \hat{k} - (g+n) = 0$

WE USE THE EQUATION GIVEN IN EXAM:

$$\Rightarrow \hat{\tilde{h}} = \frac{\dot{\tilde{h}}}{\tilde{h}} = \frac{a_k \tilde{h}^{\alpha}}{\tilde{h}} - \frac{(\delta+n+g)\tilde{h}}{\tilde{h}}$$

$$= a_k \tilde{h}^{\alpha-1} - (\delta+n+g) = 0 \quad \text{AT BGP}$$

$$\Longrightarrow \tilde{h}^{1-\alpha} = \frac{a_k}{\delta+n+g}$$

$$\Rightarrow \tilde{h} = \left[\frac{a_k}{\delta+n+g}\right]^{\frac{1}{1-\alpha}}$$

AS $\tilde{h} = \frac{h}{Ah} = \frac{k}{AhL} \Longrightarrow h = \tilde{h}\, A \cdot h.$

SO AT BGP

$$\boxed{h^*(t) = \left[\frac{a_k}{\delta+n+g}\right]^{\frac{1}{1-\alpha}} A(t) \cdot h}$$

$\boxed{\text{P } n}$

(3) $\hat{\tilde{k}}$ PATH? $\tilde{k}$ PATH ?

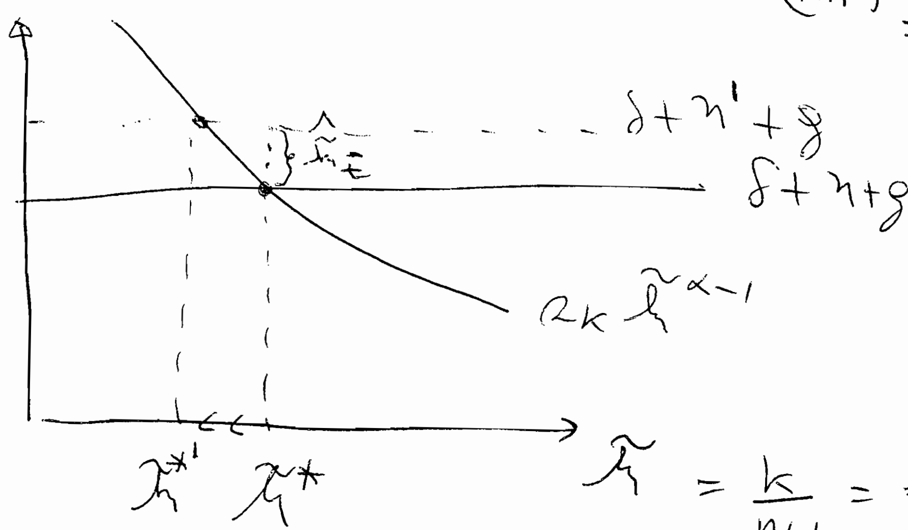
FROM EXAM QUESTIONS:

$$\dot{\tilde{k}} = s_K \tilde{k}^{\alpha} - (\delta + n + g)\tilde{k}$$

$$\Rightarrow \boxed{\hat{\tilde{k}} = s_K \tilde{k}^{\alpha - 1} - (\delta + n + g)} \text{ ALWAYS}$$

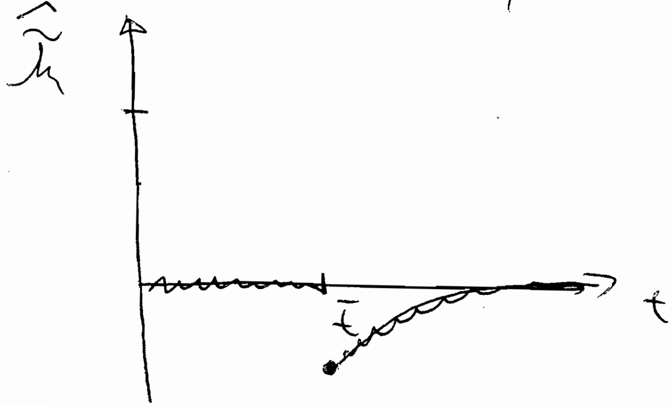
FROM QUESTIONS (1) & (2) $\Rightarrow$ $\hat{k} = g$ AT BGP

