The Role of Trade Flows in Exchange Rate Determination: A Rational Expectations Approach

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The purpose of this paper is to examine the interaction between the exchange rate and the trade balance within the framework of the portfolio approach to exchange rates and rational expectations. In a simplified linear version, it is shown that the difference between the spot exchange rate and its long-run equilibrium value is proportional to the current level of the trade-balance surplus normalized by the current stock of foreign-asset holdings. Therefore, the analysis provides some evidence in favor of the presumption that a surplus country should have an undervalued currency (relative to its long-run level). The basic idea behind the analysis is that, in a world of high capital mobility, current flow payments disequilibria can be accommodated by capital flows without need, in principle, for exchange rate movements. Only if the public expects a lasting change in the required rate of capital flows will exchange rates adjust, since in this case the expected time path of net foreign assets will be significantly affected.

This paper examines the interaction between trade flows and exchange rates within the recently developed “portfolio-balance” approach, in which the exchange rate is primarily determined by the condition that asset markets clear in a world of high capital mobility. Several models based on this analytical framework have recently been subjected to criticism by Niehans (1977) because of “... the tendency to downplay, by suitable simplifications, the flow aspects of the prob-

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lem, just as the elasticity theorists often neglected the stock aspects” (p. 1246). It is the main purpose of this paper to show that portfolio-balance models which incorporate the assumption of “rational expectations,” such as Dornbusch (1976), Kouri (1976), and Calvo and Rodríguez (1977), not only do not downplay the role of trade flows but also make them one of the fundamental factors in the determination of both the instantaneous equilibrium and the dynamic path of the exchange rate. In Section I Niehans’s criticism is considered in light of the more general issue of interdependence between stock and flow markets under rational expectations. In Section II, a simple portfolio-balance model with rational expectations is presented and then used to illustrate formally the causality relation between trade flows and exchange rates. The model concentrates on the effects of trade flows on the exchange rate—not vice versa, as was the concern of the elasticity theorists—by assuming that the trade balance is independent of the exchange rate. This assumption can be relaxed with no change in the basic conclusions, as is shown in the Appendix.

I

In order to illustrate Niehans’s point most directly, let us assume that domestic residents hold in their portfolios both domestic and foreign money in a proportion determined primarily by the expected change in the relative price between them (i.e., the exchange rate). In addition, assume that foreigners have no demand for domestic money and that there are no other internationally traded assets, so that all holdings of foreign money must be the result of past trade account surpluses. As a result, the stocks of domestic and foreign money are, for the domestic public, predetermined variables whose levels cannot be modified at a given point in time. Thus, given the existing stocks of domestic and foreign money and the expected change in their relative price, the assumption of instantaneous portfolio equilibrium is enough to determine the short-run equilibrium value of the exchange rate. If we define \( m, f, \) and \( s \) as the (natural logarithms of) existing domestic cash balances, the stock of foreign money, and the exchange rate, respectively, and \( L(s^*)^2 \) as the natural logarithm of the desired ratio of domestic money to the value of foreign currency holdings, where \( s^* \) is the expected rate of change in the exchange rate, then portfolio equilibrium requires

\[ x = \frac{dx}{dt}. \]
\[ m - s - f = L(s^*); \]  
(1)

or, given the values for \( m, f, \) and \( s^* \),

\[ s = m - f - L(s^*). \]  
(2)

Thus the exchange rate would appear to be determined without incorporating any information with respect to current or expected future developments in the flow markets of the economy.

