Econ 475  
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Answer Key Problem Set 1

(1) The per-worker production function is: \( f(k) = k^{1/2} \) since,

\[
y = F(K, L) / L - (K^{1/2} L^{1/2}) / L - (K/L)^{1/2} (L/L)^{1/2} = k^{1/2} \cdot L^{1/2} - k^{1/2}
\]

(2) We assume that:

\[ \delta = 0.05 \]

- Savings rate in country A = \( s^A = 0.10 \)
- Savings rate in country B = \( s^B = 0.20 \)

At BGP: \( s f(k^*) = \delta k^* \) so we can use this equation, the relevant savings rate and our particular functional form for \( f(k) \) to calculate \( k^* \) for each country.

Country A:

\[ s^A f(k^*) = \delta k^* \]

can be written as: \( 0.10 \cdot k^{1/2} = 0.05 \cdot k \)

We solve for \( k^* \):

\[
0.10 / 0.05 = k^{1/2} \\
2 = k^{1/2} \\
4 = k
\]

Then at the BGP:

- Capital per worker: \( k^A = 4 \)
- Income per worker: \( y^A = f(k^A) = 4^{1/2} = 2 \)
- Consumption per worker: \( c^A = (1-s^A) y^A = 0.90 \cdot 2 = 1.8 \)

Country B:

\[ s^B f(k^*) = \delta k^* \]

can be written as: \( 0.20 \cdot k^{1/2} = 0.05 \cdot k \)

We solve for \( k^* \):

\[
0.20 / 0.05 = k \cdot k^{1/2} \\
4 = k^{1/2} \\
16 = k
\]

Then at the BGP:

- Capital per worker: \( k^B = 16 \)
- Income per worker: \( y^B = f(k^B) = 16^{1/2} - 4 \)
- Consumption per worker: \( c^B = (1-s^B) y^B = 0.80 \cdot 4 = 3.2 \)

(3) Suppose at time zero (t=0) both countries have 2 units of capital per worker:

\[ k_0^A = k_0^B = 2 \]
<table>
<thead>
<tr>
<th>time</th>
<th>k^A</th>
<th>y^A</th>
<th>c^A</th>
<th>k^B</th>
<th>y^B</th>
<th>c^B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.00</td>
<td>1.414</td>
<td>1.2730</td>
<td>2.00</td>
<td>1.414</td>
<td>1.1314</td>
</tr>
<tr>
<td>1</td>
<td>2.04</td>
<td>1.428</td>
<td>1.2850</td>
<td>2.18</td>
<td>1.477</td>
<td>1.1819</td>
</tr>
<tr>
<td>2</td>
<td>2.08</td>
<td>1.442</td>
<td>1.2980</td>
<td>2.37</td>
<td>1.539</td>
<td>1.2313</td>
</tr>
<tr>
<td>3</td>
<td>2.12</td>
<td>1.456</td>
<td>1.3104</td>
<td>2.56</td>
<td>1.599</td>
<td>1.2800</td>
</tr>
<tr>
<td>4</td>
<td>2.16</td>
<td>1.469</td>
<td>1.3227</td>
<td>2.75</td>
<td>1.658</td>
<td>1.3267</td>
</tr>
</tbody>
</table>

It will take four years for consumption in country B to exceed that of country A.

We calculated the capital stock per worker in different periods as:

\[ k_{t+1} = k_t + \Delta k = k_t + s \cdot f(k_t) - \delta k_t = k_t + s \cdot k_t^{1/2} - 0.05 k_t \]

so at time 1 in Country A for instance,

\[ k^A_1 = 2.00 + 0.10 \cdot 2^{1/2} - 0.05 \cdot 2 = 2.04 \]

Income per worker (y) is just \( f(k) = k^{1/2} \), so \( y^A_1 = 2.04^{1/2} = 1.428 \) Consumption per worker c is

\[ c = (1-s) \cdot f(k) \]

so:

\[ c^A_1 = 0.90 \cdot 1.428 = 1.285 \]
(II)

Follow model (modified system)

1. \( \dot{y} = A \dot{x} \)

2. \( \dot{z} = 2AZ - (S + N) \)

3. \( \dot{z} = 2AZ - (S + N) \)

4. \( z = n \)

We could use equation (3):

AT BGP original or S.S. modified: \( \dot{x} = 0 \)

At \( t \): \( \dot{z} \) shifts curve in & - - -

Then using diagram and (3) we can conclude:

\( \dot{x} < 0 \) since \( 2AZ - (S + N) \)

This implies that \( y \) until it reaches the new S.S. at \( z^{**} \).

From diagram we can see that in absolute value the growth rates of \( z \) are decreasing as we approach
THE NEW S.S.: \( \frac{1}{\mathcal{H}} \)

\[ \begin{align*}
\mathcal{H} & \downarrow \\
\mathcal{X} & \uparrow \\
\mathcal{Y} & \downarrow \\
\mathcal{Z} & \uparrow
\end{align*} \]

(III) \( n = 0 \)
1. \( L = A \lambda^\alpha \)
2. \( L_h = 2A \lambda^\alpha - \delta \lambda \)
3. \( L_s = 2A \lambda^{\alpha-1} - \delta \)

\( 0 < \alpha < 1 \)
\( A > 0 \)
\( 0 < \delta < 1 \)

WE USE EQUATION (3)

\[ \begin{align*}
\mathcal{H}_n & \downarrow \\
\mathcal{H}_s & \uparrow \\
\mathcal{G} & \downarrow \\
\mathcal{F} & \uparrow
\end{align*} \]

AT BGP ORIGINAL OR SS MODIFIED: \( \mathcal{F} = 0 \)
At $\tau$: $\tau \leq \lambda \Rightarrow \lambda_{\tau} \rightarrow \lambda_{ss}$

From equation (3) and diagram we conclude:

$\lambda_{ss} > 0$ since $\lambda_{0} > \delta$

This implies that $\lambda_{ss}$ until it goes back to the old steady state value $\lambda_{ss}$.

From diagram we can see that growth rates of $\lambda_{ss}$ are decreasing as we approach the $ss$ value $\lambda_{ss}$.