9-3-08

MICRO REVIEW

- PROD. FUNCTION, MARG/AV. PRODUCTS
- ISOQUANTS
- COST MINIMIZATION
- PROFIT MAXIMIZATION
- PPF

- PREFERENCES
- BUDGET CONSTRAINT
- UTILITY MAXIMIZATION

PROD. FUNCTIONS

2 FACTORS $\rightarrow$ L, K

GOOD: q

$$q = f(L, K)$$

OR

1 FACTOR : L

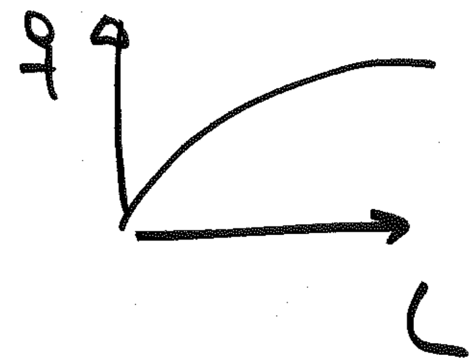
GOOD: q

$$q = f(L)$$

EXAMPLES

1) 1 FACTOR

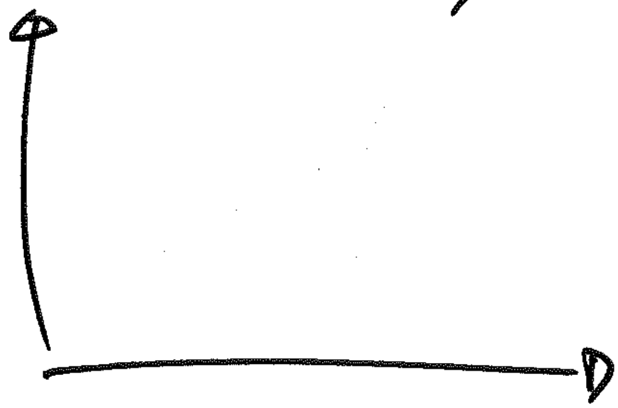
Ex1: $q = L^{1/2} = \sqrt{L}$



$\boxed{\text{MPL is } \downarrow}$

WHERE $\text{MPL} = \left.\dfrac{\Delta q}{\Delta L}\right|_{\text{SMALL CHANGES}}$

$\boxed{\downarrow \text{ APL}} = \dfrac{q}{L} = \dfrac{L^{1/2}}{L} = \dfrac{1}{L^{1/2}}$



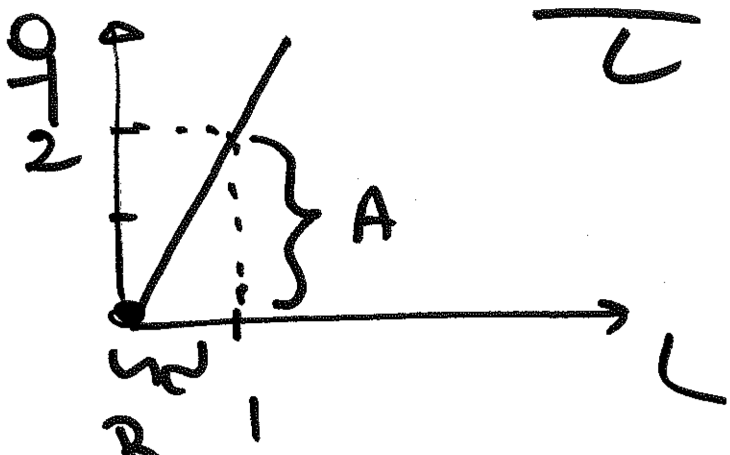
$$MPL = \frac{1}{2}\frac{1}{L^{1/2}}$$

$$APL = \frac{1}{L^{1/2}}$$

EX 2: $q = 2L$ LINEAR PROD FUNCTION

$$MPL = \frac{\Delta q}{\Delta L} = 2$$

$$APL = \frac{q}{L} = 2$$



$|\text{SLOPE}| : \frac{A}{B} = 2$

UNIT LABOR COEFFICIENT

OR

UNIT LABOR REQUIREMENT

= # OF UNITS OF LABOR NEEDED TO PRODUCE 1 UNIT OF OUTPUT

$$q = 2 \cdot L$$

$$\underbrace{\text{UNIT LABOR COEFF OR REQUIREMENT}}_{a} = \frac{1}{2}$$

NOTICE $\quad a = \dfrac{1}{APL}$

Ex 3: $q = \dfrac{1}{3} \cdot L$

$APL = \frac{1}{3} = MPL$

$\Rightarrow \quad a = 3$

2 FACTOR PROD. FUNCTION

$$q = f(L, k)$$

EX 1:

$$q = 2 \cdot L^{1/2} \cdot k^{1/2}$$

HERE $MPL = \left.\frac{\Delta q}{\Delta L}\right|$ EVERYTHING ELSE CONSTANT (PARTIAL DERIVATIVES)

$MPK = \left.\frac{\Delta q}{\Delta k}\right|$

$MPL \downarrow$

$MPk \downarrow$

RETURNS TO SCALE:

% Δq WHEN ALL INPUTS CHANGE BY THE SAME %

HERE WE HAVE
CONSTANT RETURNS TO SCALE

$\Longrightarrow$

$\% \Delta q = \% \Delta L = \% \Delta K$

$\Longrightarrow$ AVERAGE COST
IS CONSTANT

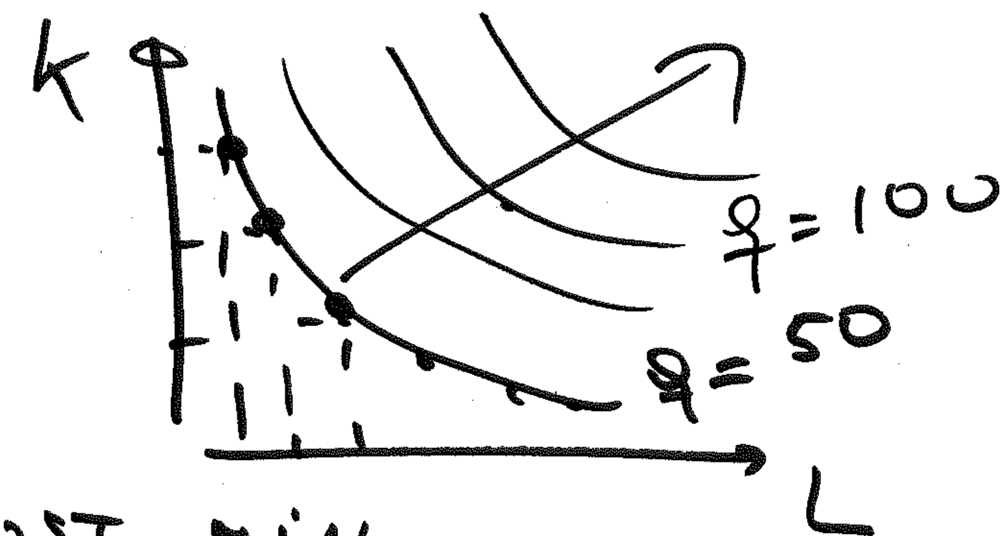
$\Longrightarrow$ "OPTIMAL" FIRM SIZE
IS UNDERMINED

$\Longrightarrow$ EASY TO WORK WITH

ISOQUANT = COMBINATIONS
OF L & K
THAT YIELD THE
SAME TOTAL OUTPUT

$\downarrow$
EQUAL

IN OUR EXAMPLE WE HAVE LOTS OF SUBSTITUTION BETWEEN INPUTS (K, L)