$$\hat{\tilde{k}} = \widehat{\left(\frac{k}{Ah}\right)} = \hat{k} - \hat{A} = \hat{k} - g = 0$$




$$\tilde{k} = \frac{K}{AhL} = \frac{k}{Ah}$$

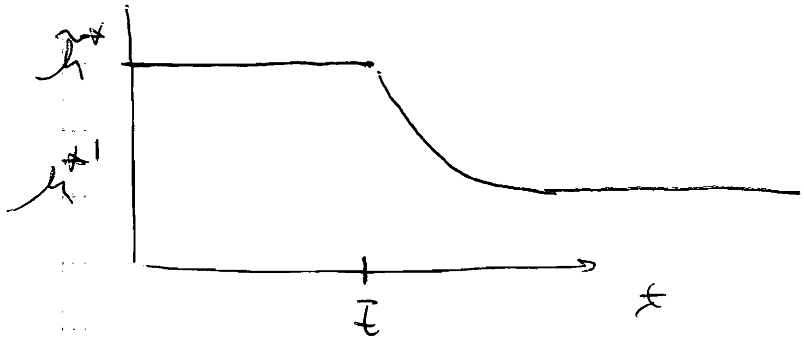
AT $\bar{t}$ : $\tilde{k} = \tilde{k}^{*}$ , AT NEW S.S. $\tilde{k} = \tilde{k}^{*\prime}$




UNTIL $\bar{t}$: $\hat{\tilde{k}} = 0$

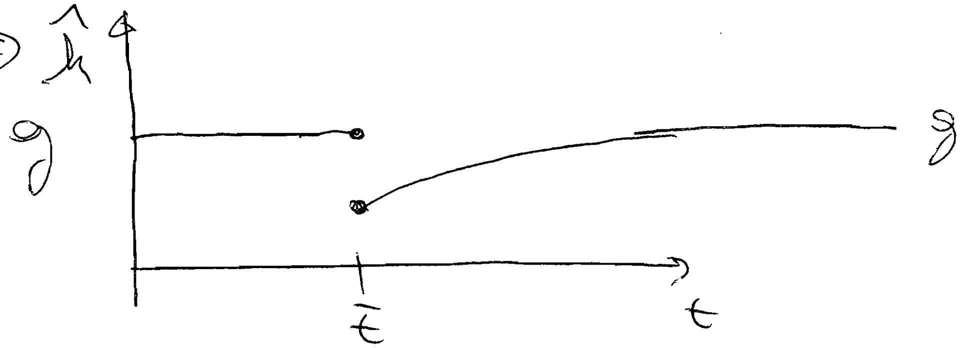
AT $\bar{t}$: $\hat{\tilde{k}} < 0$

AT NEW S.S. $\hat{\tilde{k}} = 0$




(4) $\hat{h} = \widehat{(\tilde{h}\cdot A \cdot h)} = \hat{\tilde{h}} + g$

$\Rightarrow$

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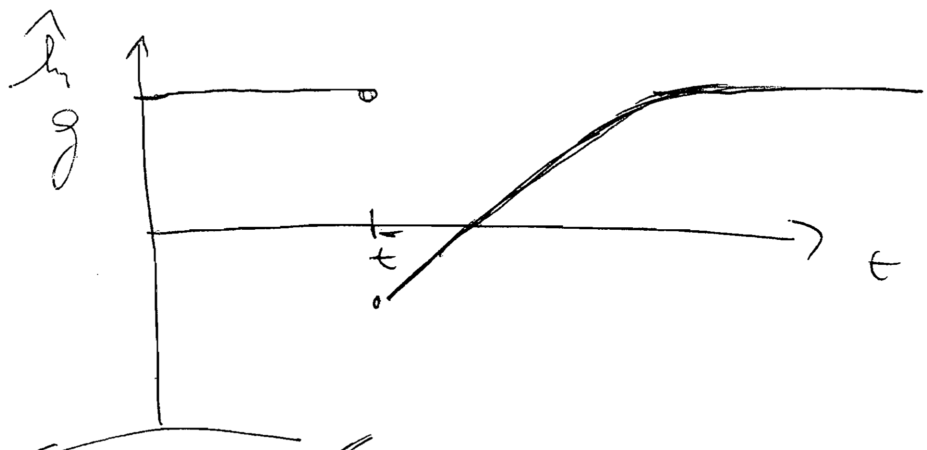


