9-17-09

2 FIRMS $\begin{array}{l} 1 \\ 2 \end{array}$

HOMOG. PRODUCTS

QUANTITY COMPETITION

<u>NE</u>: (q_1^c, q_2^c) SUCH THAT

$$\pi_1(q_1^c, q_2^c) \geq \pi_1(q_1, q_2^c)$$

FOR ALL FEASIBLE q_1

$$\pi_2(q_1^c, q_2^c) \geq \pi_2(q_1^c, q_2)$$

FOR ALL FEASIBLE q_2

OR

$$q_1^c = B_1(q_2^c)$$

$$q_2^c = B_2(q_1^c)$$

SAME

TRADITIONAL NOTATION

REACTION FUNCTIONS:

$$q_1^c = R_1(q_2^c)$$

$$q_2^c = R_2(q_1^c)$$

EXAMPLE:

$$p = 100 - q$$
$$MC_1 = MC_2 = 10$$

COMPUTE NE QUANTIES
COMPARE TO MONOPOLY OUTCOME, COLLUSIVE OUTCOME, P. COMPETITION OUTCOME

STEPS:
1) CALCULATE BEST RESPONSE FUNCTION (REACTION FUNCTIONS)
2) FIND NE (INTERSECTION OF THE 2 FUNCTIONS)

FIRM 1

$$\max_{q_1} \quad \pi_1(q_1, q_2) = \overbrace{p q_1 - 10 \cdot q_1}^{[p - MC_1] q_1}$$
$$= [100 - q_1 - q_2] q_1 - 10 q_1$$
$$= [90 - q_1 - q_2] \cdot q_1$$

$$\frac{\partial}{\partial q_1} = 0 = (-1) \cdot q_1 + 1 \cdot (90 - q_1 - q_2) =$$
$$= 90 - 2 q_1 - q_2 = 0$$

$$\Longrightarrow \boxed{q_1 = \underbrace{\frac{90 - q_2}{2}}_{R_1(q_2)}} \quad \triangle 1$$

FIRM 2

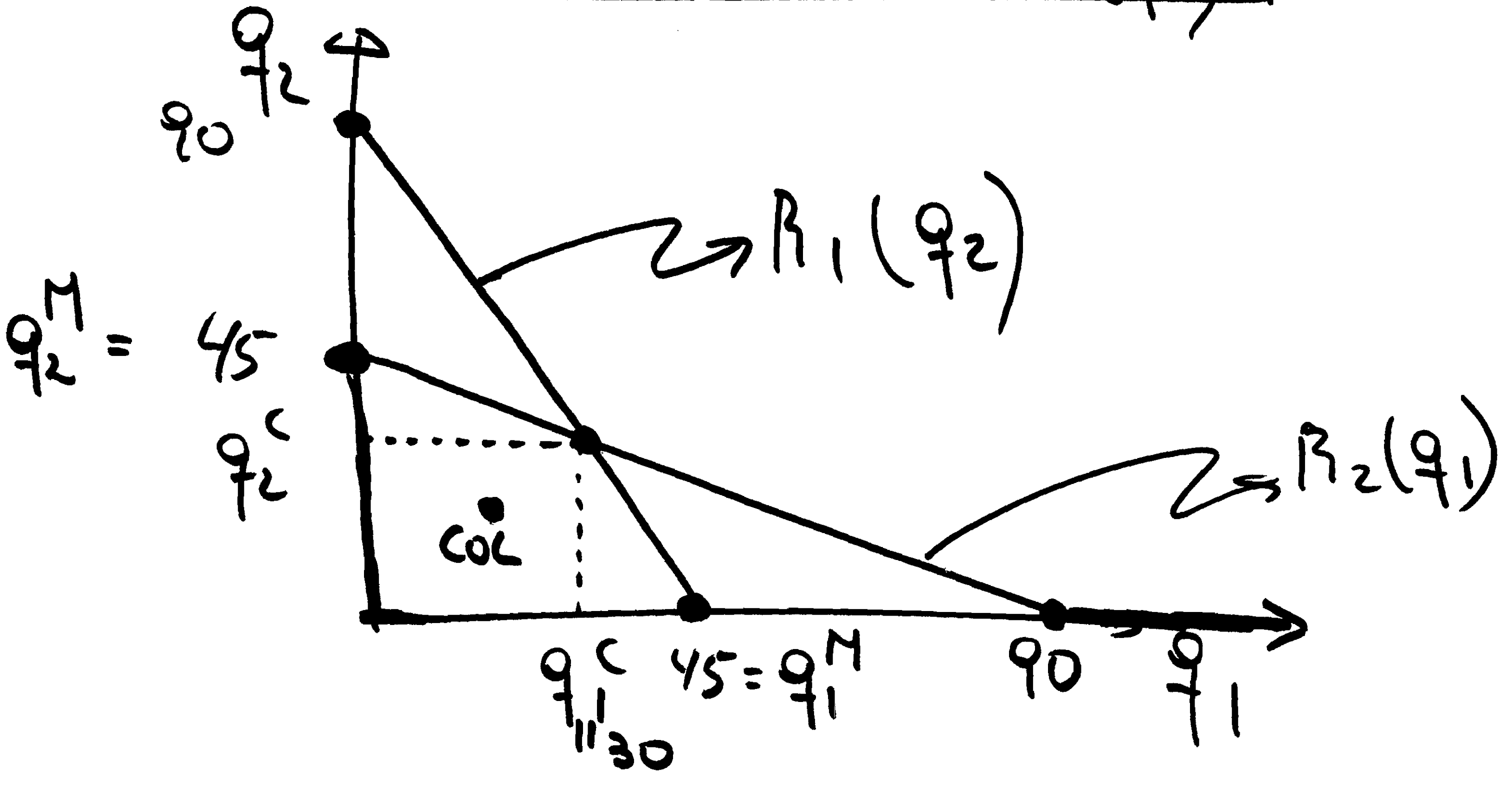
$$\max_{q_2} \quad \Pi_2(q_1, q_2) = \overbrace{p q_2 - 10 q_2}^{[p - MC_2]\, q_2}$$