COST MIN

MIN $w \cdot L + r \cdot K$

SUBJECT TO

$$q = 50$$

WHERE

w = WAGE

r = RENTAL PRICE OF CAPITAL

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ISOCOST :

$$\bar{C} = w \cdot L + r \cdot k$$

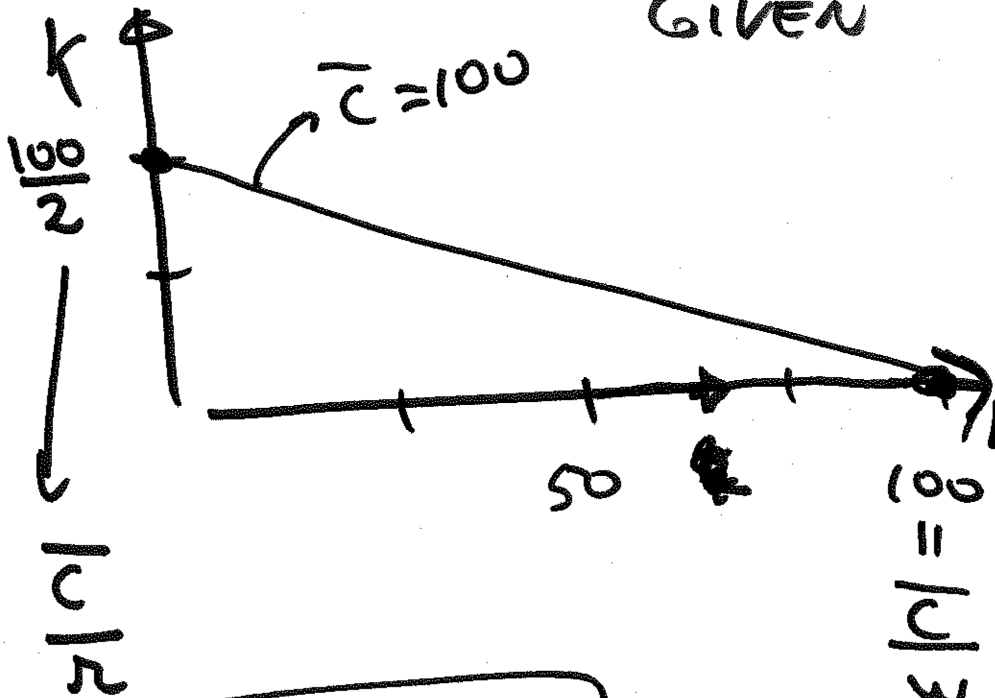
GIVEN

EX:

$\bar{C} = 100$

$w = 1$

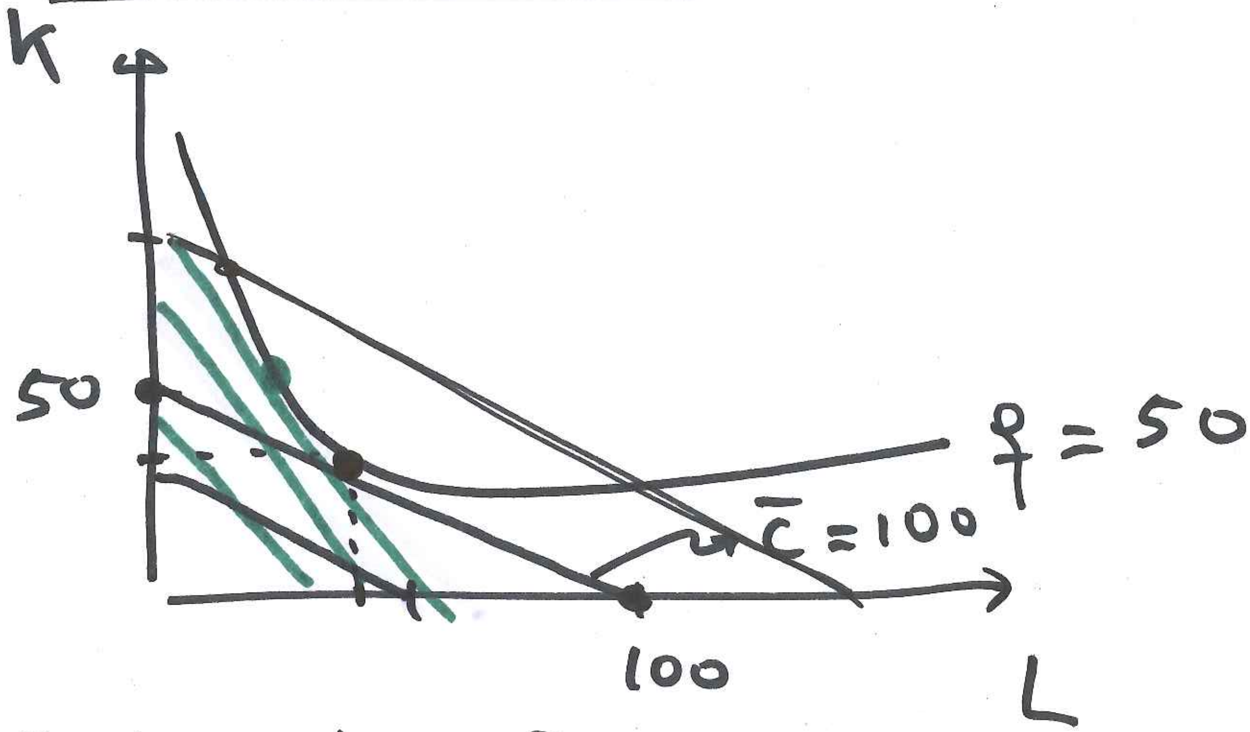
$r = 2$



$$100 = 1 \cdot L + 2 \cdot k$$

$$\boxed{|\text{SCOPE}|} = \frac{\bar{C}/r}{\bar{C}/w} = \boxed{\frac{w}{r}}$$

ISOCOST MAP



COST MIN. $\Rightarrow$

$$\left| \text{SLOPE ISOQUANT} \right| = \left| \text{SLOPE ISOCOST} \right| = \frac{w}{r}$$

$$\uparrow \frac{w}{r} \Rightarrow \downarrow \frac{L}{K} \text{ or } \uparrow \frac{K}{L}$$

PROFIT MAXIMIZATION

1) APPROACH 1

- CALCULATE "COST FUNCTION" (CHEAPEST WAY TO PRODUCE ANY OUTPUT q): $C(q)$
- USE PROFIT MAX CONDITION:

$$p = MC$$

2) APPROACH 2
PROFIT MAX. INPUT DEMANDS

MAX
PROFITS = TOTAL REVENUE - TOTAL COSTS

$$p \cdot q - [w \cdot L + r \cdot k]$$

$$q \to f(L,k)$$

$$= p \cdot f(L,k) - [wL + rk]$$

PROBLEM MAX PROFITS "BY CHOOSING K & L OPTIMALLY"

$\Rightarrow$ PROFIT MAX. INPUT DEMANDS

MATH:

$$\max_{(K,L)} \quad p \cdot f(L,K) - W \cdot L - r \cdot K$$

$$\Rightarrow \quad \frac{\partial}{\partial L} = 0$$

$$\frac{\partial}{\partial K} = 0$$

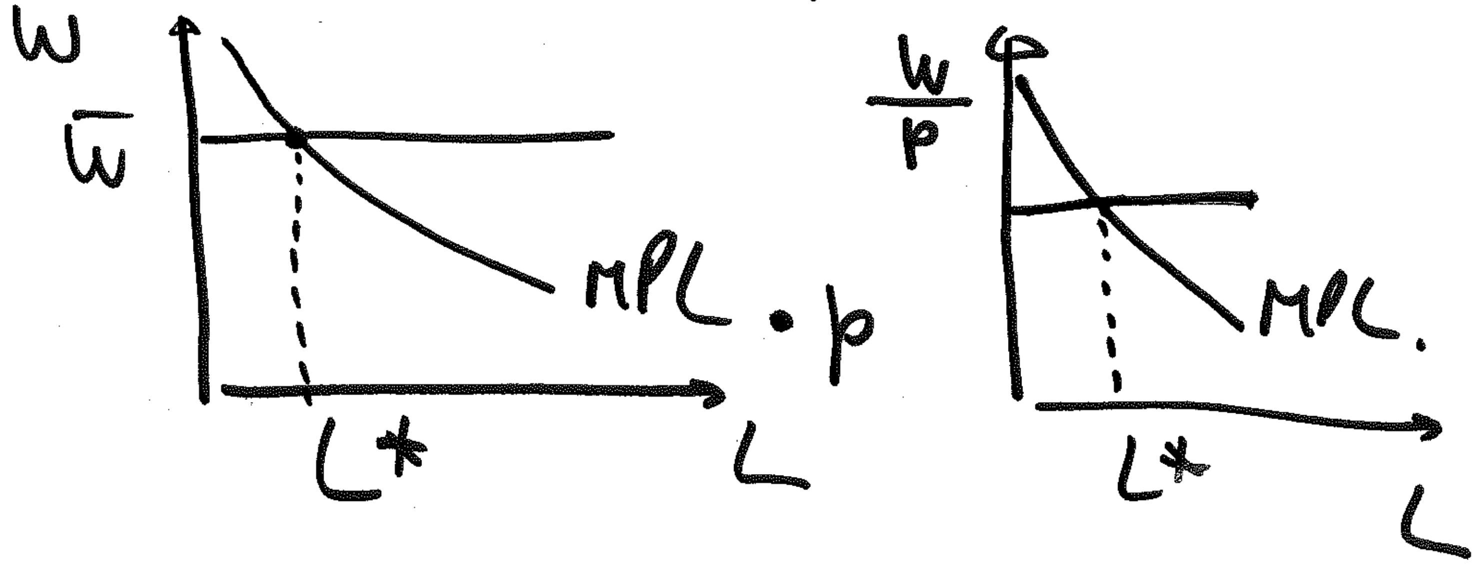
This is

$$p \cdot MPL(\ ) = W$$

$$\underbrace{p \cdot MPK(\ )}_{\text{VALUE OF MARG. PRODUCT}} = \underbrace{r}_{\text{FACTOR PRICE}}$$

EXAMPLES:

(a) "NICE" $f(L,K)$
WITH $\downarrow$ MPL, $\downarrow$ MPK



WE ASSUME THAT

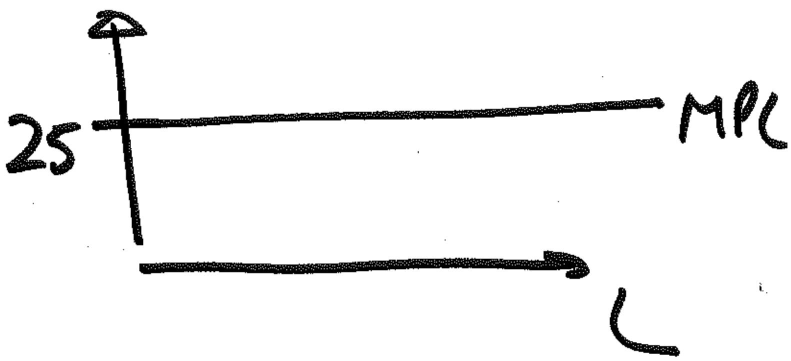
$\uparrow \frac{K}{L} \Rightarrow$ MPL $\uparrow$
MPK $\downarrow$

TECH. CHANGE SHIFTS BOTH MPL, MPK CURVES OUT

(b) LINEAR PROD. FUNCTION
ONE FACTOR: LABOR

$$q = 25 \cdot L \Rightarrow MPL = 25$$

$$a = \frac{1}{25}$$



$\Rightarrow$ EQUILIBRIUM OR PROFIT MAX IS DEFINED IFF

$$\frac{w}{p} = 25$$

- PPF

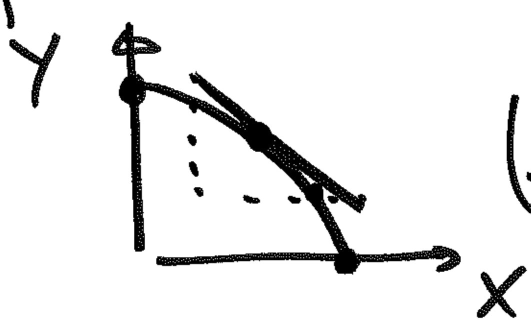
- 1 FACTOR , 2 GOODS
  L X, Y

$$\left.\begin{aligned} X &= f_X(L_X) \\ Y &= f_Y(L_Y) \end{aligned}\right] \text{TECHNOLOGY}$$

$$(*) \quad \underbrace{\bar{L}}_{\text{LABOR ENDOWMENT}} = L_X + L_Y \,] \text{ FEASIBILITY}$$

