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Answer Key Problem Set 1

(I) In the Solow model without population growth or technological change:

(1) $\Delta k = i - \delta k$,

where k is capital per worker, i investment per worker and δ the rate of depreciation of the capital stock. Since investment per worker equals savings per worker, we have that

(2) $\Delta k = s f(k) - \delta k$,

where s is the savings rate and f(k) output per worker (equal to income per worker).
Let k* be the level of capital per worker along a BGP. Since at the BGP : Δk=0, using (2) we have that:

(3) $s f(k^*) = \delta k^*$

Notice that at the BGP the function s f(k) crosses the function δk. Both functions are increasing (see Figure 4-4, page 82 in RP1).
As the savings rate increases, the function s f(k) increases for positive values of k so the intersection occurs at a higher level of k. Since output or income per worker (f(k)) is increasing in k, the **level of income per worker at the BGP increases** (see Figure 4-5 , page 86 in RP1).
At the BGP, the growth rate of capital per worker is zero and therefore the growth rate of income per worker is zero. Since this is independent of the level of k, **the savings rate has no influence on the growth rate of output/income per worker along a BGP**.

(II) Problem 1 from Problems and Applications, page 109, Mankiw.

a - This production function has constant returns to scale because:

$F(\lambda K, \lambda L) = (\lambda K)^{1/2} (\lambda L)^{1/2} = \lambda^{1/2} K^{1/2} \lambda^{1/2} L^{1/2} = \lambda K^{1/2} L^{1/2} = \lambda F(K,L)$,

for any λ different from zero. In other words, we have shown that % Δ K= % Δ L = % Δ Y.
Alternative less formal explanations are acceptable.

b- The per-worker production function is: $f(k) = k^{1/2}$ since,

$$y = F(K, L) / L = (K^{1/2} L^{1/2}) / L = (K/L)^{1/2} (L/L)^{1/2} = k^{1/2}\ 1^{1/2} = k^{1/2}$$

c- We assume that :

$\delta = 0.05$

Savings rate in country A = $s^A = 0.10$

Savings rate in country B = $s^B = 0.20$

At BGP: $s f(k^*) = \delta k^*$ (see explanation in I above before equation (3)) so we can use this equation , the relevant savings rate and our particular functional form for f(k) to calculate k* for each country.

Country A:

$s^A f(k) = \delta k$ can be written as : $0.10\ k^{1/2} = 0.05\ k$

We solve for k:

$$0.10 / 0.05 = k\, k^{-1/2}$$
$$2 = k^{1/2}$$
$$4 = k$$

Then at the BGP :

Capital per worker : $k^{A*} = 4$

Income per worker: $y^{A}* = f(k^{A*}) = 4^{1/2} = 2$

Consumption per worker: $c^{A*} = (1-s^A)\, y^{A*} = 0.90 \cdot 2 = 1.8$

Country B:

$s^B f(k) = \delta k$ can be written as : $0.20\ k^{1/2} = 0.05\, k$

We solve for k:

$$0.20 / 0.05 = k\, k^{-1/2}$$
$$4 = k^{1/2}$$
$$16 = k$$

Then at the BGP :

Capital per worker : $k^{B*} = 16$

Income per worker: $y^{B*} = f(k^{B*}) = 16^{1/2} = 4$

Consumption per worker: $c^{B*} = (1-s^B)\, y^{B*} = 0.80 \cdot 4 = 3.2$

d - Suppose at time zero (t=0) both countries have 2 units of capital per worker:

$k_0^A = k_0^B = 2$

time	k^A	y^A	c^A	k^B	y^B	c^B
0	2.00	1.414	1.2730	2.00	1.414	1.1314
1	2.04	1.428	1.2850	2.18	1.477	1.1819
2	2.08	1.442	1.2980	2.37	1.539	1.2313
3	2.12	1.456	1.3104	2.56	1.599	1.2800
4	2.16	1.469	1.3227	2.75	1.658	1.3267

It will take four years for consumption in country B to exceed that of country A.

We calculated the capital stock per worker in different periods as:
$k_{t+1} = k_t + \Delta k = k_t + s \,.\, f(k_t) - \delta k_t = k_t + s \,.\, k_t^{1/2} - 0.05\, k_t$, so at time 1 in Country A for instance, $k^A_1 = 2.00 + 0.10 \,.\, 2^{1/2} - 0.05 \,.\, 2 = 2.04$
Income per worker (y) is just $f(k) = k^{1/2}$, so : $y^A_1 = 2.04^{1/2} = 1.428$ Consumption per worker c is $c = (1-s) \,.f(k)$, so: $c^A_1 = 0.90 \,.\, 1.428 = 1.285$

(III)

a - This production function has constant returns to scale because:

$$F(\lambda K, \lambda L) = (\lambda K)^{2/3} (\lambda L)^{1/3} = \lambda^{2/3} K^{2/3} \lambda^{1/3} L^{1/3} = \lambda K^{2/3} L^{1/3} = \lambda F(K,L),$$

for any λ different from zero. In other words, we have shown that % Δ K= % Δ L = % Δ Y. Alternative less formal explanations are acceptable.

b- The per-worker production function is: $f(k) = k^{2/3}$ since,

$\hat{Y} = (14 - 12) / 12 = 1/6$

$\hat{K} = (7 - 6) / 6 = 1/6$

$\hat{L} = (4 - 3) / 3 = 1/3$

$$y = F(K, L) / L = (K^{2/3} L^{1/3}) / L = (K/L)^{2/3} (L/L)^{1/3} = k^{2/3} 1^{1/3} = k^{2/3}$$

c- We assume that :

$\delta = 0.10$

Savings rate = s= 0.30

At the steady state , $s f(k^*) = \delta k^*$ (see explanation in I above before equation (3)) so we can use this equation , the relevant savings rate and our particular functional form for f(k) to calculate k*.

$s f(k) = \delta k$ can be written as :

$$0.30 \; k^{2/3} = 0.10 \, k$$

We solve for k:

$$0.30 / 0.10 = k \, k^{-2/3}$$

$$3 = k^{1/3}$$

$$27 = k$$

Then at the steady state :

Capital per worker : $k^* = 27$

Income per worker: $y^* = f(k^*) = (27)^{2/3} = 9$

Consumption per worker: $c^* = (1-s) y^* = 0.70 \cdot 9 = 6.3$

d - We assume that $k_0 = 20$

Income per worker (y_0) is just $f(k_0) = k_0^{2/3}$, so : $y_0 = 20^{2/3} = 7.36$

Consumption per worker c is $c = (1-s) . f(k)$, so: $c_0 = 0.70 . 7.36 = 5.15$

We calculate the change in the capital stock per worker as:

$\Delta k = s . f(k_0) - \delta k_0 = 0.3 \; . 20^{2/3} - 0.10 . 20 = 2$

Since $\Delta k_0 / k_0 = 2 / 20 = 0.10$, the growth rate of capital per worker is 10 %.

Since $\Delta y_0 / y_0 = (2/3) (\Delta k_0 / k_0) = 0.066$, the growth rate of output per worker is 6.6%.

(IV) Problem 1, part b, from More Problems and Applications, page 117 from Mankiw.

In this case: $\alpha = 2/3$, so $1-\alpha = 1$

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Using the information given:

So Total Factor Productivity Growth was negative and equal to approximately 5.5%

$$\hat{A} = \hat{Y} - \alpha\hat{K} - (1-\alpha)\hat{L}$$
$$\hat{A} = 1/6 - (2/3)(1/6) - (1/3)(1/3) = -0.055$$