SS08

ANSWER KEY
EXAM 1

(I) (1) $Y = A k^{\alpha} L^{1-\alpha}$ $\qquad \dot{k} = sY - \delta k$ (2)

$C = (1-s)\, Y$

$\boxed{(1)}$ BGP $\Rightarrow$ $\hat{Y}, \hat{k}, \hat{C}$ CONSTANT

USING (2) $\Rightarrow$ $\hat{k} = s\dfrac{Y}{k} - \delta$ (3)

$\underset{\text{CONSTANT}}{(3)} \Rightarrow \dfrac{Y}{k} = \text{CONSTANT} \Rightarrow \boxed{\hat{Y} = \hat{k}}$ (4)

TAKING GROWTH RATES OF (1):

$$\hat{Y} = \underset{\substack{= \\ 0 \\ \text{SINCE A CONSTANT}}}{\hat{A}} + \alpha \underset{\substack{= \\ \hat{Y} \text{ BY (4)}}}{\hat{k}} + (1-\alpha) \underset{\substack{= \\ n}}{\hat{L}} \Rightarrow \hat{Y} = \alpha \hat{Y} + (1-\alpha) n$$

$\hat{Y}\cancel{(1-\alpha)} = \cancel{(1-\alpha)}\, n \Rightarrow$ USING (4)

$$\boxed{\hat{Y} = \hat{k} = n}$$

$\boxed{(2)}$ MODIFIED VARIABLES

$$\frac{Y}{L} = \frac{A\, k^{\alpha} L^{1-\alpha}}{L} = A\left(\frac{k}{L}\right)^{\alpha} \left(\frac{L}{L}\right)^{1-\alpha}$$

(5) $\boxed{y = A\, k^{\alpha}}$

$$\hat{k} = \hat{K} - \hat{L} \underset{\text{USING (3) ABOVE}}{=} \underbrace{s\frac{Y}{K}}_{\text{BY (3)}} - \delta - n$$

$$\hat{k} = s \frac{Y/L}{K/L} - (\delta + n) = s\frac{y}{k} - (\delta + n)$$

USING (5)

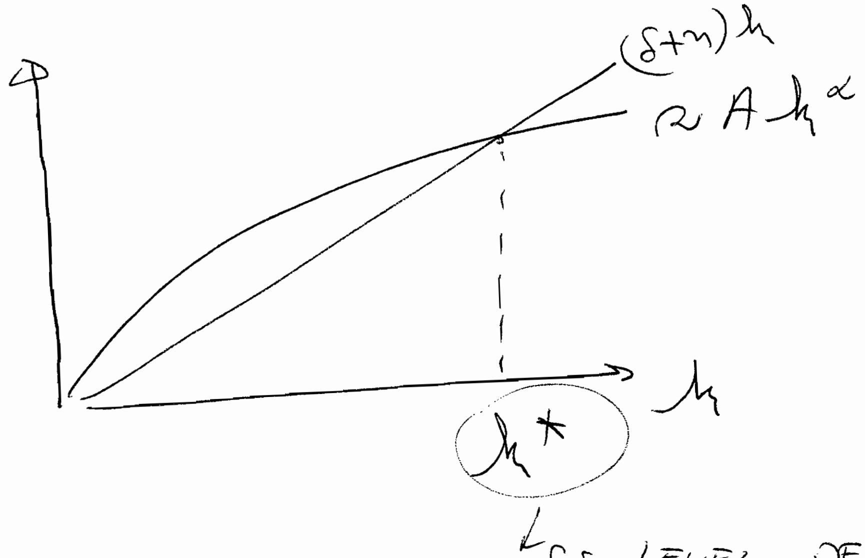
$$\hat{k} = s\, A \frac{k^{\alpha}}{k} - (\delta + n)$$

