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Answer Key Problem Set 1

(I) Consider a game with two players: Player 1 and Player 2. Player 1's action set is {u,d,s} and Player 2's action set is {U,D}. Their payoffs are given in the matrix below.

		Player 2	
		U	D
Player 1	u	0 4	0 2
	d	4 2	0 1
	s	2 6	2 6

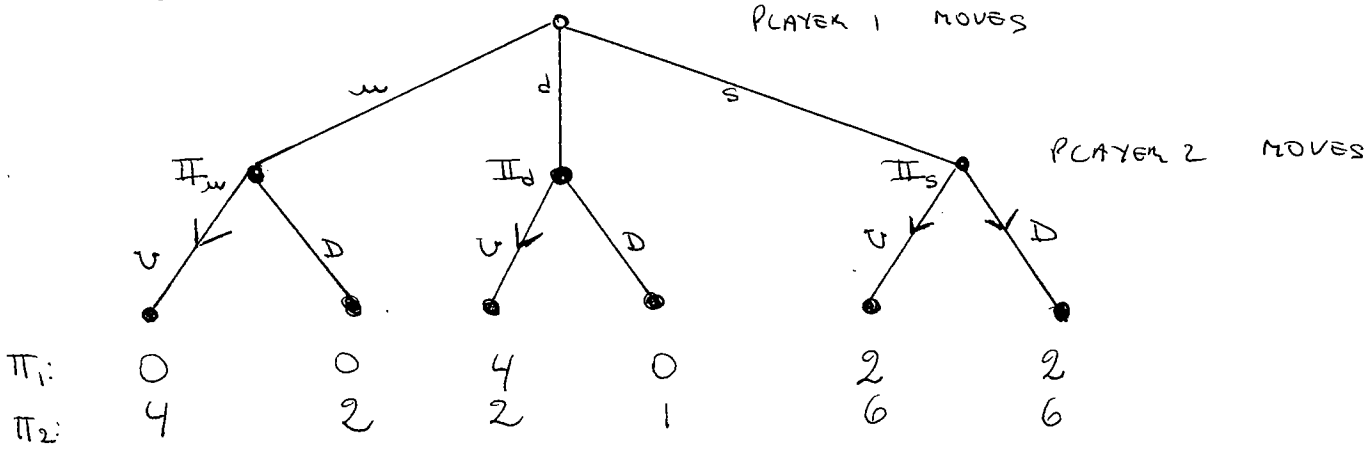
(1) Suppose that the game is played only once and that both players move simultaneously. Find the Nash Equilibrium/a.

(a) If Player 2 is playing U, Player 1 will play d (i.e. $d=B_1(U)$).
If Player 2 is playing D, Player 1 will play s (i.e. $s=B_1(D)$).
(b) If Player 1 is playing u, Player 2 will play U (i.e. $U=B_2(u)$).
If Player 1 is playing d, Player 2 will play U (i.e. $U=B_2(d)$).
If Player 1 is playing s, Player 2 will play U or D (i.e. $\{U,D\}=B_2(s)$).

So there are two N.E. in this game:
(d, U) because $d=B_1(U)$ and $U=B_2(d)$ and,
(s, D) because $s= B_1(D)$ and D belongs to $B_2(s)$.

(2) Suppose instead that Player 1 moves first and then does Player 2 (he observes Player 1's action before he moves). Construct the game tree for this game. Find the Subgame Perfect Nash Equilibrium (SPNE).

When Player 1 moves first the game tree looks as follows:



The arrows indicate the chosen action at each subgame. At the SPNE, Player 1 will use action d and Player 2 will use U at node II_u , U at node II_d and either U or D at node II_s .
So the SPNE strategy profile is : d for Player 1 and (U, U, U or D) for Player 2.

(II) Consider a game with two players (1 and 2) and three possible actions for each. Player's 1 action set is {U, M, D} and Player's 2 action set is {L, M, R}. Suppose that the game is played only once, that both players move simultaneously and that the payoffs are given below. Calculate the best response functions for each player and the Nash Equilibrium/Equilibria of the game.

		Player 2		
		L	M	R
Player 1	U	5 5	2 6	1 8
	M	6 2	4 4	2 3
	D	8 1	3 2	0 0

We calculate the Best response functions.

(a) If Player 2 is playing L, Player 1 will play D (i.e. $D=B_1(L)$).
If Player 2 is playing M, Player 1 will play M (i.e. $M=B_1(M)$).
If Player 2 is playing R, Player 1 will play M (i.e. $M=B_1(R)$).

(b) If Player 1 is playing U, Player 2 will play R (i.e. $R=B_2(U)$).
If Player 1 is playing M, Player 2 will play M (i.e. $M=B_2(M)$).
If Player 1 is playing R, Player 2 will play M (i.e. $M=B_2(R)$).

