Week 1

Swati Dhingra

dhingra@wisc.edu

Office 7316

Overview: Sections I-V of the Syllabus

Basic Questions

1. Why do countries trade with each other? Gains from Trade (GFT) for some agents.
   (a) Which agents get the GFT? Depends on the underlying structure which generates a basis for trade.
2. What generates the potential for GFT? One reason is that some goods are cheaper in Mexico than US and vice-versa. Say, corn tortillas versus whole wheat bread. Actually, cloth versus plastic (2 of the top 10 commodities by US-Mexico trade value in 2005).
   - Why is there a price wedge? Which country sells what?

Answering the Questions

Why is there a price wedge? Which country sells what?

- Production: Comparative Advantage. (Need producer theory tools here.)
  - Ricardian: Mexico has better tortilla-making technology than bread-making technology while US has better bread-making technology than tortilla-making technology.
  - Heckscher-Ohlin: Mexico has *relatively* more corn than wheat while the US has relatively more wheat than corn.
- Need producer theory tools here.
- Depends on consumption patterns too. $M^{\text{Tor}} = C^{\text{Tor}} - Y^{\text{Tor}}$ and $X^{\text{Br}} = Y^{\text{Br}} - C^{\text{Br}}$. Assume "nice" preferences.

Which agents get the GFT? Are some agents worse-off with trade?

- Depends on how an agent's consumption changes.
- Need to know how agent's income and prices in the economy change.
- Need consumer theory tools to answer this.

Some Basic Production Concepts

Returns to Scale

Think of a production function $y = f(K, L)$. Increase both inputs K and L to λK and λL respectively. A production function is homogeneous of degree d if λ times the original inputs give λ^d times the original output. In other words, a production function is homogeneous of degree d if the new output y' is $y' = f(\lambda K, \lambda L) = \lambda^d f(K, L) = \lambda^d y$.

Constant Returns to Scale

When $d = 1$. Implies that doubling inputs doubles output.

Example 1. *Cobb-Douglas Production Function:* $y = f(K, L) = K^{1/2}L^{1/2}$.

Increase inputs by λ. Now what will be the new output?

$$\begin{aligned} y' = f(\lambda K, \lambda L) = & (\lambda K)^{1/2}(\lambda L)^{1/2} \\ = & \lambda^{1/2}K^{1/2}\lambda^{1/2}L^{1/2} \\ = & \lambda^{1/2}\lambda^{1/2}(K^{1/2}L^{1/2}) \\ = & \lambda^1(K^{1/2}L^{1/2}) \\ = & \lambda^1 y \end{aligned}$$

So $d = 1$ implying that this production function is CRS.

Example 2. *Linear Production Function:* Let us think of a case where only 1 input is needed. A simple case is the linear production function $y = f(L) = aL$ where $a > 0$ is some fixed constant. For instance, when $a = 1$, $y = f(L) = L$. So 1 unit of labor can produce 1 unit of output.

Once again, increase the inputs (in this case only 1 input) by λ. What is the new output?

$$\begin{aligned} y' = f(\lambda L) = & a(\lambda L) \\ = & \lambda(aL) \\ = & \lambda y \\ = & \lambda^1 y \end{aligned}$$

So once again $d = 1$ implying the linear production function is CRS *always* (i.e. for any a).

Marginal Product

Think of a production function $y = f(K, L)$. Increase only *one* of the inputs, say L. The additional output is the contribution of input L to total output. We will refer to this as the marginal product of labor. Formally, $MPL = \partial y/\partial L = \partial f(K, L)/\partial L$ given K.

Example 1. *Cobb-Douglas Production Function:* $y = f(K, L) = K^{1/2}L^{1/2}$.

What is the marginal product of labor?

$$\begin{aligned} MPL = \partial y/\partial L = & \partial f(K, L)/\partial L \\ = & \partial(K^{1/2}L^{1/2})/\partial L \\ = & K^{1/2}(1/2)L^{1/2-1} \\ = & (1/2)K^{1/2}L^{-1/2} \\ = & (1/2)(K/L)^{1/2} \end{aligned}$$

What is the marginal product of capital? You can work this out on your own and your answer should be $MPK = (1/2)(L/K)^{1/2}$.

Example 2. *Linear Production Function:* $y = f(L) = aL$ where $a > 0$ is some fixed constant.

What is the marginal product of labor?

$$\begin{aligned} MPL = \partial y/\partial L = & \partial f(L)/\partial L \\ = & \partial(aL)/\partial L \\ = & a \end{aligned}$$

In the special case when $a = 1$, the marginal product of labor is 1 implying that increasing labor by 1 unit at the margin increases output by 1 unit.

