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Answer Key Homework 2

(I) Consider two identical firms (Firm 1 and Firm 2) that produce an homogenous product. The demand for their product is :

$$P= 200 - Q, \quad \text{where } Q= q_1 + q_2.$$

Each firm has a cost function: $C(q_i) = 20\, q_i$ (i.e. the Mc_i is 20 and there are no fixed costs)

(1) Calculate the Stakelberg equilibrium assuming that Firm 1 is the Leader and Firm 2 the Follower (i.e. calculate the SPNE assuming that Firm 1 moves first).

(a) Step 1: calculate the reaction function for firm 2.
Done in another problem:

(*) $q_2 = (180 - q_1) / 2$ **.**

(b) Step 2 : calculate the profit maximizing q_1 assuming that firm 2 will behave according to its reaction function.
Choose q_1 to maximize $\Pi_1 (q_1, q_2)$ assuming q_2 is given in equation (*):

$$\Pi_1 (q_1) = [200 - q_1 - ((180 - q_1) / 2)]\, q_1 - 20\, q_1 = [180- q_1 - ((180 - q_1) / 2)]\, q_1 .$$

Setting the first derivative equal to zero :

$$d\,\Pi_1 (\,) / d\, q_1 = (180 /2) - 2q_1 + q_1 = 0 \text{ then :}$$

$q_1= 90$, $q_2 = 45$, $p= 65$,

$\Pi_1 = 4050$
$\Pi_2 = 2025$

(II) Differentiated Products

Two firms compete by choosing price. Their demand functions are:

$$q_1 = 20 - p_1 + p_2$$
$$q_2 = 20 - p_2 + p_1,$$

where p_1 and p_2 are the prices charged by each firm, and q_1 and q_2 the resulting quantities demanded. Marginal costs are zero.

Suppose the two firms set their prices at the same time. Find the resulting Nash Equilibrium in prices. Calculate prices, quantities and profits for each firm.

To calculate the reaction function for firm i we maximize:
$\Pi_i = p_i\, q_i = p_i \; [20 - p_i + p_j]$
with respect to p_i , assuming p_j is a constant.

Reaction Functions: $p_i = (20 + p_j) /2$ for i,j=1,2
NE in prices: $p_1 = p_2 = 20$
Quantities produced at NE prices: $q_1 = q_2 = 20$
Profits: 400 for each firm.

(III) Consider two firms (Firm 1 and Firm 2) from two different countries (Country 1 and Country 2) that produce a homogeneous product for export to a third country.
Marginal costs are zero for both firms and the demand for their product is :

$$p = 120 - q_1 - q_2$$

Firms behave as Cournot duopolists choosing output simultaneously. Suppose that countries can give their firms an export subsidy of s_1 and s_2 dollars per unit respectively before both firms choose their output.
In other words, countries and firms are engaged in a sequential game. In stage 1, countries choose simultaneously their subsidy levels in order to maximize social welfare. In stage 2 , after the subsidy choices are known, both firms choose their output level simultaneously.
Calculate the subsidy levels that countries will choose at the SPNE (you are calculating the Nash Equilibrium subsidy levels).

Step 1: calculate NE in outputs in stage 2 for any subsidy pair.

To calculate the reaction function for firm i we maximize:
$\Pi_i = (p_i - Mc_i)q_i = (120 - Mc_i - q_i - q_j)\ q_i$
with respect to q_i , assuming q_j is a constant.
Setting the partial derivative equal to zero for i and j, solving the two equations and replacing Mc_i for $(-\ s_i)$ we obtain:

$$q_1 = (1/3)\ [120 + 2\ s_1 - s_2]$$

$$q_2 = (1/3)\ [120 + 2\ s_2 - s_1]$$

$$p = (1/3)\ [120 - s_1 - s_2]$$

Step 2: calculate the NE in subsidies assuming countries maximize welfare. To calculate the reaction function for country i we maximize:

$$W_i = (p_i - Mc_i)q_i - s_i\ q_i = p_i\ q_i + s_i\ q_i - s_i\ q_i = p_i\ q_i =$$
$$= (1/3)\ (120 - s_i - s_j)(1/3)\ (120 + 2\ s_i - s_j)$$

with respect to s_i , assuming s_j is a constant.
Setting the partial derivative equal to zero for i and j we get the reaction functions:
$s_1 = (120 - s_2)/\ 4$, $s_2 = (120 - s_1)/\ 4$

and solving the two equations we obtain the NE is subsidies:
$s_1 = s_2 = 24$