So the NE is (M, M) since $M= B_1(M)$ and $M= B_2(M)$.

(III) Consider two firms in the cereal market. They can decide to produce either Crispy(C) or Sweet (S) cereal and their profits are given in the matrix below.

		Firm 2	
		C	S
Firm 1	C	-5 -5	10 20
	S	20 10	-5 -5

(1) Suppose that both firms announce their decisions simultaneously. What will be outcome of the game be? (i.e. find the Nash Equilibrium/a).

We calculate the Nash Equilibrium/a looking at the Best response functions.

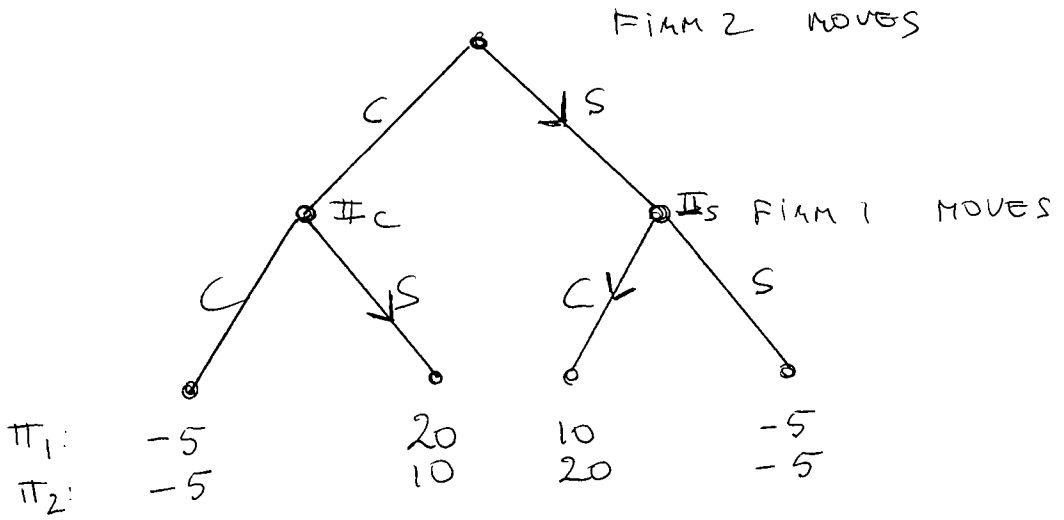
(a) If Firm 2 is playing C, Firm 1 will play S ($S= B_1 (C)$).
If Firm 2 is playing S, Firm 1 will play C ($C= B_1 (S)$).
(b) If Firm 1 is playing C, Firm 2 will play S ($S= B_2 (C)$).
If Firm 1 is playing S, Firm 2 will play C ($C= B_2 (S)$).

So the Nash equilibria are:
(C,S) since $C=B_1 (S)$ and $S= B_2 (C)$ and,

(S,C) since $S=B_1 (C)$ and $C= B_2 (S)$.

(2) Suppose Firm 2 moves first and his action is known to Firm 1 when it moves. Draw the game tree and find the SPNE.

When Firm 2 moves first the game tree looks like this:

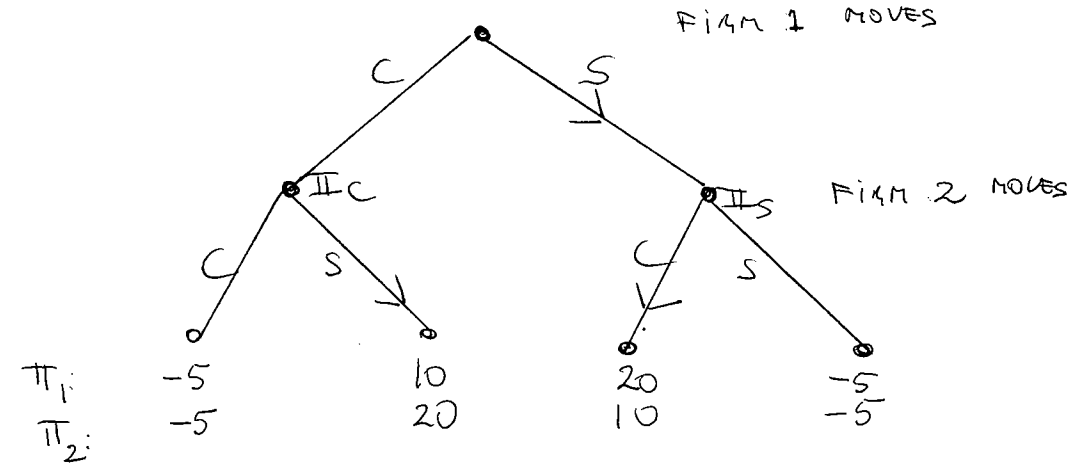


The arrows indicate the chosen action at each subgame. The following strategy profile is the SPNE:

Firm 2: Strategy S
Firm 1: (S, C) (i.e. strategy S is used in at node Π_C and strategy C is used at node Π_S).

