Derivative Review and Practice Handout

Economics 301: Intermediate Microeconomic Theory

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Functions with One Variable

For taking derivatives you will need to remember some basic equation properties and calculus rules. Here are some important properties and rules:

$e^{a+b} = e^a e^b$

$(e^a)^b = e^{ab}$

$\ln(x^a) = a \ln x$

$\ln(xy) = \ln x + \ln y$

If b is a constant, then $\frac{db}{dx} = 0$ and $\frac{d(b \cdot f(x))}{dx} = b \cdot f'(x)$.

The Power Rule:

If n is a positive integer, then $\frac{d(x^n)}{dx} = n x^{n-1}$

The Chain Rule:

For a function of a function $h(x) = g(f(x))$ we know that the derivative $dh(x)/dx = g'(f(x)) \cdot f'(x)$.

The Product Rule:

For a *function* $h(x) = f(x) \cdot g(x)$ we know that the derivative

$$\frac{dh(x)}{dx} = \frac{d(f(x) \cdot g(x))}{dx} = g(x) \cdot \frac{df(x)}{dx} + f(x) \cdot \frac{dg(x)}{dx} = g(x) \cdot f'(x) + f(x) \cdot g'(x).$$

The Quotient Rule:

$$\frac{dh(x)}{dx} = \frac{d(f(x) / g(x))}{dx} = \frac{g(x)(df(x)/dx) - f(x)(dg(x)/dx)}{g(x)^2}$$

Find the derivatives of the following functions. Solutions follow on the right half of this page.

1. $h(x) = 4x^3 + 3x^2 + x + 5$	$h'(x) = 12x^2 + 6x + 1$
2. $h(x) = (4x^2 - 1) \cdot (7x^3 + x)$	$h'(x) = (4x^2 - 1)(21x^2 + 1) + (7x^3 + x)(8x)$
3. $h(x) = (1 + x^2)^2$	$h'(x) = 2(1+x^2)\, 2x$
4. $h(x) = x^2 + 1/x$	$h'(x) = 2x - 1/x^2$
5. $h(x) = 1/(2x^4 - x^2 + 8)$	$h'(x) = -(2x^4 - x^2 + 8)^{-2} \cdot (8x^3 - 2x)$
6. $h(x) = \ln x$	$h'(x) = 1/x$
7. $h(x) = \ln ax$	$h'(x) = (1/ax) \cdot a = 1/x$
8. $h(x) = e^x$	$h'(x) = e^x$
9. $h(x) = e^{ax}$	$h'(x) = e^{ax} \cdot a$
10. $h(x) = \ln(x^3)$	$h'(x) = (1/x^3) \cdot (3x^2) = 3/x$
11. $h(x) = x^a \cdot e^x$	$h'(x) = x^a \cdot (de^x/dx) + (dx^a/dx) \cdot e^x = e^x(x^a + a \cdot x^{a-1})$
12. $h(x) = \ln(x^2 + x^3)$	$h'(x) = \frac{1}{x^2 + x^3} \cdot (2x + 3x^2)$
13. $h(x) = (5x^4 + x^2) / (x^3 + 2)$	$h'(x) = \frac{(x^3 + 2)(20x^3 + 2x) - (5x^4 + x^2)(3x^2)}{(x^3 + 2)^2}$

Functions with More than One Variable

Find f_x, f_y, f_{xx}, f_{yy}, f_{xy} and f_{yx} for the following functions. The last two partial derivatives should be equal. Solutions are provided on the bottom half of the page.

1. $f(x, y) = xy$
2. $f(x, y) = e^{xy}$
3. $f(x, y) = x^2 + 2xy + \frac{1}{3}y^3$
4. $f(x, y) = 5x^2 - 4xy^2 + 3x^2y^5 - 2x + y$
5. $f(x, y) = x^2 + y^2 \ln(x^2 + y^2)$

Answers

1. $f_x = y$ $f_{xx} = 0$ $f_{xy} = 1$
   $f_y = x$ $f_{yy} = 0$ $f_{yx} = 1$

2. $f_x = ye^{xy}$ $f_{xx} = y^2 e^{xy}$ $f_{xy} = e^{xy} + yxe^{xy}$
   $f_y = xe^{xy}$ $f_{yy} = x^2 e^{xy}$ $f_{yx} = e^{xy} + yxe^{xy}$

3. $f_x = 2x + 2y$ $f_{xx} = 2$ $f_{xy} = 2$
   $f_y = 2x + y^2$ $f_{yy} = 2y$ $f_{yx} = 2$

4. $f_x = 10x - 4y^2 + 6xy^5 - 2$ $f_{xx} = 10 + 6y^5$ $f_{xy} = -8y + 30xy^4$
   $f_y = -8xy + 15x^2y^4 + 1$ $f_{yy} = -8x + 60x^2y^3$ $f_{yx} = -8y + 30xy^4$

5. $f_x = 2x + \frac{2xy^2}{x^2 + y^2}$ $f_{xx} = 2 + \frac{2y^2(y^2 - x^2)}{(x^2 + y^2)^2}$

   $f_y = \frac{2y^3}{x^2 + y^2} + 2y \ln(x^2 + y^2)$ $f_{yy} = \frac{2y^2(3x^2 + y^2)}{(x^2 + y^2)^2} + \frac{4y^2}{x^2 + y^2} + 2\ln(x^2 + y^2)$

$f_{xy} = \frac{4x^3y}{(x^2 + y^2)^2}$ and $f_{yx} = \frac{4x^3y}{(x^2 + y^2)^2}$