In order to allow trade flows to be a direct determinant of the exchange rate, Niehans gives away the assumption of instantaneous portfolio equilibrium. In his view, equation (1) only defines the equilibrium demand for foreign currency, \( f^d \), such that

\[ f^d = m - s - L(s^*). \]  
(3)

The desired rate of accumulation of foreign currency is postulated by Niehans to be proportional to the difference between the desired and actual stocks:

\[ (f^d)^d = \lambda(f^d - f). \]  
(4)

Flow equilibrium in the asset market then requires that the desired rate of accumulation of foreign money be equal to its actual rate of change, which in turn equals the ratio of the trade balance to the actual holding of foreign money: \( \dot{f} = T/F \), where \( T \) is the trade account surplus and \( F \) is the level of foreign currency holdings. For the determination of the actual ratio of the trade balance to the stock of foreign money Niehans postulates an additional behavioral relationship of the form:

\[ \dot{f} = H(.). \]  
(5)

(The reader is left to choose his favorite argument for the \( H(\cdot) \) function). Finally, equality of (4) and (5) determines the short-run equilibrium level of the exchange rate:

\[ \lambda[m - s - L(s^*) - f] = H(.). \]  
(6)

This equilibrium condition incorporates flow considerations inasmuch as they are embedded in the \( H(\cdot) \) function. For example, a temporary deterioration in the trade balance, ceteris paribus, should decrease the value of the \( H(\cdot) \) function and lead to an instantaneous depreciation in the exchange rate, according to (6). This effect depends crucially on the assumed independence between expectations of exchange rate changes and developments in the trade account, for if the former were dependent on the latter, we would find an additional channel linking the exchange rate and the trade account which is independent from the one analyzed by Niehans. Such de-
dependence will arise if individuals form their expectations "rationally," that is, expectations of future developments are conditional forecasts based on the structure of the model describing the economy. In this case, expectations of exchange rate change must be endogenously determined. In a "rational" economy we need not resort to the assumption of portfolio disequilibrium in order to find a contemporaneous connection between the exchange rate and the level of the trade account.

Let us now explore further the relationship between the exchange rate and the trade balance under rational expectations and portfolio-balance equilibrium. Assume that the trade balance can be broken up into two components: (i) an exogenous or structural component and (ii) an endogenous component depending on wealth, income, relative prices, and so forth. For a small country, for example, an exogenous deterioration of its terms of trade or an exogenous increase in the absorption function would act as a deterioration in the structural component of the trade-balance surplus. Given enough time, it is reasonable to expect that the endogenous component of the trade balance would adjust so as to restore current payments equilibrium. To the extent, however, that the structure of the economy implies that such a shift in the structural component of the trade balance will be followed by a period of increased trade deficits, rational individuals will anticipate a decrease over time of the stock of foreign currency (required in order to finance the larger trade deficits) and will therefore expect the price of foreign exchange to increase over time in light of the portfolio-balance relationship (1). This higher expected rate of depreciation of the exchange rate induces, through portfolio balance, an instantaneous desired shift away from domestic money and into foreign money. Since both money stocks are fixed at any instant, the current exchange rate must therefore jump in order to restore portfolio balance.

Notice that this instantaneous jump in the exchange rate is triggered by the contemporaneous structural deterioration in the trade balance and does not require at all the assumption of portfolio disequilibrium. It can therefore safely be assumed that flow markets considerations do affect the exchange rate in a full equilibrium portfolio-balance model under rational expectations. Thus the dichotomy between stock and flow markets does not apply, in general, under rational expectations.3 It is well known that prices in a static general equilibrium system are, in general, dependent on the structural parameters describing the whole system and, therefore, that one

3The trivial exception occurs when the liquidity preference function does not depend on expectations at all.
cannot ascribe the determination of any one price to any single market. Introducing the time dimension, and therefore stock and flow markets, raises the possibility that some prices may be determined exclusively by one set of market equilibrium conditions (stock) which are independent from the rest of the market system. Under rational expectations, however, expectations about future asset prices are crucial elements in the market clearing process and are bound to depend on expected future developments in flow markets; the total independence of stock and flow markets is therefore not generally possible. To put it in simpler terms, a full equilibrium, portfolio-balance model of exchange rate determination under rational expectations will yield an equilibrium exchange rate determined jointly by stock equilibrium and expected developments in the flow markets.