$$\Rightarrow \hat{k} = s\, A\, k^{\alpha - 1} - (\delta + n)$$

OR

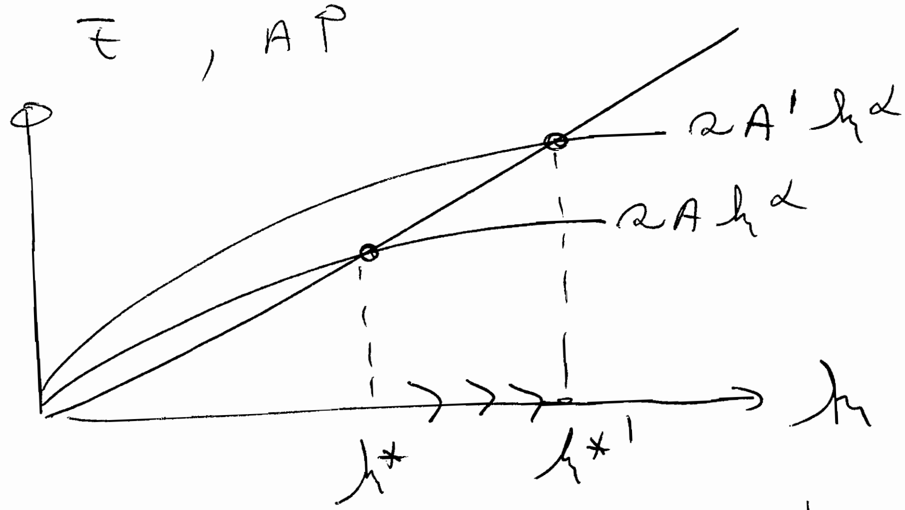
$$\boxed{\dot{k}} = \hat{k} \cdot k = \boxed{s A k^{\alpha} - (\delta + n) k}$$

(3)



A $\bar{E}$, A ↑

(4) AT $\bar{E}$, A ↑



THE S.S. LEVEL OF k ↑ TO $k^{*\prime}$

AT NEW S.S. $\hat{h} = 0 \Rightarrow \hat{y} = \widehat{(A' h^{\alpha})}$

$= \alpha \hat{h}$

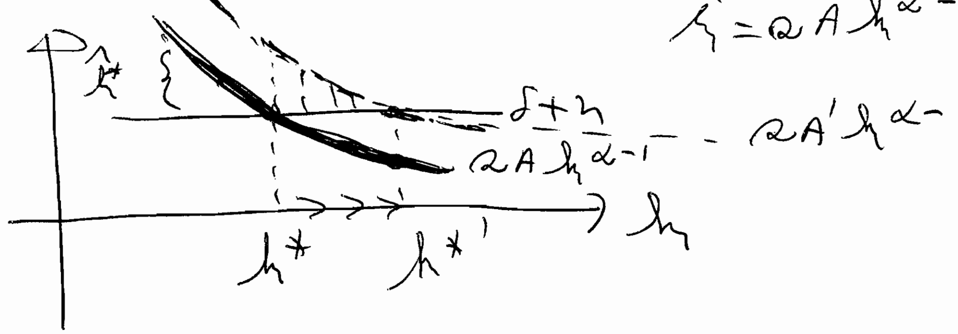
SINCE A' CONSTANT

$\Rightarrow \hat{y} = 0$ OUTPUT PER WORKER CONSTANT AT NEW S.S.

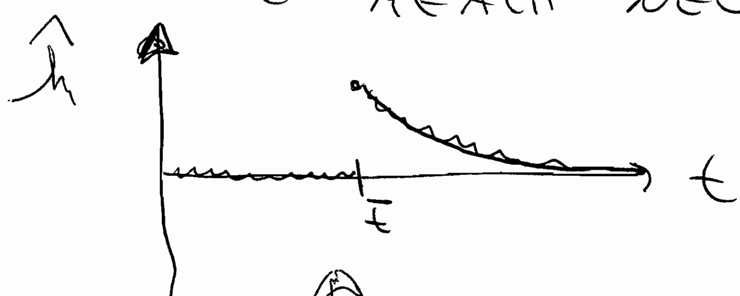
(5) ADJ. PATHS.

WE FIRST LOOK AT $\hat{h}$:

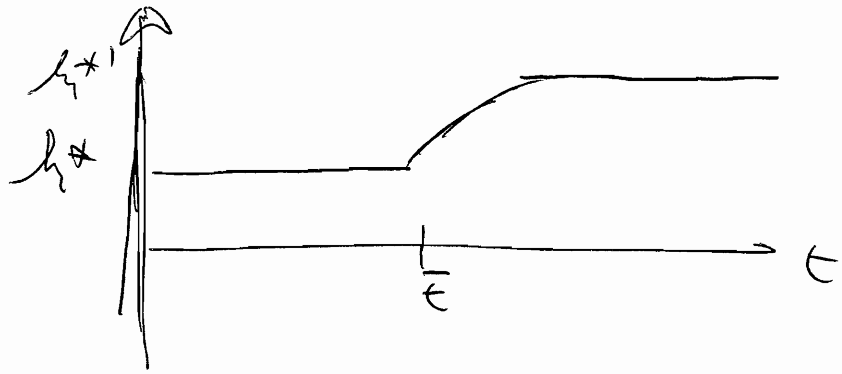
$\hat{h} = \rho A h^{\alpha-1} - (\delta+n)$



