I. Consider the following production function: \( Y = F(K, L, H) = K^{1/2} \ L^{1/4} \ H^{1/4} \) where \( Y \) is total output, \( K \) is physical capital, \( H \) is human capital, and \( L \) is labor. Show that the function has constant returns to scale.

II. Between 1975 and 2009, China’s GDP per capita grew at an average rate of 7.9% per year whereas GDP per capita in the US grew at an average rate of 1.8%. In 2009, US GDP per capita was $41,099 and Chinese GDP per capita was $7,634. Assuming that the two countries continue to grow at these rates, in what year will China overtake the US in terms of GDP per capita?

III. Consider a country whose output can be produced with 2 inputs (capital and labor). The output per worker/capita production function can be represented by \( y = k^{1/2} \), where \( y \) represents output per worker/capita and \( k \) capital per worker/capita. Assume that the fraction of output saved/invested is \( \gamma = 0.25 \), the depreciation rate is \( \delta = 0.05 \) and the assumptions of the Solow growth model hold.

(1) What are the steady-state levels of capital and output per worker?

(2) Assume the country starts in year 1 and the level of capital per worker is 16. In a table such as the one below, show how capital and output per worker change over time (the beginning is filled in as a demonstration). Continue this table up to year 8.

<table>
<thead>
<tr>
<th>Year</th>
<th>Capital per worker ( k )</th>
<th>Output per worker ( y = k^{1/2} )</th>
<th>Investment Per Worker ( \gamma y )</th>
<th>Depreciation ( \delta k )</th>
<th>Change in Capital stock per worker ( \gamma y - \delta k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.00</td>
<td>4.00</td>
<td>1.00</td>
<td>0.08</td>
<td>0.20</td>
</tr>
</tbody>
</table>

(3) Calculate the growth rate of output per worker between years 1 and 2.

(4) Calculate the growth rate of output per worker between years 7 and 8.

(5) Comparing your answers to from (1), (3) and (4), what can you conclude about the speed of output per worker growth as a country approaches its steady state?

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