### Table 1.1 Top Eleven Countries in Year 2009 According to Three Different Measures

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>GDP per Capita ($)</th>
<th>Country</th>
<th>Total GDP ($ trillions)</th>
<th>Country</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Qatar</td>
<td>159,469</td>
<td>United States</td>
<td>12.62</td>
<td>China</td>
<td>1,320</td>
</tr>
<tr>
<td>2</td>
<td>Luxembourg</td>
<td>84,525</td>
<td>China</td>
<td>10.08</td>
<td>India</td>
<td>1,160</td>
</tr>
<tr>
<td>3</td>
<td>United Arab Emirates</td>
<td>52,946</td>
<td>Japan</td>
<td>3.81</td>
<td>United States</td>
<td>307</td>
</tr>
<tr>
<td>4</td>
<td>Bermuda</td>
<td>52,090</td>
<td>India</td>
<td>3.76</td>
<td>Indonesia</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>Macao</td>
<td>51,057</td>
<td>Germany</td>
<td>2.66</td>
<td>Brazil</td>
<td>199</td>
</tr>
<tr>
<td>6</td>
<td>Norway</td>
<td>49,945</td>
<td>United Kingdom</td>
<td>2.07</td>
<td>Pakistan</td>
<td>181</td>
</tr>
<tr>
<td>7</td>
<td>Singapore</td>
<td>47,373</td>
<td>Russia</td>
<td>2.05</td>
<td>Bangladesh</td>
<td>154</td>
</tr>
<tr>
<td>8</td>
<td>Kuwait</td>
<td>46,639</td>
<td>France</td>
<td>1.98</td>
<td>Nigeria</td>
<td>149</td>
</tr>
<tr>
<td>9</td>
<td>Brunei</td>
<td>46,229</td>
<td>Italy</td>
<td>1.68</td>
<td>Russia</td>
<td>140</td>
</tr>
<tr>
<td>10</td>
<td>Australia</td>
<td>41,304</td>
<td>Brazil</td>
<td>1.62</td>
<td>Japan</td>
<td>127</td>
</tr>
<tr>
<td>11</td>
<td>United States</td>
<td>41,099</td>
<td>Mexico</td>
<td>1.29</td>
<td>Mexico</td>
<td>111</td>
</tr>
</tbody>
</table>
Figure 1.5 GDP per Capita in the United States, the United Kingdom, and Japan, 1870–2009

GDP per capita (2005 Dollars, ratio scale)

Figure 1.6 The Distribution of Growth Rates, 1975–2009

<table>
<thead>
<tr>
<th>Average annual growth rate</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5%–9.0%</td>
<td>Equatorial Guinea</td>
</tr>
<tr>
<td>8.0%–8.5%</td>
<td>China</td>
</tr>
<tr>
<td>7.5%–8.0%</td>
<td></td>
</tr>
<tr>
<td>7.0%–7.5%</td>
<td>Maldives</td>
</tr>
<tr>
<td>6.5%–7.0%</td>
<td>Taiwan, South Korea</td>
</tr>
<tr>
<td>6.0%–6.5%</td>
<td>Singapore, Vietnam</td>
</tr>
<tr>
<td>5.5%–6.0%</td>
<td>Botswana, Thailand</td>
</tr>
<tr>
<td>5.0%–5.5%</td>
<td>India, Indonesia, Egypt, Malaysia</td>
</tr>
<tr>
<td>4.5%–5.0%</td>
<td>Bulgaria, Chile, Ireland</td>
</tr>
<tr>
<td>4.0%–4.5%</td>
<td>Albania, Cambodia, Dominican Republic</td>
</tr>
<tr>
<td>3.5%–4.0%</td>
<td>Poland, Portugal, Norway, Tunisia, Uruguay</td>
</tr>
<tr>
<td>3.0%–3.5%</td>
<td>Angola, Canada, Japan, Spain, Tanzania, United States</td>
</tr>
<tr>
<td>2.5%–3.0%</td>
<td>Argentina, Ethiopia, New Zealand, Mexico, Switzerland, Syria</td>
</tr>
<tr>
<td>2.0%–2.5%</td>
<td>Afghanistan, Guatemala, Senegal, Peru, South Africa</td>
</tr>
<tr>
<td>1.5%–2.0%</td>
<td>Bolivia, Jamaica, Kenya, Nigeria</td>
</tr>
<tr>
<td>1.0%–1.5%</td>
<td>Bahrain, Iran, Sierra Leone, Venezuela</td>
</tr>
<tr>
<td>0.5%–1.0%</td>
<td>Haiti, Zambia</td>
</tr>
<tr>
<td>0.0%–0.5%</td>
<td>Brunei, Central African Republic, Iraq</td>
</tr>
<tr>
<td>-0.5%–0.0%</td>
<td>Nicaragua</td>
</tr>
<tr>
<td>-1.0%–-0.5%</td>
<td>Somalia</td>
</tr>
<tr>
<td>-1.5%–-1.0%</td>
<td>Djibouti</td>
</tr>
<tr>
<td>-2.0%–-1.5%</td>
<td>Zimbabwe</td>
</tr>
<tr>
<td>-2.5%–-2.0%</td>
<td></td>
</tr>
<tr>
<td>-3.0%–-2.5%</td>
<td></td>
</tr>
<tr>
<td>-3.5%–-3.0%</td>
<td></td>
</tr>
<tr>
<td>-4.0%–-3.5%</td>
<td></td>
</tr>
<tr>
<td>-4.5%–-4.0%</td>
<td>Liberia</td>
</tr>
</tbody>
</table>