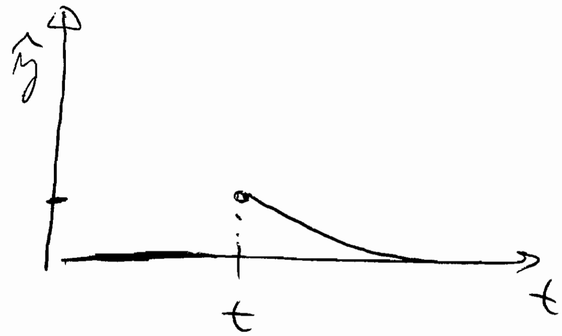
AT $\bar{t}$ (WHEN A ↑): $\hat{h}^* > 0$
BUT GROWTH RATE ↓ UNTIL
WE REACH NEW S.S.



THEN

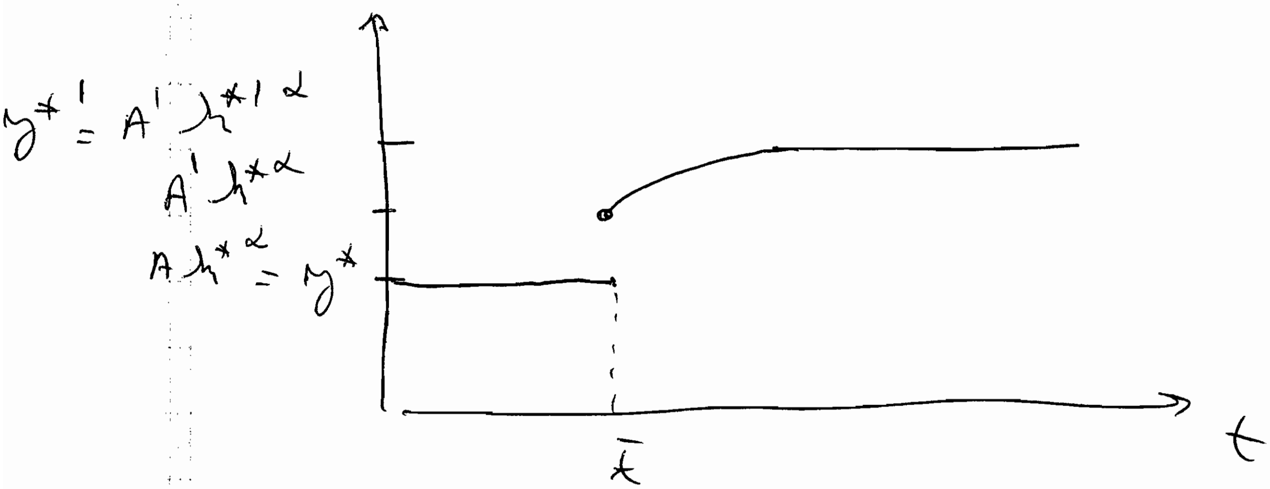


$y = A h^{\alpha} \Rightarrow \boxed{\hat{y} = \alpha \hat{h}}$



PATH OF $y(t)$

NOTICE THAT AT $\bar{t}$,
$y(t)$ JUMPS FROM $y^* = A h^{*\alpha}$
TO $A' h^{*\alpha}$
AND THEN MOVES SMOOTHLY TO NEW
S.S : $y^{*'} = A' h^{*'\alpha}$