$$= [90 - q_1 - q_2] \cdot q_2$$

$$\frac{\partial}{\partial q_2} = 0 = (-1) \cdot q_2 + 1 \cdot [90 - q_1 - q_2] =$$

$$= 90 - 2q_2 - q_1 = 0$$

$$\Longrightarrow \boxed{q_2 = \underbrace{\frac{90 - q_1}{2}}_{R_2(q_1)}} \quad \triangle 2$$



SOLVING (1) & (2)

USE (2)

$$q_1 = \frac{90}{2} - \frac{q_2}{2}$$

$$= \frac{90}{2} - \frac{1}{2}\left[\frac{90}{2} - \frac{q_1}{2}\right]$$

$$q_1 = \frac{90}{2} - \frac{1}{4}90 + \frac{q_1}{4}$$

$$q_1\left[1 - \frac{1}{4}\right] = \frac{90}{4} \Rightarrow q_1 \frac{3}{\cancel{4}} = \frac{90}{\cancel{4}}$$

$$\boxed{q_1^C} = \frac{90}{3} = \boxed{30}$$

$$\boxed{q_2^C} = R_2(30) = \frac{90-30}{2} = \boxed{30}$$

COURNOT PRICE: $\boxed{p^C} = 100 - q_1^C - q_2^C = \boxed{40}$

PROFITS: $\pi_1^C(30,30) = [p^C - MC_1]\, q_1^C$

$$= [40-10] \cdot 30 = \boxed{900}$$

$$\pi_2^C(30,30) = \boxed{900}$$

MONOPOLY

$$\max_{q} \ \pi() = [p - MC]\, q$$
$$= [100 - q - 10] \cdot q$$

$$\frac{d}{dq} = 0 \implies \boxed{q^M = 45}$$
$$\boxed{p^M = 55}$$
$$\boxed{\pi^M = 2025}$$

