Growth Accounting

Method used to identify "sources" of growth.

**Basic Setup**

1. \[ Y_t = B_t \cdot k_t \cdot L_t^{1-\alpha} \quad 0 < \alpha < 1 \]

"Related/Associated"

with Technology = Total Factor Productivity

\[ \Rightarrow B_t : \text{Assoc. with} \]

Tech. Change \[ \Rightarrow \text{Open/Closed Economy Cases} \]

Using "Hat Algebra"

\[ \hat{Y}_t = \hat{B}_t + \alpha \hat{k}_t + (1-\alpha) \hat{L}_t \]

"Input Growth"

Where \( \alpha \) = Capital Share

(with CRS & P.C. see next)

Data Availability

"Can be calculated"

Idea of \( B_t \) as a Residual:

\[ \hat{B}_t = \hat{Y}_t - [\alpha \hat{k}_t + (1-\alpha) \hat{L}_t] \]

i.e. Growth in output that cannot be explained by input growth
Other names for $B_t$:

- Solow Residual (Measure of our ignorance)
- TFP Growth (Total Factor Productivity)

Data comes in 2 forms:

→ Contributions in % terms

\[
\frac{\Delta Y_t}{\Delta t} = \frac{\Delta B_t}{\Delta t} + \Delta L_t \cdot \Delta K_t + (1 - \lambda) \Delta L_t
\]

\[
\frac{\Delta Y_t}{\Delta t} = \Delta B_t + \Delta L_t \cdot \Delta K_t
\]

% of % of output growth due to input growth due to TFP growth

→ Taking factors separately

\[
\frac{\Delta L_t}{\Delta t} = \% \text{ of output growth due to } K\text{-accumulation}
\]

\[
(1 - \lambda) \Delta L_t = \% \text{ of output growth due to } L\text{-increase}
\]

→ Using per worker variables

\[
y_t = B_t \cdot L_t \cdot \frac{1}{L_t}
\]

\[
\Delta y_t = B_t \cdot \Delta L_t \cdot \frac{1}{L_t}
\]

[2]
\[ \Delta g_t = B_t + \alpha \Delta h_t \]
\[ \Rightarrow B_t = \Delta g_t - \alpha \Delta h_t \]
\[ 1 = \frac{\Delta h_t}{\Delta g_t} = \frac{B_t}{\Delta h_t} + \frac{\Delta h_t}{\Delta g_t} \]

**Language:** Capital deepening refers to \( \frac{K_t}{L_t} \)

Then
\[ \frac{\Delta h_t}{\Delta g_t} = \% \text{ growth in P.C. output due to capital deepening} \]

\[ \Delta K_t \& \Delta L_t \]

are calculated taking into consideration in many cases changes in composition (later we will see the case of \( \Delta L_t \) taking into account changes in skill level/education)
Growth Rates Calculations:

Average Better

Growth Rates between periods 0 and T:

\[ g = \left( \frac{X_T}{X_0} \right)^{\frac{1}{T}} - 1 \]

Growth Rate:

\[ \frac{X_T - X_0}{X_0} \]

Remarks:

- Small # in parameters (i.e. \( \alpha \)) make big # in outcomes of growth accounting.

- Also estimates of "k" controversial because of the arbitrary distinction between K-accumulation and core notion of tech. change.

  Ex: New machine

  \[ \text{k-acc} \rightarrow \text{tech. change} \]

  (Embodied tech. change)

- Quality issues of data, calculations etc.
Show: "L" is K-share

with P.C. & Codd Douglass
CRS Technology

\[ Y = k^x L^{1-x} \]

\[ \frac{\partial Y}{\partial L} = (1-x) k^x L^{-x} k^x \]

\[ \frac{\partial Y}{\partial K} = x k^{-x-1} L^{1-x} \]

Profit Maximization & P.C.:

\[ \text{Max} \quad P \cdot Y - WL - RK \]

\[ L, K \] output price wage real price of capital

\[ = P \cdot \left[ k^x L^{1-x} \right] - WL - RK \]

\[ \frac{\partial}{\partial L} = P \cdot \frac{\partial Y}{\partial L} - W = 0 \]

\[ \frac{\partial}{\partial K} = P \cdot \frac{\partial Y}{\partial K} - R = 0 \]

\[ \Rightarrow \quad \left\{ \begin{array}{l}
W = \text{MPL} \cdot P \\
R = \text{MPK} \cdot P
\end{array} \right. \]
L-shape: \[ \frac{W \cdot L}{P \cdot Y} \]

K-shape: \[ \frac{L \cdot K}{P \cdot Y} \]

Since \( W/P = NPC \)

L-shape: \( NPC \), \[ \frac{L}{Y} = \]

\[ = (1 - \alpha) \frac{L - \alpha}{NPC} \cdot \frac{L}{Y} \]

\[ = (1 - \alpha) \frac{L \cdot K}{Y} = (1 - \alpha) \]

K-shape: \( NPC \), \[ \frac{K}{Y} = \]

\[ = \alpha \frac{K^{a - 1} \cdot L - \alpha}{Y} \]

\[ = \alpha \frac{K^{a - 1} \cdot Y}{Y} = \alpha \]
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<tbody>
<tr>
<td>Output per hour</td>
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<td>3.3</td>
<td>1.5</td>
<td>2.9</td>
<td>2.7</td>
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<td>Contributions from:</td>
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<td>Capital per hour worked</td>
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<td>0.1</td>
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<td>Other capital services</td>
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<tr>
<td>Multifactor productivity</td>
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<td>2.1</td>
<td>0.6</td>
<td>1.5</td>
<td>1.3</td>
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