MODELS I & II DETAILS

PAYOFF FUNCTIONS:
- FIRMS 1, 2: PROFITS
- GOVERNMENTS 1 & 2: WELFARE IN THE MARKET CONSIDERED (PARTIAL EQUILIBRIUM):

\[ W_i = CS_i + PS_i + \text{GOV. REVENUE}_i \]

\[ \text{COUNTRY } i = 1, 2 \]

"TOTAL PROFITS FIRM i" (INCLUDES TAXES/SUBSIDIES)

Since all output exported
\[ CS_i = 0 \]

\[ i = 1, 2 \]

\[ \text{PER UNIT EXPORT/PROD SUBSIDY} \]

Assume \( S_i > 0 \) SUBSIDY < TAX

\[ \Rightarrow \]

\[ \text{GOV. REVENUE} = -S_i \cdot q_i \]
MODEL I: Homogeneous Products
Quantity Competition

Country 3 Demand:

\[ p = 36 - \frac{q}{9} = 36 - \frac{q_1 + q_2}{9} \]

Effective Costs for Firms 1, 2:

\[ C_1 = \text{EFF. MC Firm 1} \]
\[ C_2 = \text{EFF. MC Firm 2} \]

MC\(1\) = Marginal Cost of Production Firm 1

MC\(2\) = Marginal Cost of Production Firm 2

\[ \Rightarrow C_1 = MC_1 - S_1 \]
\[ C_2 = MC_2 - S_2 \]

Where \( S_i > 0 \)

Remark:

MODEL I
\[ S_1 > 0 \text{ or } < \]
\[ S_2 = 0 \]

MODEL II
\[ S_1 > 0 \text{ or } < \]
\[ S_2 > 0 \text{ or } < \]
S P U E : M O D E L I

STEP 1: FIND NE QUANTITIES $q_1, q_2$ FOR ANY GIVEN $S_1$.

STEP 2: FIND $S_1$ THAT MAXIMIZES WEFAKE FOR COUNTRY 1 ($W_1$) ASSUMING THE NE WILL PREVAIL IN THE LAST STAGE.

REMARK: FOR STEP 1 WE WILL DO TWO THINGS:
(i) FIND NE IN $q$'S FOR ANY $C_1, C_2$
(ii) REPLACE AS FOLLOWS: $C_e = MC_1 - S_1$ $C_2 = MC_2$
**STEP 1**

1. Find NE q_i's for any c_1, c_2
2. Find best responses
3. **Intersection** of best responses

**Firm 1:**

\[
\max \; \Pi_1(q_1, q_2) = [p - c_1] \cdot q_1 \\
q_1 = \left[ 3c - q_1 - q_2 - c_1 \right] \cdot q_1 \\
= \left\{ (3c - c_1) - q_1 - q_2 \right\} \cdot q_1
\]

\[
\frac{\partial \Pi_1}{\partial q_1} = 0 \quad \Rightarrow \quad \hat{q}_1 = B_1(q_2) = \frac{3c - c_1 - q_2}{2}
\]

**Firm 2:**

\[
\max \; \Pi_2(q_1, q_2) = [p - c_2] \cdot q_2
\]

\[
\frac{\partial \Pi_2}{\partial q_2} = 0 \quad \Rightarrow \quad \hat{q}_2 = B_2(q_1) = \frac{3c - c_2 - q_1}{2}
\]

**Find "Intersection" of best responses (i.e. solve 2 equations, 2 unknowns)**
We have that 

\[ f_1(s) = \frac{1}{3} \begin{bmatrix} 36 - 2s \\ 36 - s \end{bmatrix} \]

and 

\[ f_2(s) = \frac{1}{3} \begin{bmatrix} 36 - 2s \\ 36 - s \end{bmatrix} \]

In this case we reduce 

\[ c_1 \neq c_2 \] 

and 

\[ \frac{t_1}{t_2} = \frac{1}{3} \begin{bmatrix} 3c - 2c \\ c \end{bmatrix} \]
STEP 2: FIND $s_1^*$ that max. $W_1$