COLLUSIVE

$$\max_{q_1, q_2} \ \pi_1 + \pi_2$$

$$\implies q = q_1 + q_2 = q^M = 45$$

SHARING RULE : $\boxed{q_1^{COL} = q_2^{COL} = \frac{q^M}{2} = 22.5}$

$$p^{COL} = 55$$

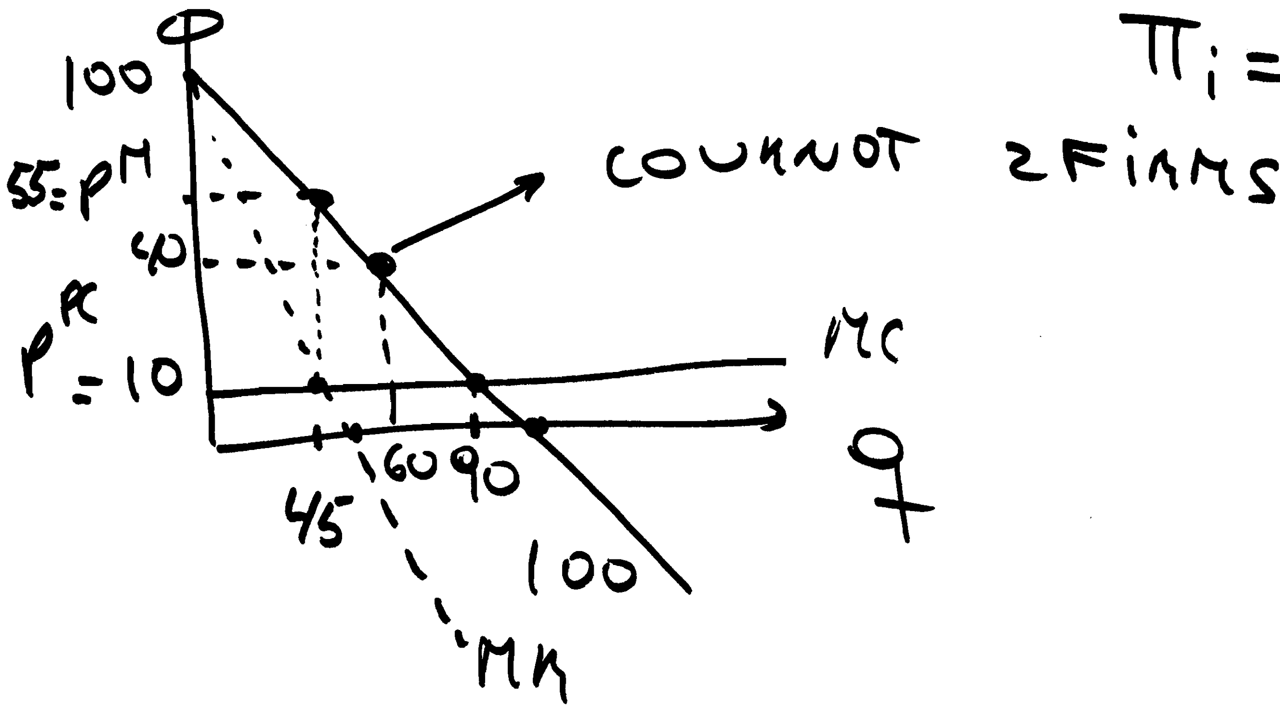
$$\pi_1^{COL} = \pi_2^{COL} = \frac{\pi^M}{2} = \frac{2025}{2}$$
$$= \boxed{1012.5}$$

P.C:

$p = MC = 10$

$\Rightarrow \quad p^{PC} = 100 - q \Rightarrow q^{PC} = 90$

$\pi_i = 0$



REMARK

PRISONNER'S DILEMMA IDEA

1 \ 2	C	NC
C	−1, −1	1, −3
NC	−3, 1	0, 0

SIMILAR IDEA

COLLUSION IS NOT A NE IN A ONE PERIOD GAME

WHY? FIRMS HAVE INCENTIVES TO DEVIATE

EXAMPLE: IF $q_2^{COL} = 22.5$

$$q_1^{CHEAT} = R_1(q_2^{COL}) = \frac{90 - q_2^{COL}}{2} = \boxed{33.7}$$

$$\Rightarrow q = q_1^{CHEAT} + q_2^{COC} =$$

$$= 33.75 + 22.5 = \boxed{56.25}$$

$$\boxed{p = 43.75}$$

$$\Pi_1^{CHEAT}(33.75, 22.5) = 1139.1$$

$$\Pi_2 \quad (33.75, 22.5) = 759$$

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