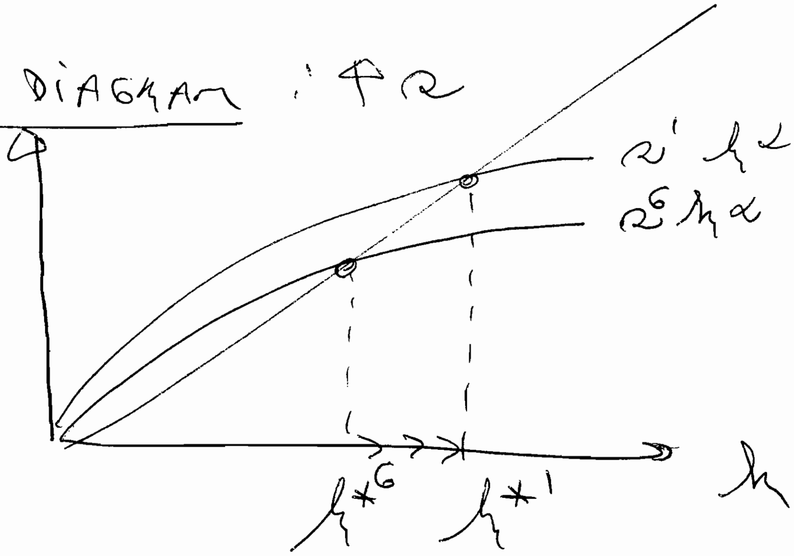
GOLDEN RULE $\left[\begin{array}{l} A=1 \quad \alpha=0.5 \quad n=0 \\ \delta=0.10 \end{array}\right.$

$s^G = 0.5$

ASSUME COUNTRY AT S.S. WITH $s^G = 0.5$
AND AT $\bar{t}$: $\boxed{s\uparrow}$

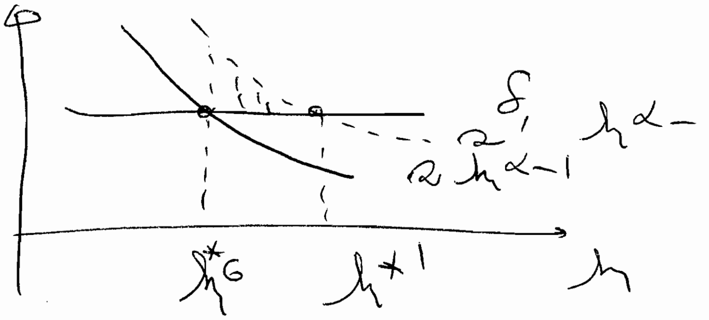
(6) $h(t)$? $y(t)$?

SOLOW DIAGRAM: ↑ a

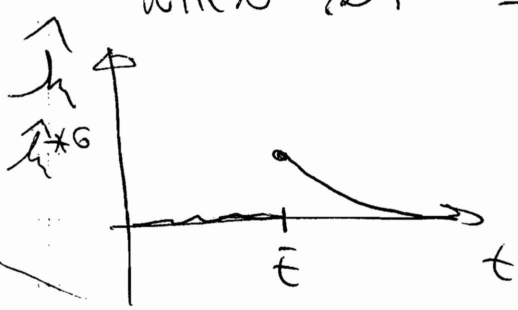


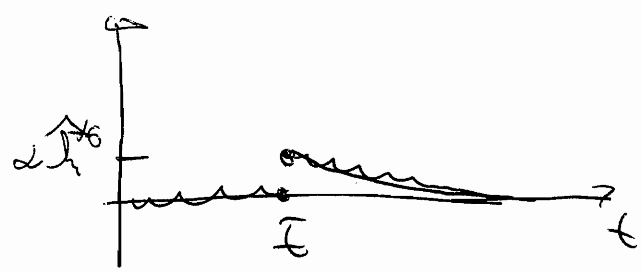
NOT NEEDED BUT USEFUL

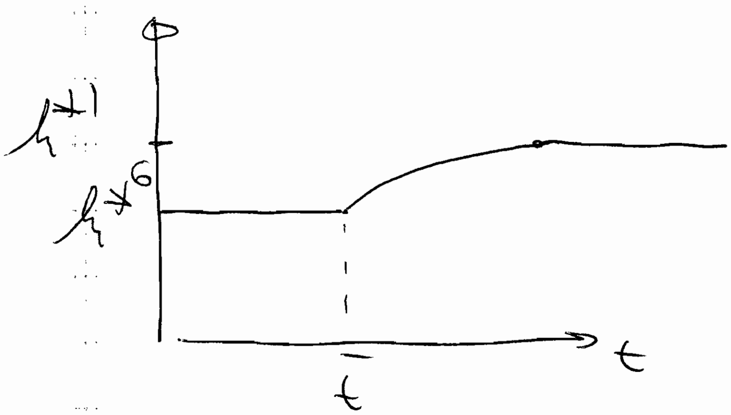
$$y = k^\alpha \Rightarrow \hat{y} = \alpha \hat{k}$$