A deterioration in the trade balance expected to last only for an instant would not have a significant effect on the exchange rate (zero effect in continuous time), but why should it in a world of rational speculators who are aware of the extremely temporary nature of the shock? By analogy, we do not expect the price level to move significantly in response to a transitory change in the rate of output during any given day (a Sunday?) if there are inventories managed by rational agents. Most observed developments in the trade balance are, however, of a more lasting nature, as has been the case, for example, with the recent U.S. deficits. In this case it seems that the dollar exchange rates have been determined largely by expectations that the trade deficits will persist rather than by the need to equilibrate desired current flow payments. In other words, the observed weakness of the dollar is likely to be due to an important structural deficit component in the U.S. trade balance that is expected to continue into the near future, coupled with a slow response of the trade balance to its endogenous determinants (so far the devaluation of the dollar has apparently failed to reduce the deficit). These characteristics have generated the expectation of a deteriorating net foreign asset position for the United States and, in turn, the fall of the dollar.

To summarize the argument of this section, in a world of high capital mobility current flow payments disequilibria can be easily accommodated by capital flows with no need, in principle, for exchange rate movements. Only if there is the expectation of a lasting change in the required rate of capital flows will exchange rates adjust, since in this case the expected time path of net foreign assets is affected.

The next section will rely on what is essentially a simplified version of Kouri's model to illustrate formally the main points made in the previous discussion.
II

The essential elements of the intuitive argument given above can be most directly and simply demonstrated by recourse to the "small country" assumption according to which the prices of all internationally traded goods are given to the country in terms of foreign exchange. For simplicity assume that there are no nontraded goods so that the domestic price level is fixed in terms of foreign currency and moves in line with the exchange rate when measured in terms of domestic currency. Also for simplicity assume that full employment always prevails.

At time $t$ there are in the economy stocks of domestic and foreign money which for simplicity are assumed to be demanded by domestic residents according to the following linear portfolio-balance relation ($a$ and $b$ are positive constants):

$$m(t) - s(t) - f(t) = a - \frac{1}{b} \cdot [ds(t)/dt], \quad (7)$$

where the opportunity cost of holding domestic money in terms of foreign money is equal to the actual rate of change in the exchange rate, given the assumption of rational expectations and the absence of uncertainty.\footnote{The inclusion of nontraded goods would only complicate the exposition without adding any substance to the present argument. A model incorporating nontraded goods and rational expectations is presented in Calvo and Rodriguez (1977).}

Given the time paths for the stocks of domestic and foreign money, the differential equation (7) can be solved to yield the equilibrium value for the spot exchange rate:

$$s(t) = s(t + T)e^{-bT} + h \int_{0}^{T} [m(t + x) - a - f(t + x)]e^{-bx}dx. \quad (8)$$

According to (8), which is formally equivalent to (7), the spot exchange rate at time $t$ is determined by its expected future value at time $t + T$ and by the time path of both domestic and foreign money during the time interval between $t$ and $t + T$. Allowing $T$ to go to infinity and assuming that the term $s(t + T)e^{-bT}$ converges to zero as $T$ approaches infinity, the equilibrium exchange rate $s(t)$ approaches the long-run expected exchange rate $s_{e}$.\footnote{Uncertainty is incorporated in an extended version of this paper (Rodriguez 1978) and does not lead to any fundamental change in the main results. For analytical simplicity I have therefore preferred to concentrate only on the limiting case of perfect foresight. For other models of exchange rate determination incorporating uncertainty under rational expectations, see the papers by Mussa (1978) and Bilson (1978) in Frenkel and Johnson (1978) and Black (1972). Extending the model to allow foreign money to earn a fixed rate of interest would be straightforward, although it could affect the stability properties for a large enough foreign rate of interest. For a more detailed analysis of the role of the service account see, e.g., Frenkel and Rodriguez (1975) and Rodriguez (1979).}
goes to infinity, we obtain the rational expectations equilibrium level for the spot rate:

\[ s(t) = b \int_0^\infty [m(t + x) - \alpha]e^{-βx} - b \int_0^\infty f(t + x)e^{-βx}dx. \]  

(9)

According to (9), the spot rate is determined as a linear function of the discounted values of all future levels of domestic and foreign money.