(3) Suppose Firm 1 moves first and his action is known to Firm 2 when it moves. Draw the game tree and find the SPNE.

When Firm 1 moves first, the game tree looks like this:



The arrows indicate the chosen action at each subgame. The following strategy profile is the SPNE:

Firm 1: Strategy S
Firm 2: (S, C) (i.e. strategy S is used in at node Π_C and strategy C is used at node Π_S).

(IV) Consider two identical firms (Firm 1 and Firm 2) that produce an homogenous product . The demand for their product is :

$$P= 200 -Q, \quad \text{where } Q= q_1 + q_2.$$

Each firm has a cost function: $C(q_i) = 20\ q_i$ (i.e. the Mc_i is 20 and there are no fixed costs)

(1) Calculate the Cournot Equilibrium (price and quantities, profits). Graph the reaction functions and show the Cournot equilibrium in the graph.

Choose q_1 to maximize $\Pi_1 (q_1, q_2)$ assuming q_2 is fixed:

$$\Pi_1 (q_1, q_2) = (200 - q_1 - q_2)\ q_1 - 20\ q_1 = (180 - q_1 - q_2)\ q_1 .$$

Setting the first derivative equal to zero:

$\partial \Pi_1 (q_1, q_2) / \partial q_1 = 180 - 2q_1 - q_2 = 0$, then the reaction function for firm 1 is :

(1) $q_1 = (180 - q_2) / 2$

Similarly, choosing q_2 to maximize $\Pi_2 (q_1, q_2)$ and assuming q_1 is fixed we obtain the reaction function for firm 2 :

(2) $q_2 = (180 - q_1) / 2$.

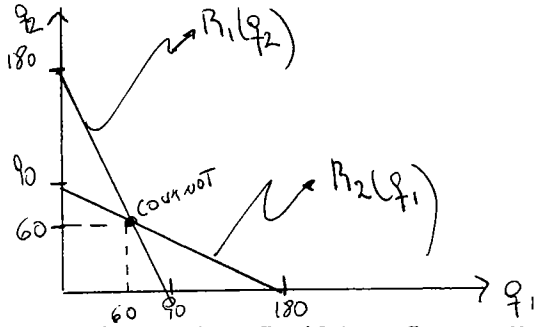
Solving equations (1) and (2) we get the Cournot quantities:

$q_1^C = q_2^C = 60$.

Using the demand function we get the Cournot price and the profits:

$p^C = 200 - 120 = 80$; $\Pi_1^C = \Pi_2^C = 3{,}600$.

Reaction Functions:



(2) Calculate output, prices and profits if the 2 firms collude. Assume that each firm is going to produce half of the collusive output.

We maximize the sum of profits, so we choose q_1 and q_2 to maximize $\Pi_1 + \Pi_2$, where:

$\Pi_1 + \Pi_2 = [200 - (q_1 + q_2)] (q_1 + q_2) - 20 (q_1 + q_2)$.

Notice that since $q_1 + q_2 = q$ this is equivalent to solve the monopoly problem when the monopolist cost and demand are:

$p = 200 - q$ **and** $MC = 20$

The monopolist maximizes:

$\Pi (q) = (200 - q) q - 20 q$,

so:

$\partial \Pi (q) / \partial q = 200 - 2q - 20 = 0 \Rightarrow q^M = 90$, $p^M = 200 - q^M = 110$.

If we choose the symmetric solution: $q_1 = q_2 = q / 2 = 45$, then $\Pi_1 = \Pi_2 = \Pi^M / 2 = 4050$.

(3) Suppose Firm 1 is producing at the collusive output level. Calculate the output level that Firm 2 will choose if it wants to maximize current period profits. Calculate profits for both firms . What that this imply for the survival of a collusive agreement?

If Firm 1 is producing 45 units, and Firm 2 want to maximize current period profits it will choose its output according to its reaction function (equation (2)). Then:

$q_2 = (180 - 45) / 2 = 67.5$

Since total output is :$q_1 + q_2 = 45 + 67.5 = 112.5$, $p = 87.5$ and

$\Pi_1 = (87.5 - 20) . 45 = 3037.5$

$\Pi_2 = (87.5 - 20) . 67.5 = 4556.25$

Since firms have an strong incentive to cheat, a collusive agreement wil not survive if the firms meet only once or for a finite number of periods.