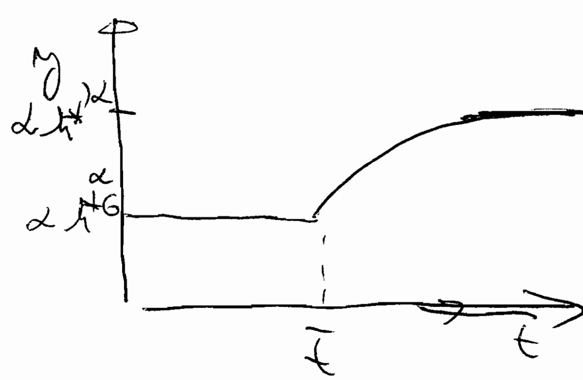


WHEN $a \uparrow \Rightarrow k \uparrow$ TO APPROACH NEW S.S.









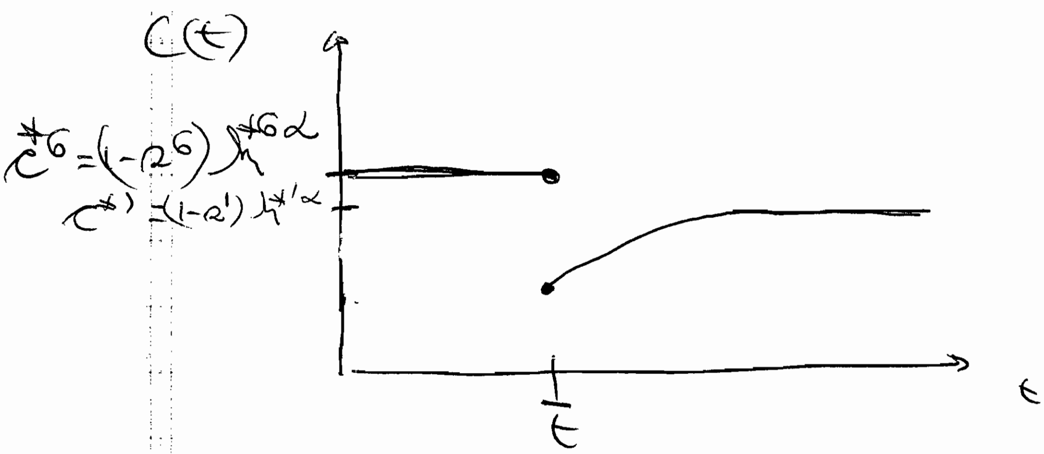
$\boxed{(7)}$ $c(t)$?

$$c = (1-s)\,y = (1-s)\,k^{\alpha}$$

AT $\bar{t}$, $s \uparrow \Rightarrow c$ <u>DROPS</u>
AND THEN STARTS TO $\uparrow$
BECAUSE $k \uparrow$

$$\hat{c} = \underbrace{\widehat{(1-s)}}_{\text{JUST INITIAL DROP}} + \alpha\,\hat{k}$$

BECAUSE $s^{NEW} > s^{G}$ WE KNOW THAT NEW S.S. CONSUMPTION PER WORKER WILL BE LOWER THAN THE ONE AT s^{G}:



<u>SUMMARY</u>

AT $\bar{t}$: C DROPS

AT NEW S.S. $c^{*}(t)$ IS LOWER THAN $\underbrace{c^{*G}(t)}_{\text{CONS. PER WORKER AT GOLDEN RULE SAVINGS RATE}}$

(II)