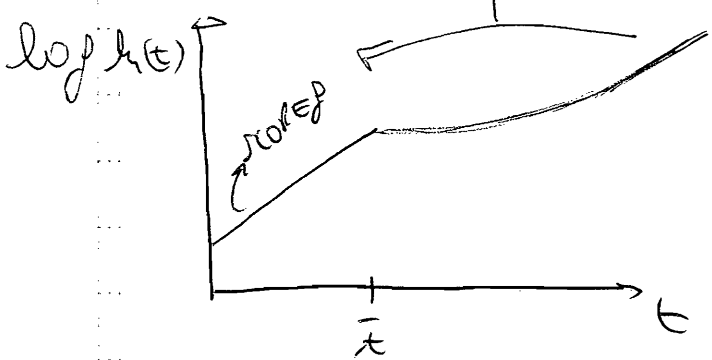

(II) (1) $Y = K^{\alpha}(A\,L_Y)^{1-\alpha}$

(2) $\dot{A} = \delta A^{\phi} L_A^{\lambda}$

(3) $\dot{K} = s_K Y - dK$

(4) $\hat{k} = s_K\, s_Y^{1-\alpha}\left(\frac{k}{A}\right)^{\alpha-1} - (n+d)$

(1)

e $\hat{Y} = \alpha\hat{K} + (1-\alpha)\,[\hat{A} + \underbrace{\hat{L}_Y}_{n}] = \alpha\hat{K} + (1-\alpha)[\hat{A}+n]$

$\boxed{\hat{y}} = \widehat{\left(\frac{Y}{L}\right)} = \hat{Y} - \hat{L} = \hat{Y} - n = \alpha\hat{K} + (1-\alpha)(\hat{A}+n) - n$

$= \alpha\hat{K} + (1-\alpha)\hat{A} - \alpha n$

$= \alpha(\hat{K} - n) + (1-\alpha)\hat{A}$

5 $\Longrightarrow \hat{y} = \alpha\,\widehat{\left(\frac{K}{L}\right)} + (1-\alpha)\hat{A} = \boxed{\alpha\,\hat{k} + (1-\alpha)\hat{A}}$

USING (2):

a $\boxed{\hat{A}} = \dfrac{\delta A^{\phi} L_A^{\lambda}}{A} = \boxed{\dfrac{\delta L_A^{\lambda}}{A^{1-\phi}}}$

(2) ALONG BGP: $\hat{Y}, \hat{K}, \hat{A}$ CONSTANT

cf. (3) $\Longrightarrow \hat{K} = s_K \dfrac{Y}{K} - d \quad \Longrightarrow \dfrac{Y}{K}$ CONSTANT

$\Longrightarrow \boxed{\hat{Y} = \hat{K}}$ c

$\Longrightarrow \hat{Y} - n = \boxed{\hat{y}} = \hat{K} - n = \boxed{\hat{k}}$ d

METHOD 1:

USING (a) & (c)

$$\hat{Y} = \alpha \underbrace{\hat{K}}_{=\hat{Y}} + (1-\alpha)(\hat{A}+n)$$

$$\Rightarrow \hat{Y}(1-\alpha) = (1-\alpha)(\hat{A}+n)$$

$$\Rightarrow \hat{Y} = \hat{A}+n \Rightarrow \widehat{\left(\frac{Y}{L}\right)} = \hat{y} = \hat{Y} - \underbrace{\hat{L}}_{=n}$$

$$= \boxed{\hat{A}}$$

$\hat{y} = \hat{A}$ AT BGP

METHOD 2

USING (b) & (d)

$$\hat{y} = \alpha \underset{\hat{y}}{\underset{\|}{\hat{k}}} + (1-\alpha)\hat{A} \Rightarrow \hat{y}(1-\alpha) = (1-\alpha)\hat{A}$$

$$\boxed{\hat{y} = \hat{A}}$$

CAPITAL PER WORKER: $\hat{k}$

GROWTH AT BGP.