Figure 1.2 GDP per Capita in the United States, 1870–2009
Figure 1.4 GDP per Capita in the United States, 1870–2009 (Ratio Scale)

GDP per capita (2005 Dollars, ratio scale)

Year

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REAL PER CAPITA GDP IN THE UNITED STATES, 1870-2008

Per capita GDP (1990 dollars, log scale)

Year

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Figure 1.7 GDP per Capita by Country Group, 1820–2008

GDP per Capita (2005 Dollars, ratio scale)

Country group and population in 2009 (in millions)
- Western Europe 408
- Western Offshoots 366
- Eastern Europe 114
- Former USSR 275
- Latin America 580
- China 1,320
- India 1,150
- Japan 127
- Africa 908
- World 6,810

Sources: Maddison (2008), Heston, Summers, and Aten (2011).
Figure 1.3 The Effect of Using a Ratio Scale
Table 1.3. The Effect of Using PPP on Comparisons of GDP

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP per Capita in 2009 Using Market Exchange Rates (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>41,099</td>
</tr>
<tr>
<td>Japan</td>
<td>36,651</td>
</tr>
<tr>
<td>Germany</td>
<td>36,702</td>
</tr>
<tr>
<td>Argentina</td>
<td>6,519</td>
</tr>
<tr>
<td>Mexico</td>
<td>7,257</td>
</tr>
<tr>
<td>India</td>
<td>1,041</td>
</tr>
</tbody>
</table>
Growth Facts according to Jones Ch. 1, 3rd. edition 2013

1- There is enormous variation in per capita income across economies. The poorest countries have per capita incomes that are less than 5% of per capita incomes in the richest countries.

2- Rates of economic growth vary substantially across countries.

3- Growth rates are not generally constant over time. For the world as a whole, growth rates were close to zero over most of history but have increased sharply in the 20th century. For individual countries, growth rates also change over time.

4- A country’s relative position in the world distribution of per capita incomes is not immutable. Countries can move from being “poor” to being “rich” and vice versa.

5- In the US over the last century:
   1- the real rate of return to capital, r, shows no trend upward or downward.
   2- the shares of income devoted to capital, r.K/Y, and labor, w.L/Y, show no trend; and
   3- the average growth rate of output per person has been positive and relatively constant over time—that is, the US exhibits steady, sustained per capita income growth.

6- Growth in output and growth in the volume of international trade are closely related.

7- Both skilled and unskilled workers tend to migrate from poor to rich countries.

Kaldor Facts (1961)

1. Labor productivity has grown at a sustained rate.

2. Capital per worker has also grown at a sustained rate.

3. The real interest rate or return on capital has been stable.

4. The ratio of capital to output has also been stable.

5. Capital and labor have captured stable shares of national income.

6. Among the fast growing countries of the world, there is an appreciable variation in the rate of growth “of the order of 2-5 percent.”
New Kaldor Factor: Jones and Romer (2009)

1. **Increases in the extent of the market.** Increased flows of goods, ideas, finance, and people — via globalization as well as urbanization — have increased the extent of the market for all workers and consumers.

2. **Accelerating growth.** For thousands of years, growth in both population and per capita GDP has accelerated, rising from virtually zero to the relatively rapid rates observed in the last century.

3. **Variation in modern growth rates.** The variation in the rate of growth of per capita GDP increases with the distance from the technology frontier.

4. **Large income and TFP differences.** Differences in measured inputs explain less than half of the enormous cross country differences in per capita GDP.

5. **Increases in human capital per worker.** Human capital per worker is rising dramatically throughout the world.