(1) $Y = K^{\alpha}(AL)^{1-\alpha}$ $\hat{A} = g$, $\hat{L} = n$

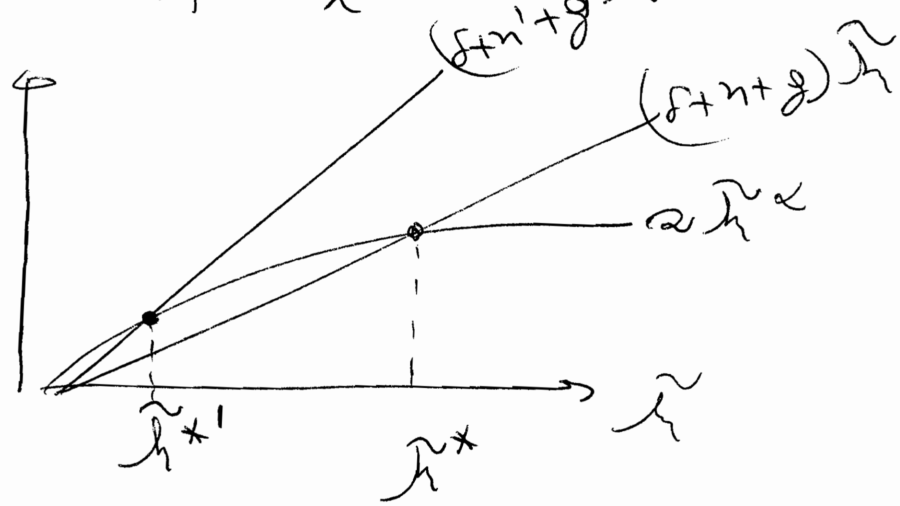
(2) $\dot{K} = sY - \delta K$

MODIFIED SYSTEM:

$$\tilde{y} = \tilde{k}^{\alpha}$$
$$\dot{\tilde{k}} = s\tilde{k}^{\alpha} - (\delta + g + n)\tilde{k}$$

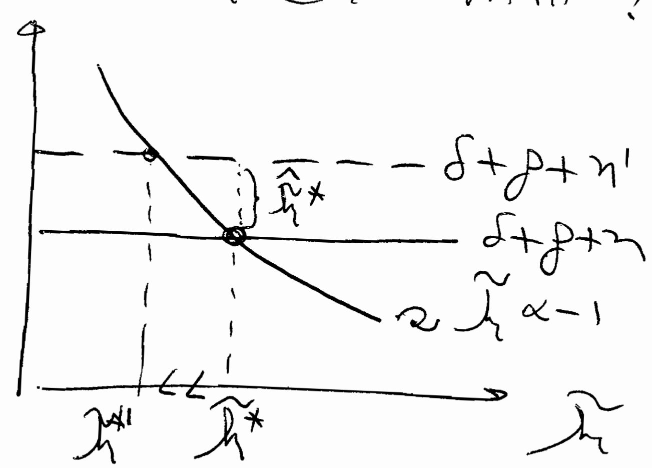
$\boxed{\uparrow n}$ AT $\bar{t}$

$\boxed{(6)}$



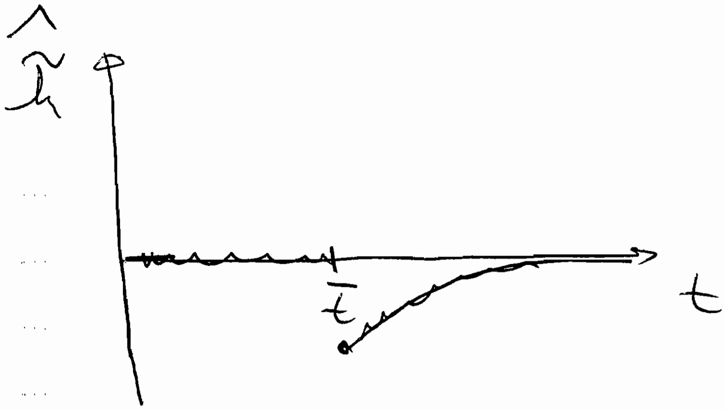
S.S. LEVEL OF $\tilde{k} \downarrow$ to $\tilde{k}^{*\prime}$

$\boxed{(7)}$ $\tilde{k}(t)$ PATH ?