PPF: MAX $X = f_X(L_X)$
SUBJECT TO
$Y = f_Y(L_Y)$
& (*)

IF f'S ARE "NICE"
↓ ↓ MPRODUCTS



$$|\text{SLOPE}| = \left|\frac{\Delta Y}{\Delta X}\right| = \text{MRTS}$$

- ASSUME f's ARE LINEAR

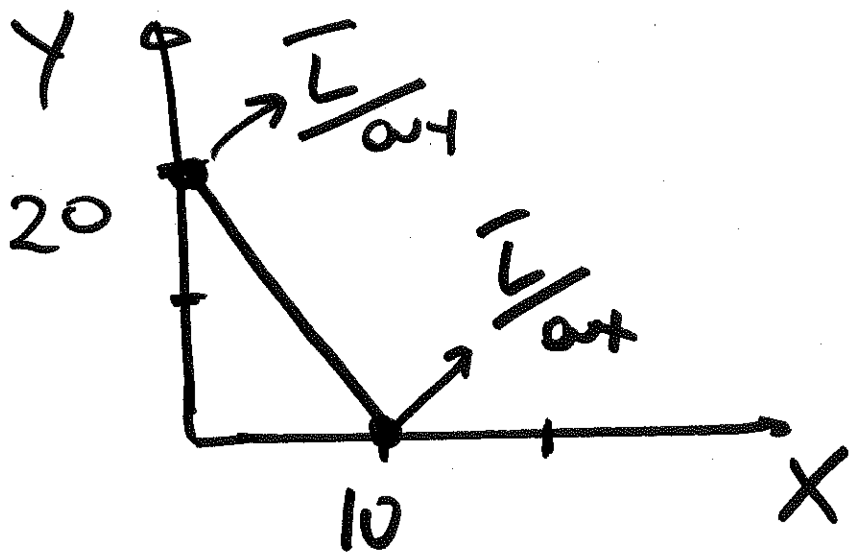
① $X = \frac{1}{10} \cdot L_X$

$\bar{L} = 100$

② $Y = \frac{1}{5} \cdot L_Y$

FEASIBILITY:

③ $100 = L_X + L_Y$



IF $Y = 0$

$\Rightarrow X = 10$

IF $X = 0$

$\Rightarrow Y = 20$

<u>MATH</u>: ① $\Rightarrow L_X = 10 \cdot X$

② $\Rightarrow L_Y = 5 \cdot Y$

$\boxed{100 = 10X + 5 \cdot Y}$ PPF EQ.

IN THIS CASE:

$MPL_X = 1/10 \ , \ a_X = 10$

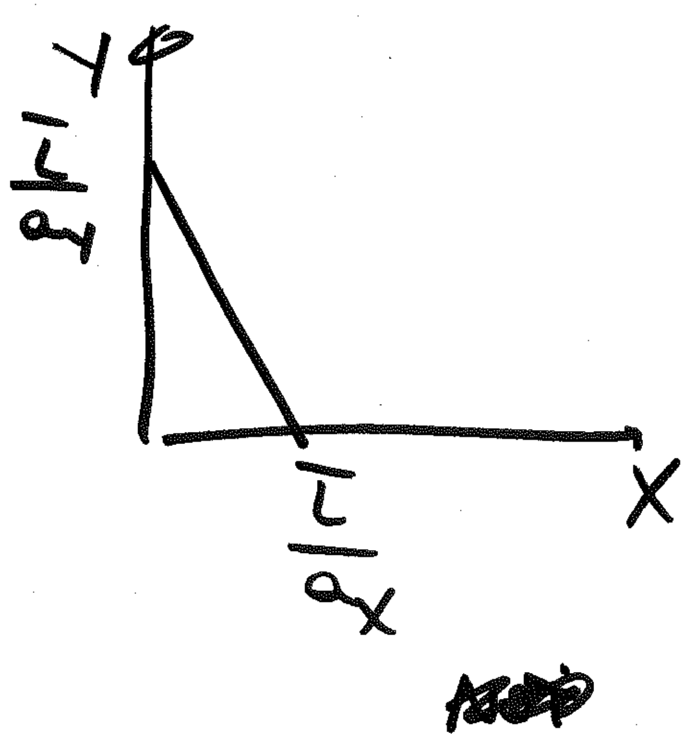
$MPL_Y = 1/5 \ , \ a_Y = 5$

IF NO X IS PRODUCED:

$$\bar{Y} = \frac{\bar{L}}{a_Y} = \frac{100}{5} = 20$$

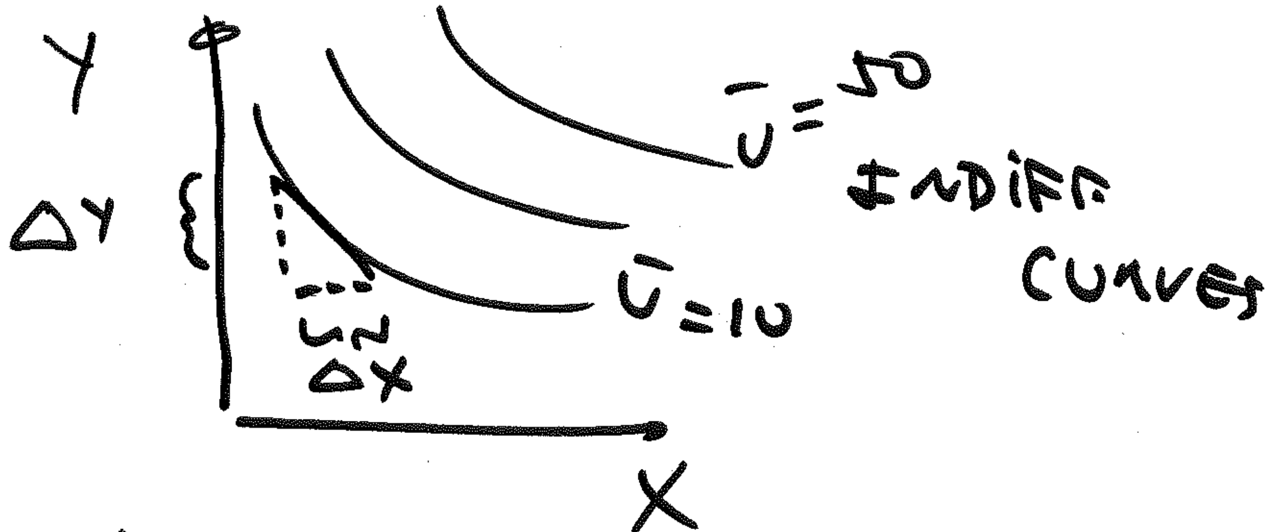
IF NO Y IS PRODUCED:

$$\bar{X} = \frac{\bar{L}}{a_X} = \frac{100}{10} = 10$$



$$\left|\begin{array}{c}\text{SLOPE} \\ \text{PPF}\end{array}\right| = \frac{\frac{\bar{L}}{a_Y}}{\frac{\bar{L}}{a_X}} = \frac{a_X}{a_Y} = \frac{10}{5} = \boxed{2}$$

PREFERENCES



$$\left|\frac{\Delta Y}{\Delta X}\right| = \left|\text{SLOPE OF IC}\right| = \text{MRS}$$

