Problem Set 2

Consider the version of the Solow model with labor augmenting technological change. The production function is:

\[ Y = K^{\alpha} (AL)^{1-\alpha} \]

where \( A \) is positive, \( \alpha \) is a parameter between zero and one, \( K \) is capital and \( L \) the number of workers or labor force (equal to population).

Assume that labor (or population) grows at a rate \( n \), that \( A \) grows at a rate \( g \), that physical capital depreciates at a rate \( \delta \) (between zero and one) and that the savings rate \( s \) is a constant between zero and one. Physical capital is then accumulated according to:

\[ \dot{K} = sY - \delta K \]

and consumption is: \( C = (1-s)Y \).

(1) What are the conditions that need to be satisfied along a Balanced Growth Path (BGP)? Use these conditions and equations (1) and (2) to derive the growth rates of \( Y \) and \( K \) along a BGP.

(2) To solve the model it is convenient to divide (1) by \( AL \), so that we work with variables per effective number of workers: \( (Y/AL) \), \( (K/AL) \), and similarly.

Derive the per effective unit of labor production function and the law of motion of the capital per effective unit of labor.

**BGP in the per effective unit of labor variables:**

(3) Derive the growth rate of capital per worker, output per worker and consumption per worker at the BGP of the modified system.

**Comparative Statics: decrease in \( L \) (due to a virus)**

Assume that the country has been at the BGP for many years and that at time \( t \) the number of workers/population decreases due to a deadly virus. Answer question (6):

(4) Adjustment paths of \( k = K/L \) and \( y = Y/L \). Draw a diagram showing how \( k \), \( y \) and their growth rates move over time (use the horizontal axis for the variable \( t \) and the vertical axis for the relevant variable).

Hint: first look at adjustment path of \( K/AL \), \( Y/AL \) and then derive the paths of \( k \) and \( y \).

**Comparative Statics: decreases in the rate of population growth \( (n) \)**

Assume that the country has been at the BGP for many years and that at time \( t \) the rate of population growth decreases. Answer question (7):

(5) Adjustment paths of \( k = K/L \) and \( y = Y/L \). Draw a diagram showing how \( k \), \( y \) and their growth rates move over time (use the horizontal axis for the variable \( t \) and the vertical axis for the relevant variable).

Hint: first look at adjustment path of \( K/AL \), \( Y/AL \) and then derive the paths of \( k \) and \( y \).