Problem Set 1

(I) Consider the version of the Solow growth model without technological change covered in lecture. Assume there are two countries (Country A and Country B) with closed economies and identical production functions: \( Y = F(K, L) = K^{1/2} L^{1/2} \).

1. Write down the per worker production function \( y = f(k) \)? (where \( y = Y/L, \ k = K/L \))

2. Assume that neither country has population growth or technological progress and that 5% of capital depreciates each year. Assume further that Country A saves 10% of output each year and Country B saves 20% of output each year.

   Find the level of capital per worker for each country at the BGP of the original system (or steady state of the modified system). Then find the levels of income per worker and consumption per worker at the BGP.

3. Suppose that both countries start off (at \( t=0 \)) with a capital stock per worker of 2. What are the levels of income per worker and consumption per worker at the initial period?

   Use a calculator (or Excel) to show how capital stock per worker, output per worker and consumption per worker will evolve over time in both countries.

   How many years will it be before the consumption per worker in Country B is higher than the one in Country A? You can stop your calculations after this happens.

(II) Consider the version of the Solow growth model without technological change covered in lecture. Assume that one country has been at the BGP for many years and suddenly at time \( t^- \) there is a permanent decrease in the savings rate. Show how the economy will adjust to a new BGP by working with the modified system (per capita/worker variables). Show how capital per worker adjusts to the new Steady State level and how its growth rate changes over time.

(III) Consider a version of the Solow growth model without technological change covered in lecture with a rate of population growth of zero (i.e. \( n=0 \)). Assume that the country has been at the BGP for many years and that suddenly at time \( t^- \) there is a onetime increase in its population. Show how the economy will adjust to a new BGP by working with the modified system (per capita/worker variables). Show how capital per worker adjusts to the new Steady State level and how its growth rate changes over time.