$$(b) \Rightarrow \quad \alpha\hat{k} = \underset{\hat{A}}{\underset{\|}{\hat{y}}} - (1-\alpha)\hat{A} = \alpha\hat{A}$$

$$\Rightarrow \boxed{\hat{k} = \hat{A}}$$

$\hat{A}$ AT BGP

FROM PREVIOUS PAGE (a): $\hat{A} = \dfrac{\delta L_A^{\lambda}}{A^{1-\phi}}$

AT BGP $\hat{A}$ CONSTANT $\Rightarrow$ $\widehat{NUM} = \widehat{DEN}$

$$\widehat{\left(\delta L_A^{\lambda}\right)} = \widehat{\left(A^{1-\phi}\right)}$$

$$\lambda \hat{L}_A = (1-\phi)\,\hat{A} \implies \boxed{\hat{A}_{BGP} = \frac{\lambda n}{1-\phi}}$$

$$\hat{L}_A = n$$

$\boxed{\text{CONCLUSION}}$ AT BGP : $\boxed{\hat{y} = \hat{k} = \hat{A} = \frac{\lambda n}{1-\phi}}$

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COMP. STATICS

NOW: $\lambda = 1 \quad \phi = 0$

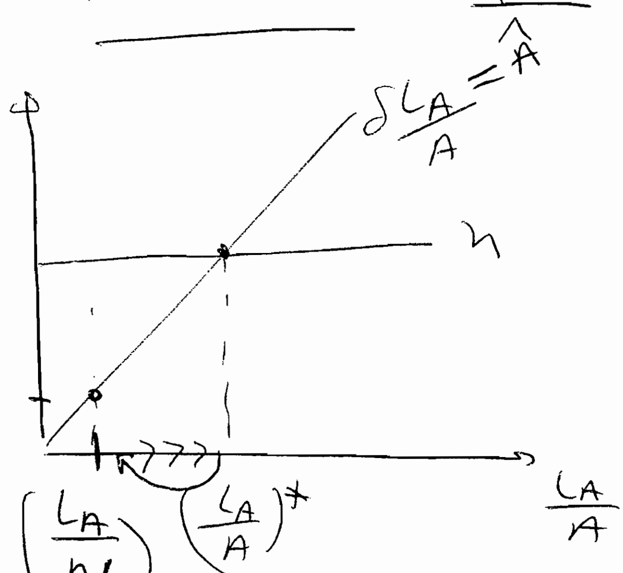
$\implies$

$$\hat{A} = \delta \frac{L_A}{A} \qquad \text{EVERYWHERE}$$

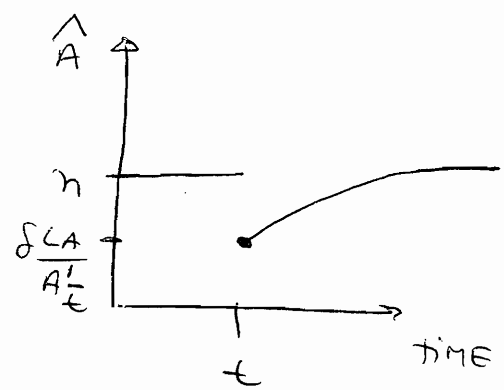
$$\hat{A}_{BGP} = n$$

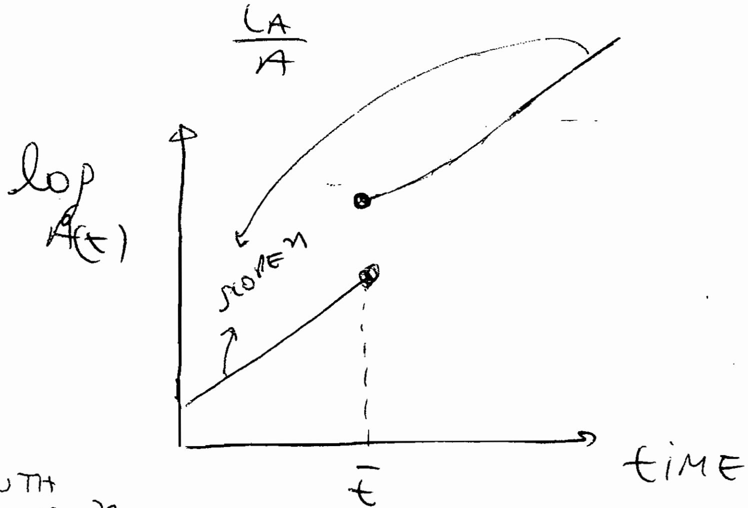
BLUEPRINT GIFT : ↑A AT $\bar{t}$

(3)