ASSUMING NE IN LAST STAGE

(i.e. $p_1^{NE}, q_1^{NE}, q_2^{NE}$)

$$W_1(s_1) = cS_1 + R S_1(s_1) + \text{Gov. Rev}_1(s_1)$$

"TOTAL"
PROFITS Firm 1

$$\pi_1^{NE}(s_1)$$

$$\Rightarrow$$

$$W_1(s_1) = [p^{NE} - c_1].q_1^{NE} - s_1.q_1^{NE}$$

$$= [p^{NE} - (mc_1 - s_1)].q_1^{NE} - s_1.q_1^{NE}$$

$$= (p^{NE} + s_1).q_1^{NE} - s_1.q_1^{NE}$$

$$= \underbrace{p^{NE}.q_1^{NE} + s_1.q_1^{NE}}_{\text{production profits for Firm 1: } (p - mc_1).q_1^{NE}}$$

$$= \frac{1}{3} [36 - s_1]$$

$$= \frac{1}{3} [36 + 2s_1]$$
\[
\max_{s_1} w_1(s_1) = \frac{1}{q} \left[ 36 - s_1 \right] \cdot \left[ 36 + 2s_1 \right]
\]

\[
\Rightarrow \frac{d}{ds_1} = \frac{1}{q} \left\{ (-1) \left[ 36 + 2s_1 \right] + 2 \cdot \left[ 36 - s_1 \right] \right\} = 0
\]

\[
0 = 36 - 4s_1 = 0 \Rightarrow s_1^* = 9 > 0
\]

Profit shifting export subsidy

SPNE:

GOV 1: \[ s_1^* = 9 \]

FIRM 1: \[ q_1^{NE}(s_1) = \frac{1}{3} \left[ 36 + 2s_1 \right] \]

FIRM 2: \[ q_2^{NE}(s_1) = \frac{1}{3} \left[ 36 - s_1 \right] \]

SPNE outcome:

\[ s_1^* = 9 \]

\[ q_1^{NE}(9) = \frac{36 + 18}{3} = 18 \]

\[ q_2^{NE}(9) = \frac{36 - 9}{3} = 9 \]

\[ p_2^{NE}(9) = \frac{1}{3} \left[ 36 - 9 \right] = 9 \]

\( q^* = 27 \leq \)
\[ w_1(q) = \left[ P^{NE} - m_{C1} \right] \cdot q_{NE} \]
\[ = 9 \times 18 = 162 \]
\[ w_2(q) = C_{S2} + P_{S2} + \text{Gov. Rev. 2} \]
\[ = \left[ P^{NE} - m_{C2} \right] \cdot q_{NE} = 9 \times 9 = 81 \]
\[ w_3(q) = C_{S3} + P_{S3} + \text{Gov. Rev. 3} \]
\[ = \frac{[3C-q]}{2} \cdot q_{NE} \]
\[ = \frac{27 \times 27}{2} \]

Diagram:
- **Country 3 Market**
- **CS3**
- **3C**
- **q**
- **p = 3C - q**
Comparison of outcomes of SpnE for different scenarios for our example: $p = 3G - f_1 - f_2$.

<table>
<thead>
<tr>
<th>No Gov.</th>
<th>Model I: $s_i &gt; 0, s_j &gt; 0$</th>
<th>Model II: $s_i &gt; 0, s_j &lt; 0$</th>
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<tbody>
<tr>
<td>$f_1$</td>
<td>12</td>
<td>18</td>
</tr>
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<td>$f_2$</td>
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<td>9</td>
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<tr>
<td>$f$</td>
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<tr>
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<td>81</td>
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<tr>
<td>$w_3$</td>
<td>27x27</td>
<td>$\frac{27x27}{2}$</td>
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</table>

Remarks:

**Model II**: $s_i > 0$ shifts profits from Firm 2 to Firm 1; $T_1^+ T_2^-$ compared to no intervention.

Country 3 benefits from $s_i > 0$ (i.e., $W_3^+$ compared to no intervention).
Assume

\[ p = BC - q_1 - q_2 \]
\[ MC_1 = MC_2 = 0 \]

SPNE:
Work backwards

**STEP 1**: NE in last stage

- Find best responses for plans with effect, marginal costs \( C_1 \geq C_2 \)
- Find "intersection"

Calculate NE outcome:

\[ q_1(C_1, C_2) \quad p(C_1, C_2) \]

**STEP 2**: NE in \( s_1, s_2 \) assuming NE in \( q \)'s holds in last stage

- Find best responses for welfare max., governments

\[ s_1 = B_1(s_2) \]
\[ s_2 = B_2(s_1) \]
(2) Find "INTERSECTION" 
Solving 2 Eq. 1: (b/n)

