HW #3 ANSWER KEY

Two identical MKTs: $p_i = 100 - Q_i$

Two identical firms: $MC_i = 10,\ FC_i = 0$

Transport costs: $t = 5$

(I) Free trade

1) $\Pi_i = q_i^H p_i + q_i^F p_j - 10(q_i^H + q_i^F) - 5q_i^F$

$= q_i^H\left[(100 - q_i^H - q_j^F) - 10\right] + q_i^F\left[(100 - q_i^F - q_j^H) - 10 - 5\right]$

$= q_i^H(90 - q_i^H - q_j^F) + q_i^F(85 - q_i^F - q_j^H)$

$\frac{\partial \Pi_1}{\partial q_1^H} = q_1^H(-1) + (90 - q_1^H - q_2^F) = 0$

$\rightarrow q_1^H = \frac{90 - q_2^F}{2}$ (1)

$\frac{\partial \Pi_1}{\partial q_1^F} = q_1^F(-1) + (85 - q_1^F - q_2^H) = 0$

$\rightarrow q_1^F = \frac{85 - q_2^H}{2}$ (2)

$\frac{\partial \Pi_2}{\partial q_2^H} = q_2^H(-1) + (90 - q_2^H - q_1^F) = 0$

$\rightarrow q_2^H = \frac{90 - q_1^F}{2}$ (3)

$\frac{\partial \Pi_2}{\partial q_2^F} = q_2^F(-1) + (85 - q_2^F - q_1^H) = 0$

$\rightarrow q_2^F = \frac{85 - q_1^H}{2}$ (4)

Solving for country 1 ((1) + (4)):

$$q_1^H = \frac{90}{2} - \frac{1}{2}\left(\frac{85 - q_1^H}{2}\right)$$

$$q_1^H = \frac{180 - 85}{4} + \frac{1}{4} q_1^H$$

$$\frac{3}{4} q_1^H = \frac{95}{4}$$

$$\boxed{q_1^H = \frac{95}{3}} = 31,\overline{6}$$

$$\rightarrow q_2^F = \frac{1}{2}\left(85 - \frac{95}{3}\right)$$

$$q_2^F = \frac{1}{2}\left(\frac{255 - 95}{3}\right)$$

$$q_2^F = \frac{1}{2}\left(\frac{160}{3}\right)$$

$$\boxed{q_2^F = \frac{80}{3}} = 26,\overline{6}$$

$$\Rightarrow q_1 = q_1^H + q_2^F$$

$$q_1 = \frac{95 + 80}{3}$$

$$\boxed{q_1 = \frac{175}{3}} = 58,\overline{3}$$

$$p = 100 - \frac{175}{3}$$

$$\boxed{p_1 = \frac{125}{3}} = 41,\overline{3}$$

By the symmetry of the model:

$$q_1^F = \frac{80}{3}$$

$$q_2^H = \frac{95}{3}$$

$$q_2 = \frac{175}{3}$$

$$p_2 = \frac{125}{3}$$

$$\Rightarrow \pi_1 = \pi_2 = \frac{125}{3}\left(\frac{95}{3} + \frac{80}{3}\right) - 10\left(\frac{95}{3} + \frac{80}{3}\right) - 5\left(\frac{80}{3}\right)$$

$$= \frac{95}{3}\left(\frac{175}{3}\right) - \frac{400}{3}$$

$$= \frac{16,625 - 1,200}{9}$$

$$\boxed{\pi_i = \frac{15,425}{9}} = 1713,\hat{8}$$

2) YES Since $p_1 = p_2$, and $\tau = 5$, then we have $p^F < p^H + \tau$

(II) Autarky

$q_i = q_i^H$, $q_i^F = 0$

$\rightarrow P_i = 100 - q_i$

$MR_i = 100 - 2q_i$

1) To max Π, $MR = MC$: $100 - 2q_i = 10$

$90 = 2q_i$

$\boxed{q_i^A = 45}$

$\rightarrow P_i = 100 - 45$

$\boxed{P_i^A = 55}$

2) $W_i^A = \underbrace{\frac{(100-55)45}{2}}_{CS_i} + \underbrace{(55-10)45}_{\Pi_i}$

$= \frac{45^2}{2} + 45^2$

$= \frac{3 \cdot 45^2}{2}$

$= \frac{3 \cdot 2025}{2}$

$= \frac{6075}{2}$

$\boxed{W_i^A = 3037,5}$ ONE COUNTRY

$\boxed{W_{AUTARKY}^{WORLD} = 2 \times 3037.5 = 6075}$

With free trade:

$$W_i^{FT} = \frac{1}{2}\left(100 - \frac{125}{3}\right)\frac{175}{3} + \frac{15,425}{9}$$

$$= \frac{1}{2}\left(\frac{175}{3}\right)^2 + \frac{15425}{9}$$

$$= \frac{1}{2}\left(\frac{30625}{9}\right) + \frac{15425}{9}$$

$$= \frac{30625 + 30850}{18}$$

$$= \frac{61475}{18}$$

$$\boxed{W_i^{FT} = 3415,27\overline{7}} \qquad \boxed{W_{FT}^{world} = 2 \times 3415.27 = 6830.54}$$

⇒ Despite the waste of resources, the lower price & higher quantity traded in each mkt as a result of increased competition, lead to an increase in welfare when there is trade btw countries.