$$\delta \frac{L_A}{A'_{\bar{t}}} = \hat{A}_{\bar{t}}$$



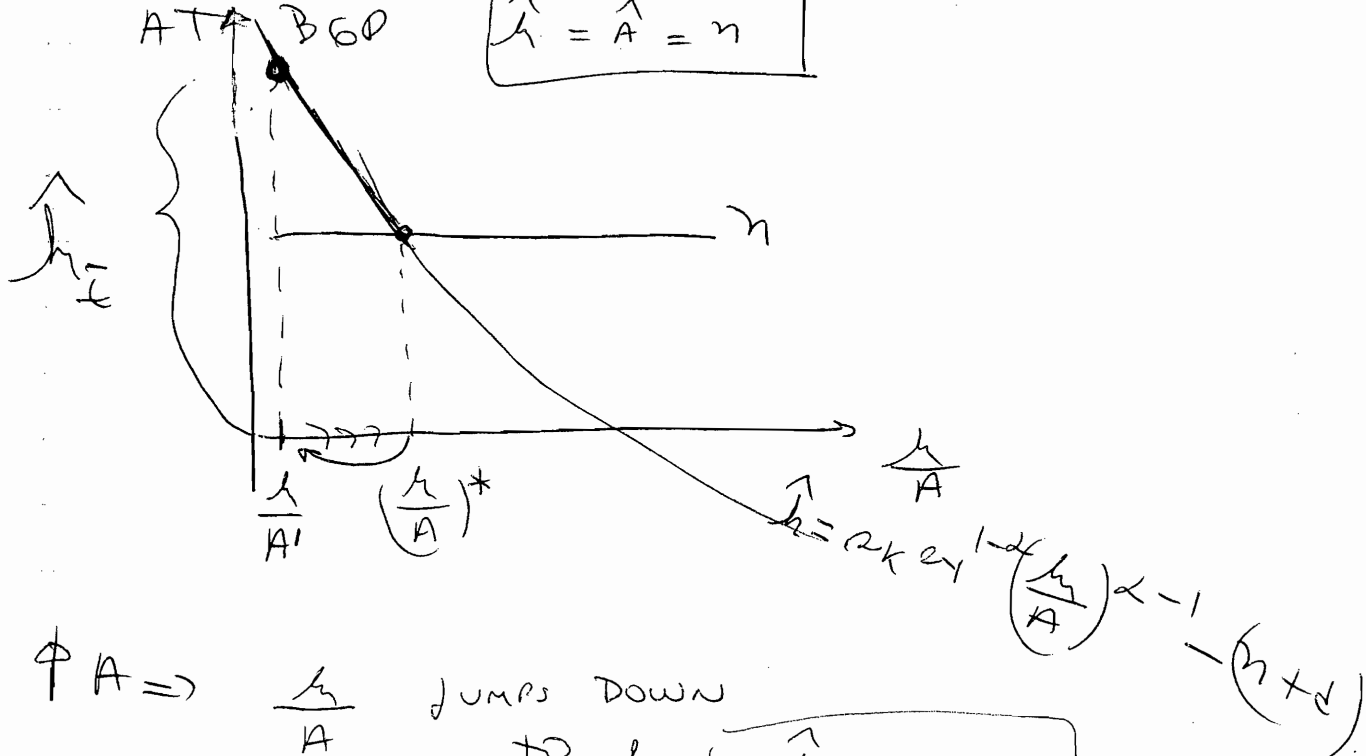

↑A ⇒ MORE BLUEPRINT
↑ LEVEL AT $\bar{t}$
BUT RATE OF GROWTH SMALLER
THEN BACK TO GROWTH RATE OF $n$

(4) $k$ & $\hat{k}$

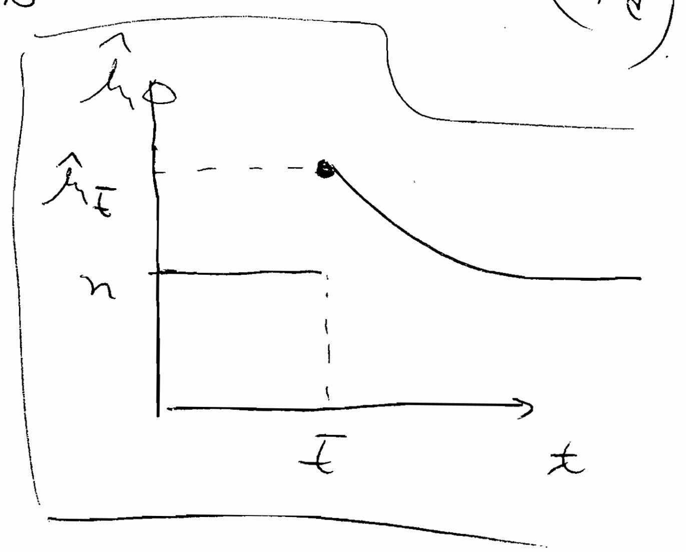
FROM EXAM INFORMATION

$$\hat{k} = r_K \, r_Y^{1-\alpha} \left(\frac{k}{A}\right)^{\alpha-1} - (n+d)$$

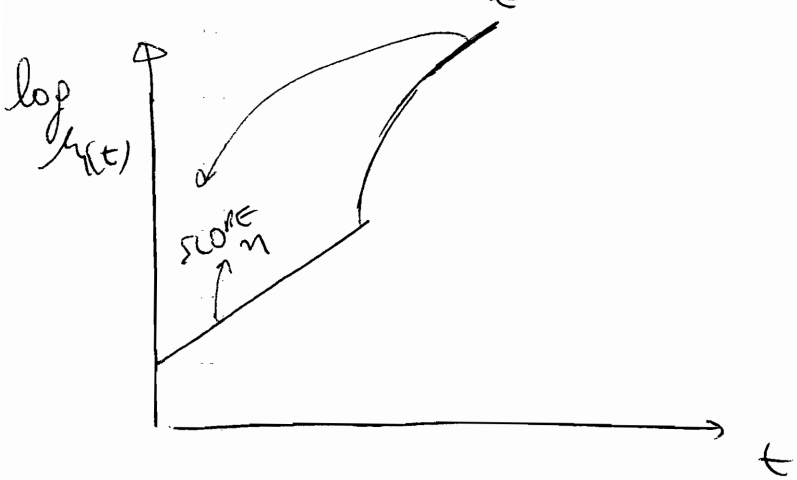
AT BGP $\boxed{\hat{k} = \hat{A} = n}$




At $\bar{t}$: $\uparrow A \Rightarrow \frac{k}{A}$ JUMPS DOWN to $\frac{k}{A'}$




$\Rightarrow \hat{k}_{\bar{t}} > n$



(III)

(1) PC $\Rightarrow$ $p = MC$

$\exists$ ideas/R&D $\Rightarrow$ $FC > 0$ $\Rightarrow$ $MC < AC$

$\Rightarrow$ PC $\Rightarrow$ NEG. PROFITS

WHEN COST FUNCTION: $C(q) = F + MC \cdot q$

WE CAN'T HAVE AN EQUIL. WITH P.C.