Marginal Returns

An interesting question is what happens to MPL when you keep increasing L. In other words, what is the sign of $\partial MPL/\partial L = \partial^2 y/\partial L^2 = \partial^2 f(K,L)/\partial L^2$ given K?

Example 1. *Cobb-Douglas Production Function:* $y = f(K,L) = K^{1/2}L^{1/2}$.

What is happening to the marginal product of labor as more labor is used?

$$\begin{aligned}\partial MPL/\partial L = \partial^2 y/\partial L^2 = & \quad \partial^2 f(K,L)/\partial L^2 \\ = & \quad \partial((1/2)K^{1/2}L^{-1/2})/\partial L \\ = & (1/2)K^{1/2}(-1/2)L^{-1/2-1} \\ = & \quad -(1/4)K^{1/2}L^{-1/2-1} \\ < & \quad 0\end{aligned}$$

So as long as K and L are positive, the marginal product of labor is falling. This production function shows **Diminishing Marginal Returns**.

Example 2. *Linear Production Function:* $y = f(L) = aL$ where $a > 0$ is some fixed constant.

What is happening to the marginal product of labor as more labor is used?

$$\begin{aligned}\partial MPL/\partial L = \partial^2 y/\partial L^2 = & \partial^2 f(L)/\partial L^2 \\ = & \quad \partial(a)/\partial L \\ = & \quad 0\end{aligned}$$

Here the MPL is constant. So as more and more labor is used, the contribution of the marginal unit of labor to output does not fall. The Linear production function always shows **Constant Marginal Returns**.

Production Possibilities Frontier

We will look at the earlier examples to see what happens when the economy produces 2 goods. We are interested in exploring the tradeoff between the 2 goods, X and Y. The PPF illustrates this tradeoff. It shows the different combinations of goods X and Y that the economy can produce when it has a given amount of inputs L and K.

Think of a production function $x = g(K, L)$ for good X and a production function $y = f(K, L)$ for good Y. We want to know how many units of good Y have to be given up to increase units of good X.

Example 1. *Cobb-Douglas Production Function:* $x = g(K, L) = K^{1/4}L^{3/4}$ and $y = f(K, L) = K^{1/2}L^{1/2}$. Suppose the economy has 81 units of K and 16 units of L.

Step 1: Suppose all 81 units of K and 16 units of L are devoted to producing good X. Then $x = K^{1/4}L^{3/4} = (81)^{1/4}(16)^{3/4} = (3)(8) = 24$. This is the maximum amount of X that the economy can produce. Since all inputs are spent on production of X, there is no Y produced in the economy.

Step 2: Suppose all 81 units of K and 16 units of L are devoted to producing good Y. Then $y = K^{1/2}L^{1/2} = (81)^{1/2}(16)^{1/2} = (9)(4) = 36$. This is the maximum amount of Y that the economy can produce. Since all inputs are spent on production of Y, there is no X produced in the economy.

Step 3: Now think of other combinations. Say, 16 units of K and 1 unit of L is given for production of X. Then $x = K^{1/4}L^{3/4} = (16)^{1/4}(1)^{3/4} = (2)(1) = 2$. How many units of K and L are left over? The answer is $81 - 16 = 65$ units of K and $16 - 1 = 15$ units of L are left over. The economy does not want to waste these resources so it uses them to produce good Y. Then $y = (65)^{1/2}(15)^{1/2} = 31.2$.

Step 4: Do the same for the case of Y. 16 units of K and 1 unit of L is given for production of Y. Then $y = K^{1/2}L^{1/2} = (16)^{1/2}(1)^{1/2} = (4)(1) = 4$. How many units of K and L are left over? The answer is $81 - 16 = 65$ units of K and $16 - 1 = 15$ units of L are left over. The economy does not want to waste these resources so it uses them to produce good X. Then $x = (65)^{1/4}(15)^{3/4} = 21.6$.

Let's do one more set of points. 40 units of K and 1 unit of L are given to X. $x = (40)^{1/4}(1)^{3/4} = 40^{1/4} = 2.5$. We still have 41 units of K and 15 units of L left. Use them to produce Y. We will get $y = 41^{1/2}15^{1/2} = 24.8$ units of good Y.

Now reverse the input allocations. 40 units of K and 1 unit of L are given to Y. So $y = (40)^{1/2}(1)^{1/2} = 40^{1/2} = 6.3$ units of good Y. The left over inputs are given to X so $x = (41)^{1/4}(15)^{3/4} = 19.3$ units of X.

Step 5: Collect the various (x, y) combinations.