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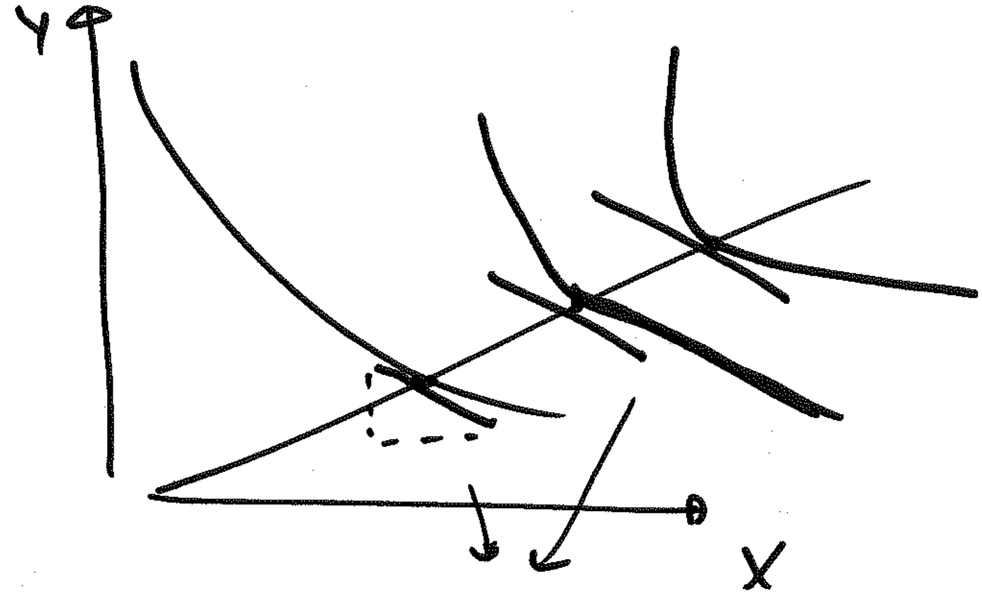
PREFERENCES

"NICE"



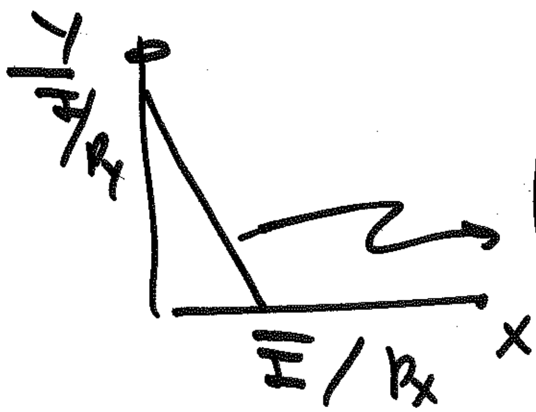
SAME SLOPE

"LINEAR EXPANSION PATH"

$MRS = \left| \begin{array}{l} \text{SLOPE} \\ \text{INDIFF. CURVE} \end{array} \right|$

BUDGET CONSTRAINT

$\bar{I} = p_Y \cdot Y + p_X \cdot X$



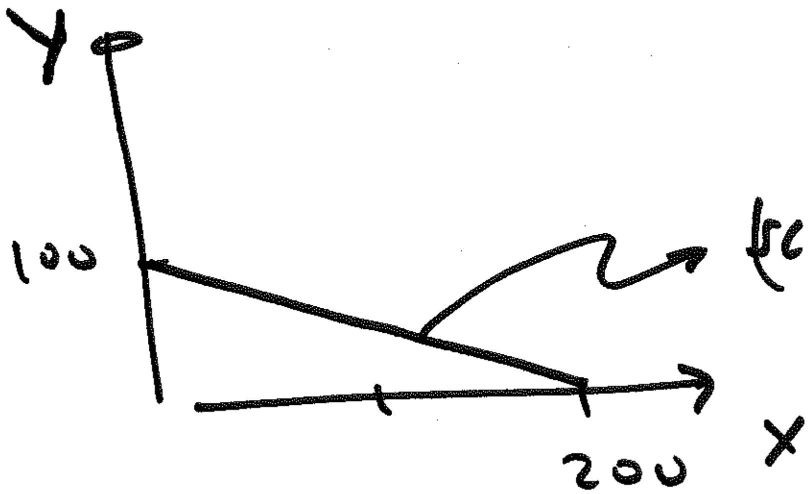
$|\text{SLOPE}| = \frac{p_X}{p_Y}$

ISOVALUE LINE

$$\bar{V} = p_x X + p_y Y$$

$\bar{V}$ → REVENUE = VALUE

Ex: $\bar{V} = 200$ $p_x = 1$ $p_y = 2$



$$|\text{SLOPE}| = \frac{\bar{V}/p_y}{\bar{V}/p_x} = \frac{p_x}{p_y}$$