Econ 464

Example of Cobb Douglas Production Function

The Cobb Douglas production function :
$y = F(K, L) = K^{\alpha} L^{1-\alpha}$ where $0<\alpha<1$
has all the properties we assumed in the H-O Model.
We will let $\alpha = 1/3$ in this example so:

$y = K^{1/3} L^{2/3}$

(I) Returns to scale: constant
Proof:
Let λ be any positive number, then

$F(\lambda K, \lambda L) = (\lambda K)^{1/3} (\lambda L)^{2/3} = \lambda^{1/3} K^{1/3} \lambda^{2/3} L^{2/3} = \lambda^{1/3+2/3} K^{1/3} L^{2/3} = \lambda K^{1/3} L^{2/3} = \lambda F(K,L)$,
We have shown that if $\% \Delta K= \% \Delta L$ then $\% \Delta y=\% \Delta K= \% \Delta L$

Ex: Let $\lambda =1.10$, in this case both L and K increase by 10 %. As a result y increases by 10%.
Let $\lambda =0.50$, in this case both L and K decrease by 50 %. As a result y decreases by 50%.

(II) Marginal Products

$MPL = \partial y/ \partial L = 2/3 L^{2/3-1} K^{1/3} = 2/3 (K/L)^{1/3}$

$MPK = \partial y/ \partial K = 1/3 K^{1/3-1} L^{2/3} = 1/3 (L/K)^{2/3}$

Implications:

If K constant, increases in L decrease MPL (standard negative sloped MPL curve).
If L constant, increases in K decrease MPK (standard negative sloped MPK curve).

If K/L increases, then MPL increases and MPK decreases.

If MPL increases, then K/L has increased.
If MPK increases, then K/L has decreased.