(I) Duopoly with homogeneous product: quantity and price competition

Consider a market with two firms (firms 1 and 2). The (inverse) demand curve is:
\[ p = 30 - 2q \]
where \( q = q_1 + q_2 \)

Suppose Firm 1 has a marginal cost \((c_1)\) of 6 and Firm 2 has a marginal cost \((c_2)\) of 12. Assume neither firm has a fixed cost.

(1) Suppose firms choose quantities sequentially: Firm 1 chooses \(q_1\) and after that decision is known Firm 2 chooses \(q_2\). Calculate the SPNE.

(2) Suppose firms choose prices sequentially: Firm 2 chooses its price first and after that decision is known Firm 1 chooses its price. What is the SPNE? Justify.

(II) Duopoly with Homogenous products and Fixed Costs: price competition

Consider a market with two firms and a market inverse demand:
\[ p = 90 - q \]
where \( q \) is the total market output

Firms have different marginal and fixed costs:
Firm 1: \( c_1 = 50 \) and \( FC_1 = 0 \)
Firm 2: \( c_2 = 0 \) and \( FC_2 = 50 \)

Assume the two firms choose prices simultaneously:
(1) What is the Nash Equilibrium in prices? Justify.
(2) How many units of output will each firm produce at the NE?

(III) Duopoly with differentiated products, quantity competition

Consider the case of two firms (Firm 1 and Firm 2) that produce differentiated products, compete in quantities and have zero marginal and fixed costs.
The demands they face are:
\[ p_1 = 100 - q_1 - \frac{1}{2} q_2 \]
\[ p_2 = 100 - q_2 - \frac{1}{2} q_1 \]
where \( q_i \) and \( p_i \) are the quantity sold and price charged by firm \( i \) (\( i = 1, 2 \)).

(1) Suppose firms choose quantities simultaneously. Calculate the NE in quantities.
(2) Suppose firms choose quantities sequentially: Firm 2 chooses \( q_2 \) and after that decision is known Firm 1 chooses \( q_1 \). Calculate the SPNE.