**STEP 1:**

1. **Best Responses (Last Class Page 4)**
   \[ q_1 = B_1(q_2) = (36 - c) - q_2 \]
   \[ q_2 = B_2(q_1) = \frac{2}{36 - c_2} - q_1 \]

2. **Solving (Last Class Page 3)**
   \[ q_{1, NE} = \frac{1}{3} [36 - 2c_1 + c_2] \]
   \[ q_{2, NE} = \frac{1}{3} [36 - 2c_2 + c_1] \]
   In this case
   \[ c_1 = 3c_1 - s_1 = -s_1 \]
   \[ c_2 = 3c_2 - s_2 = -s_2 \]

3. **Replacing**
   \[ q_{1, NE}(s_1, s_2) = \frac{1}{3} [36 + 2s_1 - s_2] \]
   \[ q_{2, NE}(s_1, s_2) = \frac{1}{3} [36 + 2s_2 - s_1] \]
   \[ p_{NE}(s_1, s_2) = \frac{1}{3} [36 - s_1 - s_2] \]
Step 2:

Find best responses for government.

Gov. 1:

\[
\text{Max } W_1(S_1, S_2) = CS_1 + PS_1 + \text{Gov. rev}_1
\]

\[
0 \text{ "Effective profits"}
\]

\[
S_1 \neq C_1 - S_1 \cdot q_{1, \text{NE}}
\]

\[
= \left[ p_{1, \text{NE}} - C_1 \right] \cdot q_{1, \text{NE}} - S_1 \cdot q_{1, \text{NE}}
\]

\[
= \left[ p_{1, \text{NE}} - MC_1 + S_1 \right] \cdot q_{1, \text{NE}} - S_1 \cdot q_{1, \text{NE}}
\]

\[
= \frac{p_{1, \text{NE}} \cdot q_{1, \text{NE}}}{P_1, \text{NE}} + S_1 \cdot q_{1, \text{NE}} - S_1 \cdot q_{1, \text{NE}}
\]

Production profits

Using previous results, \( \Rightarrow \)

\[
\text{Max } S_1 = \frac{1}{3} \left[ 3G - S_1 - S_2 \right] \cdot \frac{1}{3} \left[ 3G + 2S_1 - S_2 \right]
\]

\[
\frac{\partial}{\partial S_1} = 0 = \frac{1}{9} \left\{ (-1)[3G + 2S_1 - S_2] + 2 \cdot [3G - S_1 - S_2] \right\} = 0
\]
\[ S_1 = B_1(S_2) = \frac{3C - S_2}{4} \]

**Similarily for country 2:**

\[ S_2 = B_2(S_1) = \frac{3C - S_1}{4} \]

\[ S_1^* = 7.2 \]
\[ S_2^* = 7.2 \]

**Conclusion**

**SPNE:**

\[ S_1^* = S_2^* = 7.2 \]

\[ \varphi_1^{NE} = \frac{1}{3} \left[ 3C + 2S_1 - S_2 \right] \]
\[ \varphi_2^{NE} = \frac{1}{3} \left[ 3C + 2S_2 - S_1 \right] \]

**Outcome at SPNE:**

\[ S_1^* = S_2^* = 7.2 \]
\[ \varphi_1^* = \varphi_2^* = 14.4 \]
\[ p^* = 2.2 \]
\[ \omega_1^* = p_1^* \cdot \varphi_1^* = 103.7 \]
\[ \omega_2^* = p_2^* \cdot \varphi_2^* = 103.7 \]

**See page 9 from last class for comparison**
**Big Result:**

Outcome is worse than when \( s_1 = s_2 = 0 \) for countries 1 and 2.

Outcome is better for country 3.

**Next Topics**

1. What changes if firms compete in prices (differentiated products) instead of quantities?
   - \( s_1 < 0 \)
   - \( s_2 < 0 \)

2. International agreements motivated by our big result.
A) With Prod/Export Subsidies 
competition \( \rightarrow \) \( s_1 > 0, s_2 > 0 \) 
Subsidies! 
\( p \) competition \( \rightarrow \) \( s_1 < 0, s_2 < 0 \) 
Taxes!

Problem!
But if we consider 
R&D Subsidies/Taxes
Instead
the result does not change with \( p \) or \( q \)
competition in the output market.
Subsidies!

B) GATT/WTO
Agreement US / EU.
Figure 1: Intervention by Europe

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Airbus

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---------|-----------|
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Not enter| 0.56      | 0.0       |

Figure 2: Intervention by both governments

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