6. **Long-run stability of relative wages.** The rising quantity of human capital relative to unskilled labor has not been matched by a sustained decline in its relative price.
See Also

CH2: Diagrams from ACEDCU
"Introduction to Econ. Growth"
(CCLOSES)
**Math Tools** / Dot, Hat Notation

- Variables are functions of time \((t)\)

  *Example:
  \[
  y_t \quad \text{or} \quad y(t) \\
  c_t \quad \text{or} \quad c(t) \\
  k_t \quad \text{or} \quad k(t) \\
  l_t \quad \text{or} \quad l(t)
  *

- Example using \(x(t), y(t)\)

  \[
  \frac{dx(t)}{dt} = \dot{x}(t) \\
  \frac{dy(t)}{dt} = \dot{y}(t)
  \]

- Growth Rates: "Hats":

  \[
  \frac{d}{dt} \frac{x(t)}{x(t)} = \frac{\dot{x}(t)}{x(t)} = \hat{x}(t)
  \]

  \[
  \Rightarrow \quad \dot{x}(t) = \hat{x}(t) \cdot x(t)
  \]

  \[
  \left[ \frac{d \log x(t)}{dt} \right] = \frac{1}{x(t)} \frac{dx(t)}{dt} = \hat{x}(t)
  \]
Example:

\[ X(t) = \text{constant} \cdot e^{a \cdot g \cdot t} \]

= \frac{a}{e^{g \cdot t}}

Taking logs

\[ \log X(t) = \log a + g \cdot t \]

\[ \frac{d}{dt} \log X(t) = \frac{1}{X(t)} \cdot \frac{dX(t)}{dt} = g = \frac{1}{X(t)} \]

Diagrams:

\[ X(t) \]

\[ a \]

\[ \log X(t) \]

\[ \log a \]

\[ t \]

\[ X(t) \text{ in "nano scale"} \]

\[ \log X(t) = \log a + g \cdot t \]

[Scale 1 = g]!
1) **Product**

\[ Z(t) = X(t) \cdot Y(t) \]

\[ \hat{Z}(t) = \hat{X}(t) + \hat{Y}(t) \]

*Why?*

\[ \log Z(t) = \log X(t) + \log Y(t) \]

\[ \frac{d}{dt} \log Z(t) = \frac{d}{dt} \log X(t) + \frac{d}{dt} \log Y(t) \]

\[ \frac{d}{dt} \hat{Z}(t) = \frac{d}{dt} \hat{X}(t) + \frac{d}{dt} \hat{Y}(t) \]

2) **Quotient**

\[ Z(t) = \frac{X(t)}{Y(t)} \]

\[ \hat{Z}(t) = \hat{X}(t) - \hat{Y}(t) \]

3) \[ \hat{Z}(t) = \hat{X}(t)^a \quad 0 < a < 1 \]

\[ \hat{Z}(t) = \hat{X}(t)^{1-a} \]

**Combined Example:**

\[ Y(t) = K(t) \cdot L(t) \]
\[ Y_t = k_t^x + \hat{Y}_t \]
\[ = \hat{2} k_t + (1-\hat{2}) \hat{C}_t \]

**SYSTEMS**

**VARIABLES**: \( X_t, Y_t, C_t, L_t \)

**PARAMETERS**:
- Technological Parameter
- Preference Parameter
- Other

**Steady State of a System (S.S.)**

Situation where the values of the variables do not change

**Example**

**Discrete Time**: \( t, t+1, t+2, \ldots \)

\[ X_t = X_{t+1} = X_{t+2} = \ldots = X_{t+N} = \ldots \]

\[ Y_t = Y_{t+1} = Y_{t+2} \ldots \]

\[ C_t = C_{t+1} = C_{t+2} \ldots \]

\[ L_t = L_{t+1} = L_{t+2} \ldots \]

**Continuous Time**:

\[ \frac{dX_t}{dt} = 0 \quad \text{for all} \quad t \]

\[ Y_t = C_t = L_t = 0 \]

\[ \Rightarrow X_t = 0 \quad \Rightarrow Y_t = C_t = L_t = 0 \]
Assume system evolves to a "steady state" at $x$

where $x^*$ is the S.S. value or $x_t$

"Balanced Growth Path" or a system

situation where all variables
grow at "constant rates"

Remarks:
- These constant rates can be positive, negative or zero
- The constant growth rates can be different between variables.
- A BGP with constant growth rates equal to zero is a steady state.
EVENY S.S. IS A BGP
BUT NOT EVERY BGP IS
A S.S.

EXAMPLE OF SYSTEM: $X_t, Z_t, Y_t$

AT BGP:

$X_t = \Delta x \rightarrow \text{constant!}$

$\Delta t = \Delta y \rightarrow \text{constant!}$

$\Delta t = \Delta z \rightarrow \text{constant!}$

BCP WITH POSITIVE GROWTH
\[ y(t) \]

\[ \frac{dy}{dt} \]

\[ t \]

\[ \text{slope} = \frac{y}{t} \]