Units of X	Units of Y
24	0
0	36
2	31.2
21.6	4
2.5	24.8
19.3	6

Plot these on a graph with units of good X on the x-axis and units of good Y on the y-axis. A picture like Figure 1(a) will emerge.

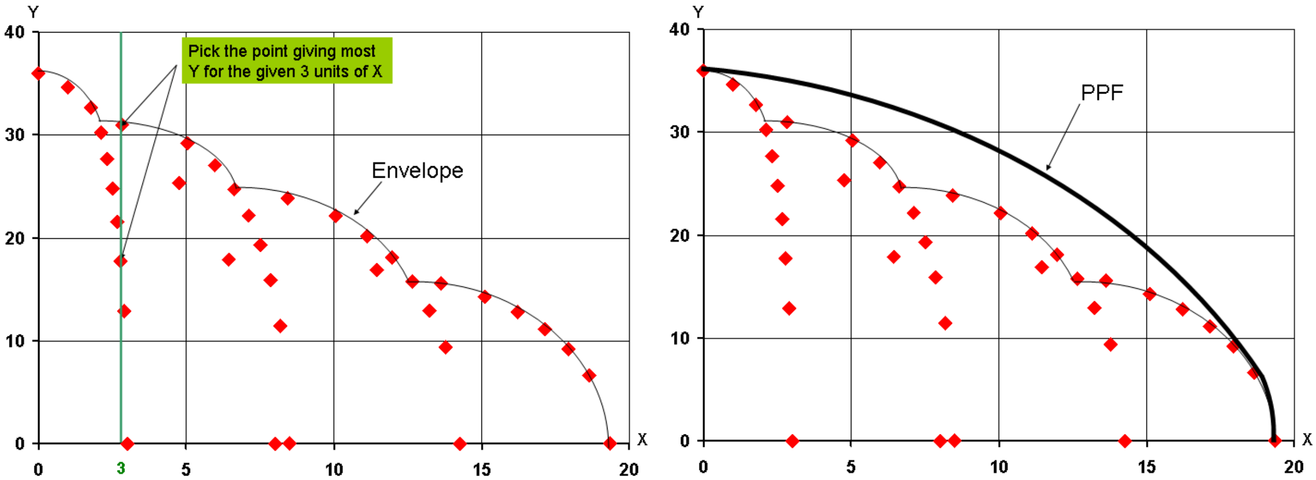
Step 6: For the PPF pick the largest Y given some X shown in Figure 1 (a). Now trace out all such points and you get an upper envelope of x and y combinations. Suppose we had plotted many more such points. then there would be a smooth PPF like that shown in Figure 1 (b). The PPF tells us what is the maximum units of Y that the economy can produce if we hold the units of X fixed. In other words, what is the best way to allocate inputs so that we can produce good Y without sacrificing any of the fixed amount of X.

Observations

1. Why is the PPF downward sloping? Because in order to increase X, we must sacrifice some Y. Due to the fact that the production functions are increasing in inputs, i.e. MPL and MPK are positive for both goods. So any increase in inputs for X increases its output. The increase in inputs must be counterbalanced by a reduction in inputs for Y because of the resource constraint of the economy. Since the production function of Y is also increasing in its inputs, this implies that reducing inputs for Y reduces the output of Y.

2. Why is the PPF concave? Because as we keep diverting more and more inputs to X, we get less and less of Y. This property hold due to the law of **diminishing marginal returns**. Adding more and more of an input to produce good X increases the output of good X more and more slowly.

Figure 1: Cobb-Douglas Production Functions



(a) (b)

Example 2. *Linear Production Function:* Once again, let us think of a case where only 1 input is needed. The production function for good X is $x = g(L) = 0.5L$ and the production function for good Y is $y = f(L) = 2L$. We want to know how many units of good Y have to be given up to increase units of good X. As earlier, suppose the economy has 16 units of L. (The economy has only 1 input so we need not worry about K).

Follow Steps 1-4.

Step 1: Suppose all 16 units of L are devoted to producing good X. Then $x = 0.5L = 0.5(16) = 8$. This is the maximum amount of X that the economy can produce. Since all inputs are spent on production of X, there is no Y produced in the economy.

Step 2: Suppose all 16 units of L are devoted to producing good Y. Then $y = 2L = 2(16) = 32$. This is the maximum amount of Y that the economy can produce. Since all inputs are spent on production of Y, there is no X produced in the economy.

Step 3: Now think of other combinations. Say, 1 unit of L is given for production of X. Then $x = 0.5(1) = 0.5$. How many units of L are left over? The answer is $16 - 1 = 15$ units of L are left over. The economy does not want to waste these resources so it uses them to produce good Y. Then $y = 2(15) = 30$.