(2)

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha \, dj$$

(FINAL GOODS PROD. FUNCTION)

$0 < \alpha < 1$

$x_j$ : INT. GOODS

NOTICE:

$$\int_0^A x_j^\alpha \, dj \approx \sum_{j=1}^{A} x_j^\alpha$$

IF EVERY $x_j$ IS SOLD AT THE SAME PRICE (WHAT IS THE CASE HERE), A FIRM WILL DEMAND THE SAME QUANTITY OF EACH $x_j$ BECAUSE $0 < \alpha < 1$ (i.e. WE HAVE DECREASING MARG. PRODUCT) TO EACH OF THE $x_j$'s.

i.e. $x_j = x$

$$\Rightarrow \int_0^A x_j^\alpha \, dj \approx \sum_{j=1}^{A} x_j^\alpha = \boxed{A \cdot x^\alpha}$$

AND THE PROD. FUNCTION CAN BE WRITTEN AS:

$$Y = L_Y^{1-\alpha} \left( A X^{\alpha} \right)$$

SO YP AS AP

(IV)

GROWTH ACCOUNTING: METHOD TO IDENTIFY SOURCES OF GROWTH
(PROD. GROWTH? INPUT GROWTH? ETC)

≠ FROM CAUSALITY

TFP GROWTH CALCULATION = RESIDUAL = PART OF OUTPUT OR OUTPUT PC GROWTH THAT CAN'T BE EXPLAINED BY INPUT GROWTH

USUAL FORMULATION: $Y = L^{1-\alpha} K^{\alpha} B$

$$\hat{Y} = \hat{B} + \alpha \hat{K} + (1-\alpha)\hat{L}$$

$\hat{B}$ = TFP GROWTH

$$\Longrightarrow \hat{B} = \hat{Y} - \left[ \alpha \hat{K} + (1-\alpha)\hat{L} \right]$$

~~TAKE~~ OBSERVED = CAPITAL SHARE

OBSERVED / CALCULATED

(2) H-K: CONTRIBUTION OF TFP GROWTH TO P.C. OUTPUT GROWTH is SIGNIFICANT

ANNUAL GROWTH TFP (66-91) : $\boxed{2.3\%}$

SINGP: TFP GROWTH ALMOST NIL

⇒ GROWTH DUE TO INPUT GROWTH

ANNUAL GROWTH OF TFP (66-90) = $\boxed{0.2\%}$

DIFFERENCES

<u>K-GROWTH</u> is HIGHER in SINGAPORE ! 10.8% (RAW 66-90 ANNUAL)

THAN H-K $\boxed{7.7\%}$ (RAW 66-91 ANNUAL)

<u>L-GROWTH</u> is HIGHER IN SING 4.5% (RAW 66-90 ANNUAL)

THAN H-K $\boxed{2.6\%}$ (RAW 66-91 ANNUAL)

<u>SIMILARITIES</u> : BOTH VERY HIGH OUTPUT GROWTH IN PERIOD

$\boxed{\text{H-K}}$ 66-91
7.3%

SING 66-90
8.7%

(3) CONVERGENCE

"POOR" /BACKWORD COUNTRIES WILL GROW FASTER THAN "RICH" COUNTRIES SO THAT THE GAP BETWEEN THE 2 GROUPS WILL CLOSE

$$\frac{GDP_{PC}^{\text{BACKWARD COUNTRY}}}{GDP_{PC}^{\text{ADV. COUNTRY}}}$$

MEASURE OF HOW BACKWARD

FIG 3.6: LACK OF CONVERGENCE FOR THE WORLD

I.E. NOT ALL POOR/BACK. COUNTRY GROW FASTER THAN ADVANCED ONES

CONDITIONAL CONV.: THE FURTHER AN ECONOMY IS "BELOW" ("ABOVE") ITS STEADY STATE, THE FASTER ("SLOWER") THE ECONOMY SHOULD GROW

FIG. 3.8 SHOWS CONDITIONAL CONVERGENCY FOR THE COUNTRIES CONSIDERED —