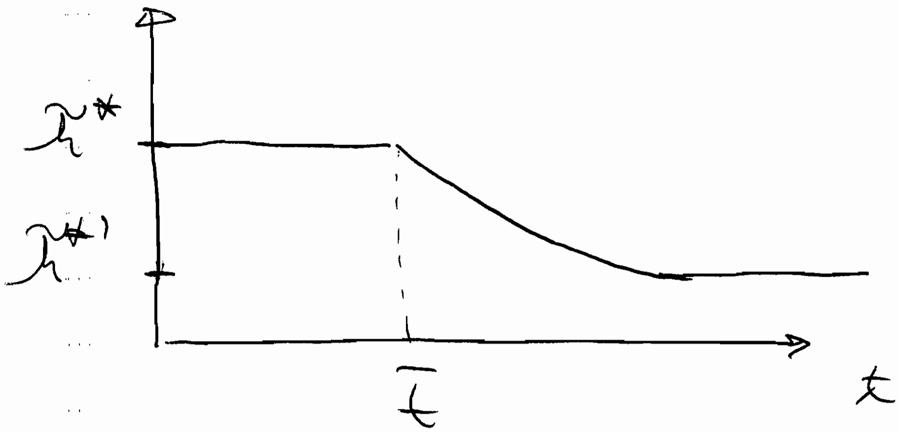


$(5) \Rightarrow \hat{\tilde{k}} = s\tilde{k}^{\alpha-1} - (\delta+g+n)$

At $\bar{t}$, $n\uparrow \Rightarrow s\tilde{k}^{*\alpha-1} < (\delta+g+n')$
$\Rightarrow \hat{\tilde{k}} < 0$



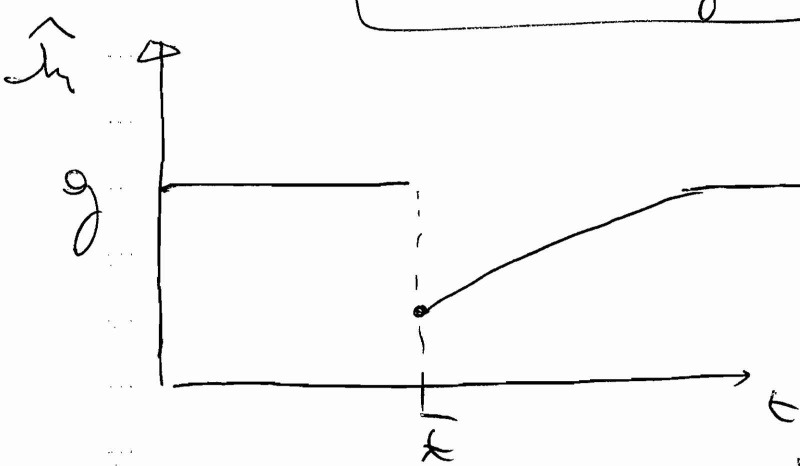
THEN IT ADJUSTS TO NEW S.S.



(8) $h(t)$ PATH ?

NOTICE : $\tilde{h} = \frac{h}{A} \Rightarrow \hat{\tilde{h}} = \hat{h} - \overbrace{\hat{A}}^{=g}$
$= \hat{h} - g$