Step 4: Do the same for the case of Y. 1 unit of L is given for production of Y. Then $y = 2$. How many units of L are left over? The answer is $16 - 1 = 15$ units of L are left over. The economy does not want to waste these resources so it uses them to produce good X. Then $x = 7.5$.

You can go ahead and get more such points by changing the L allocations for each good.

Step 5: Collect the various (x, y) combinations.

Units of X	Units of Y
8	0
0	32
0.5	30
7.5	2

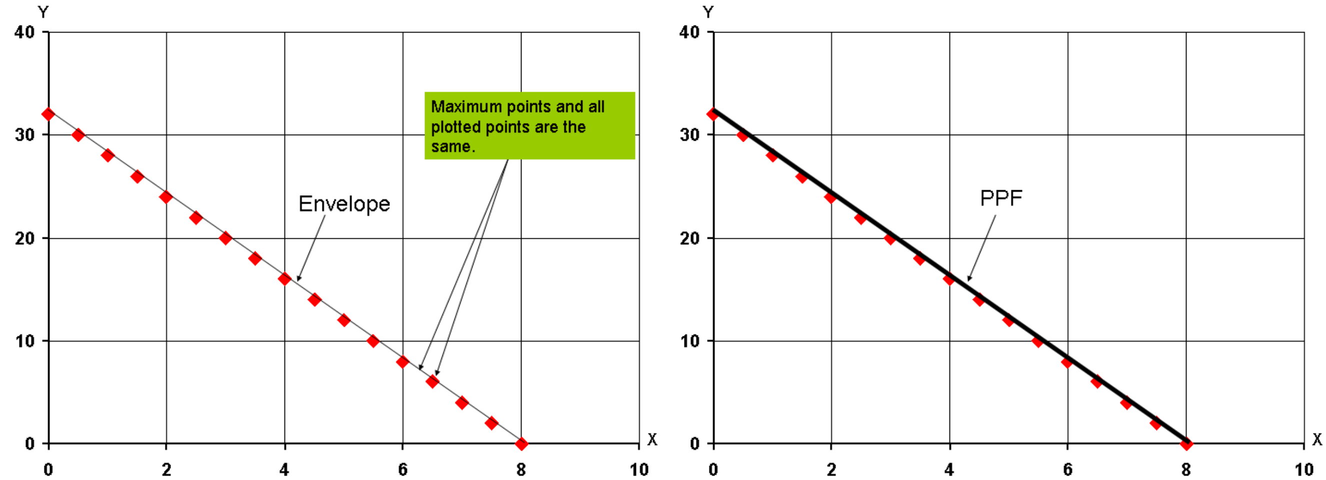
Plot these on a graph with units of good X on the x-axis and units of good Y on the y-axis. A picture like Figure 2(a) will emerge.

Step 6: For the PPF pick the largest Y given some X shown in Figure 2(a). Now trace out all such points and you get an upper envelope of x and y combinations. Note that the upper envelope is exactly the points plotted. So the PPF is the same as the plotted points and is already smooth as shown in Figure 2(b). The PPF tells us what is the maximum units of Y that the economy can produce if we hold the units of X fixed. In other words, what is the best way to allocate inputs so that we can produce good Y without sacrificing any of the fixed amount of X.

Observations

1. Why is the PPF downward sloping? Because in order to increase X, we must sacrifice some Y. Same reason as earlier: $MPL = 0.5 > 0$ for X and $MPL = 2 > 0$ for Y.

Figure 2: Linear Production Functions



(a) (b)

2. Why is the PPF a straight line? Because as we keep diverting more and more inputs to X, we do **not** get less and less of Y. Recall that the linear production function yields constant marginal returns ($\partial MPL/\partial L = a =$ fixed constant). So adding more and more of an input to produce good X increases the output of good X at the same rate.

3. What is the slope of the PPF? The slope of the PPF is straightforward. Try to remember how you derive the slope of a straight line. If you know 2 points on the line then the slope is simply $m = (y_2 - y_1)/(x_2 - x_1)$. In this case, we know two points $(x_1, y_1) = (8, 0)$ and $(x_2, y_2) = (0, 32)$. So the slope of the PPF is $m = (8 - 0)/(0 - 32) = -0.25$. We will call the absolute value of the slope of the PPF its MRT or the Marginal Rate of Transformation because it tells us how Y can be transformed into X through reallocation of inputs between these two goods.

4. What else do we notice about the MRT? Here the MRT is *constant*. So *for any value of* X, if we want to increase X by 1 unit, we must give up 0.25 units of Y.

Practice Question

- What can you say about the slope of the PPF for the Cobb-Douglas example?