GROWTH MODEL

ASSUME $\hat{A}_N = g$ (TECH. CHANGE IN K – GOODS SECTOR IN NORTH)

- TO KEEP THE NORTH'S COMPARATIVE ADVANTAGE IN K – GOODS (OR TECHNOLOGICAL ADVANTAGE) WE ASSUME $\hat{A}_S < \hat{A}_N$
- ASSUME
  - TRANSPORT COSTS CONSTANT (i.e. d's DO NOT CHANGE)
  - $\hat{L} = 0$ (NO POP. GROWTH)
  - SOLOW'S ASSUMPTION:
    A COUNTRY SPENDS A FRACTION a OF ITS INCOME ON K – GOODS (SINCE SAVINGS RATE CONSTANT & BECAUSE SAVINGS EQUALS VALUE OF INVESTMENT)

(11) $$a Y_t = p^K_t I_t$$

$\Rightarrow$ (11') $$I_t = a Y_t / p^K_t$$

LAW OF MOTION OF K:

$$\dot{K}_t = I_t - \delta K_t$$

OR

(12) $$\hat{K}_t = \frac{I_t}{K_t} - \delta$$

USING (11') & (12)

(12') $$\boxed{\hat{K}_t = \frac{a Y_t}{p^K_t K_t} - \delta}$$

IMPLICATIONS :

(I) GROWTH RATES OF PRICES

PRICES ARE GIVEN BY:

$$p^C_S = 1 \qquad p^C_N = d^C$$

$$p^k_S = \frac{d^k d^c}{A^N} \qquad p^k_N = \frac{d^c}{A_N}$$

TAKING GROWTH RATES:

$$\hat{p}^C_S = 0 \qquad \hat{p}^C_N = 0$$

$$\boxed{\hat{p}^k_S = -\hat{A}^N = -g} \quad \boxed{\hat{p}^k_N = -\hat{A}^N = -g}$$

$$\Rightarrow (13) \quad \boxed{\hat{p}^k = -g} \quad \text{in BOTH COUNTRIES}$$

(II) GROWTH RATE OF INVESTMENT

BY EQUATION (11') $\quad I_t = \dfrac{a Y_t}{p^k_t}$

TAKING GROWTH RATES:

$$(14) \qquad \hat{I} = \hat{Y} - \underset{-g}{\underset{\parallel}{\hat{p}^k}} = \hat{Y} + g$$

BY (10) $\quad y_N = \dfrac{Y_N}{L_N p^c_N}$, $\quad y_S = \dfrac{Y_S}{L_S p^c_S}$

TAKING GROWTH RATES $\quad \boxed{\begin{array}{l} \hat{y}_N = \hat{Y}_N \\ \hat{y}_S = \hat{Y}_S \end{array}}$

Let $\hat{y} = g_y = \hat{Y}$ then : (14) $\boxed{\hat{I} = g_y + g}$

(III) LAW OF MOTION OF CAPITAL PER WORKER -

USING (11')

(15) $$\hat{k} = \hat{K} - \hat{L} = \frac{sY}{p^k K} - \delta$$

WE WORK NOW WITH Y/K.

(16) $$\frac{Y}{K} \underset{\text{DIV. \& MULTIP. BY } L\,p^c}{=} \frac{Y/L \cdot p^c}{K/L \cdot p^c} \underset{\text{USING (10)}}{=} \frac{y}{k/p^c} = \frac{f(k)}{k/p^c}$$

THEN

(17) $$\boxed{\hat{k} = \frac{s\, f(k)}{k\, \frac{p^k}{p^c}} - \delta}$$

ASSUME SPECIFIC PROD. FUNCTION

(18) $$F(K, L) = K^{\alpha} L^{1-\alpha} \Rightarrow \boxed{y = f(k) = k^{\alpha}}$$ (18')

THEN (17) IS NOW:

(19) $$\boxed{\hat{k}} = \frac{s\, k^{\alpha}}{\frac{p^k}{p^c}\, k} - \delta = \boxed{\frac{s\, k^{\alpha-1}}{\frac{p^k}{p^c}} - \delta}$$

BGP : WE CALCULATE <u>GROWTH RATE OF OUTPUT PER WORKER</u> AND <u>OUTPUT PER WORKER</u>

ALONG BGP : $\hat{Y}, \hat{K}, \ldots$ CONSTANT

EQ. (12'): $\hat{K} = \dfrac{sY}{p^K K} - \delta$

$\Rightarrow$

$$\frac{Y}{p^K K} \text{ CONSTANT} \Rightarrow \hat{Y} = \underbrace{\hat{p}^K}_{-g} + \hat{K}$$

$$\Rightarrow \hat{Y} = -g + \hat{K} \Rightarrow \hat{K} = \underbrace{\hat{Y}}_{g_y} + g$$

$$\Rightarrow \boxed{\hat{h}} = \hat{k} - \overset{=0}{\hat{L}} = \boxed{g_y + g} \quad (20)$$

TAKING GROWTH RATES IN (18'):

$$\hat{y} = \alpha \hat{h} \quad (21)$$

USING (20) & (21):

$$\hat{h} = \alpha \hat{h} + g \Rightarrow \boxed{\hat{h} = \frac{g}{1-\alpha}} \quad (22)$$

$$\& \quad \boxed{\hat{y} = \frac{\alpha}{1-\alpha} g} \qquad \underline{\text{GROWTH RATES}}$$

OUTPUT PER WORKER AT BGP

USING (19) & (22)

$$\hat{k} = \frac{\alpha k^{\alpha-1}}{\frac{p^k}{p^c}} - \delta = \frac{g}{1-\alpha}$$

$$\Rightarrow \quad \frac{\alpha}{\frac{p^k}{p^c}} = k^{1-\alpha}\left[\frac{g}{1-\alpha} + \delta\right]$$

$$\Rightarrow \quad k = \left[\frac{\alpha}{\frac{p^k}{p^c}\left[\frac{g}{1-\alpha}+\delta\right]}\right]^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \quad \boxed{y^* = k^\alpha = \left[\frac{\alpha}{\left(\delta+\frac{g}{1-\alpha}\right)\frac{p^k}{p^c}}\right]^{\frac{\alpha}{1-\alpha}}}$$

RELATIVE PRICES OF K-GOODS $\left(\frac{p^k}{p^c}\right)$

AFFECT <u>NEGATIVELY</u> <u>OUTPUT PER WORKER</u>

<u>CONCLUSION</u>:

- $\uparrow \frac{p^k}{p^c} \Rightarrow \downarrow y^*$
- $\hat{y}^* = \frac{\alpha}{1-\alpha} g$ → RATE OF TECH. CHANGE IN K-GOODS SECTOR

  ↳ GROWTH RATE OF OUTPUT PER WORKER AT BGP

EMPIRICAL:

• ENRICH MODEL BY ADDING MORE COUNTRIES & HETEROGENOUS K-GOODS
⟹ FOLLOWING EQUATIONS OBTAINED:

(A) K-GOODS IMPORT SHARES DEPEND ON: TECHNOLOGIES, PROD. COSTS, TRANSPORT COSTS (OR TRADE BARRIERS) —

(B) K-GOODS PRICES DEPEND ON: TECHNOLOGIES, PROD. COST, TRANSPORT COSTS

(C) OUTPUT PER WORKER DEPENDS ON: SAVINGS RATE, RELATIVE PRICES OR K-GOODS (BASICALLY EQ. 21)

THEY PLUG (A) INTO B TO ESTIMATE RELATIVE PRICES OF K-GOODS —

USE THOSE ESTIMATED RELATIVE PRICE TO ESTIMATE (C) FOR A SET OF COUNTRIES —

RESULT: DIFFERENCES IN RELATIVE PRICES OF K-GOODS EXPLAIN ABOUT 26% OF DIFFERENCES IN INCOME PER WORKER —