$\Rightarrow \boxed{\hat{h} = \hat{\tilde{h}} + g}$

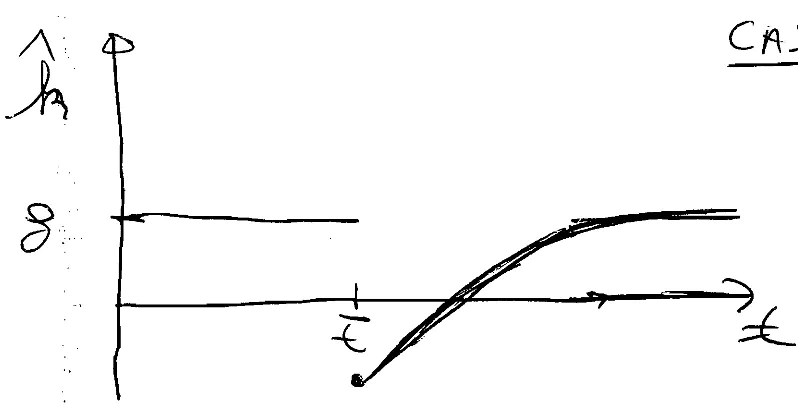


CASE 1

ASSUMING $|\hat{\tilde{h}}^*| < g$

IT IS OK TO ASSUME $|\hat{\tilde{h}}^*| > g$ & GET INSTEAD →

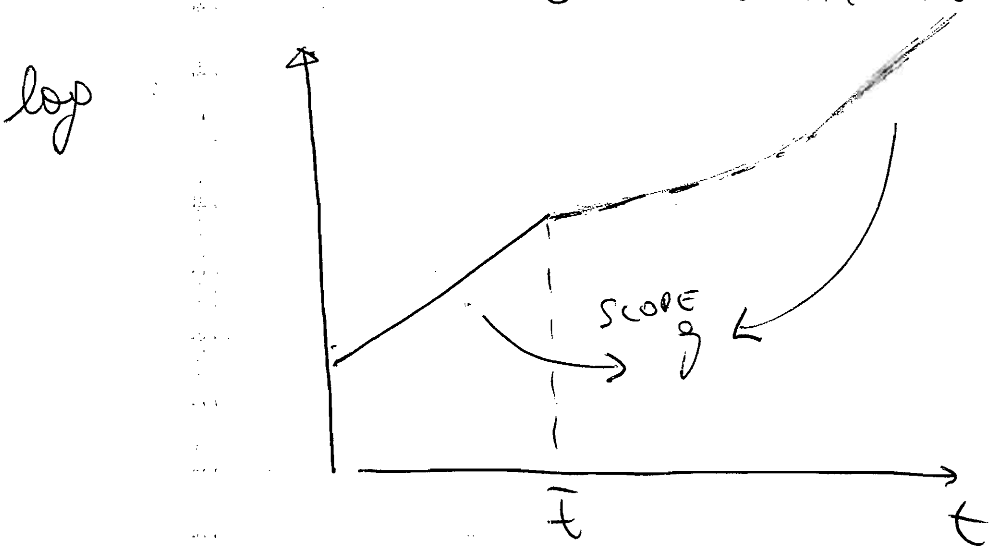
CASE 2



ONLY ONE NEEDED

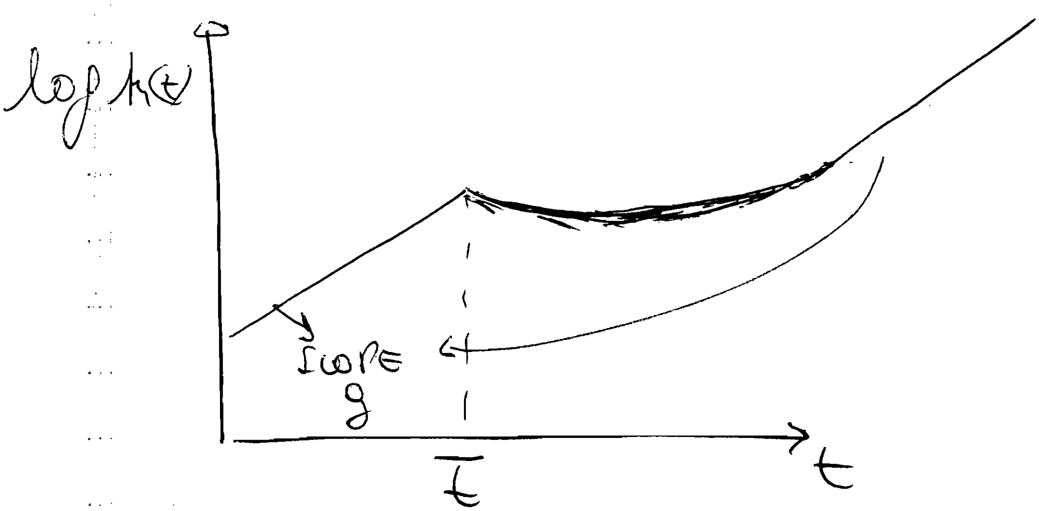
$\boxed{(9)}$

ASSUMING WE ARE IN CASE 1



NOTICE NEW PATH AT BGP WILL BE LOWER

ASSUMING WE ARE IN CASE 2



(III)

HOME (H): $g = 0.03 = 3\%$
$s^H = 0.15 = 15\%$

FOREIGN 1 (F1): $g = 0.02 = 2\%$
$s^{F1} = 25\%$

FOREIGN 2 (F2): $g = 0.02 = 2\%$
$s^{F2} = 15\%$

REMARK 1:

F1 & F2 WILL AT BGP
GROW THE SAME GROW RATE
OF OUTPUT PER WORKER:
$\hat{y}^{F1} = \hat{y}^{F2} = 2\%$

HOME WILL GROW FASTER:
$\hat{y}^{H} = 3\%$

SLOPE OF $\log y(t)$ AT BGP WILL BE IDENTICAL FOR F1 & F2 & HIGHER FOR HOME

LEVEL OF PATH

HIGHER SAVINGS RATES $\Longrightarrow$
HIGHER PATH OF $y(t)$ AT BGP
SO PATH FOR FOREIGN 1 WILL BE HIGHER THAN PATH OF F2

$\Longrightarrow$