The future time path of domestic money holdings is fully determined by the monetary authorities; in order to avoid complications arising from domestic monetary disturbances, assume that the domestic money supply is set at the constant level \( \bar{m} \) for all \( t \). With this additional assumption, (9) simplifies further to

\[ s(t) = \bar{m} - \alpha - b \int_0^\infty f(t + x)e^{-βx}dx. \]  

(10)

It is clear from (10) that determination of the spot rate requires knowledge about the present level and the whole future time path of foreign currency holdings. While the present level of the foreign currency stock depends on the past trajectory of the trade balance (since the trade balance surplus equals the rate of acquisition of foreign currency), the future time path of foreign currency depends on the future time path of the trade balance. Without knowledge of the latter, therefore, it is impossible, in this model, to determine the spot exchange rate.

To illustrate further the relation between the exchange rate and the trade balance we now assume a specific structural form for the trade balance and proceed to derive an expression linking the spot exchange rate with its long-run equilibrium value and the current value of the trade balance. The rate of change in the stock of foreign currency \( (df/dt) \) equals the ratio of the trade balance surplus to the actual holdings of foreign currency:

\[ df/dt = T(t)/F(t), \]  

(11)

a ratio which we shall call the Normalized Trade Balance (NTB). In general, the NTB can be expected to depend on structural parameters as well as all other endogenous variables of the economy, that is, the stock of foreign currency and the exchange rate. In order to focus on the causality from the trade balance to the exchange rate, assume that the NTB does not depend directly on the exchange rate and that it is negatively related to the stock of foreign money (due to a positive

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*In terms of our model, this implies that rational speculators will only consider those perfect foresight paths which converge to the steady state. On the relevance of this assumption see, e.g., Brock (1974), Sargent and Wallace (1975), and Calvo (1979).*
wealth effect which raises absorption and therefore deteriorates the trade balance:

\[ \frac{df}{dt} = T_0 - \theta f(t), \quad \theta > 0. \quad (12) \]

In terms of our previous discussion in Section I, the constant term \( T_0 \) stands for the "structural" component of the trade balance while the term \( \theta f \) is the "endogenous" component. Niehans assumes that the endogenous component is a function of the exchange rate, but his alternative formulation does not lead to any significant change in our conclusion. In the Appendix, the more general case in which the NTB depends on both the stock of foreign currency and the exchange rate is analyzed.

By integration of (12) we derive the equivalent expression:

\[ f(t + x) = \frac{T_0}{\theta} + \left( f(t) - \frac{T_0}{\theta} \right) e^{-\theta x}. \quad (13) \]

It is clear from (13) that as time grows large (\( x \) increases), \( f(t + x) \) converges to its long-run value, \( \hat{f} = \frac{T_0}{\theta} \), which is independent from the initial value, \( f(t) \). Notice also from (12) that as \( f = \hat{f} \), the NTB equals zero so that the whole system reaches the steady state.

Substituting (13) into (10) and integrating, we obtain the following expression for the value for the spot exchange rate:

\[ s(t) = \bar{m} - \alpha - \frac{T_0}{\theta} + \frac{b}{\theta(b + \theta)} \left[ T_0 - \theta \hat{f}(t) \right]. \quad (14) \]

The second term in the right-hand side of (14) incorporates the effects of the current NTB on the spot exchange rate since \( \text{NTB}(t) = T_0 - \theta \hat{f}(t) \). We thus obtain

\[ s(t) = \bar{m} - \alpha - \frac{T_0}{\theta} + \frac{b}{\theta(b + \theta)} \text{NTB}(t). \quad (15) \]

As we have seen, in the long run the NTB tends to zero. As a consequence, from (15), the exchange rate tends in the long run to the value \( \bar{s} \), given by

\[ \bar{s} = \bar{m} - \alpha - \frac{T_0}{\theta}. \quad (16) \]

Substituting (16) into (15) we obtain our final expression for the determination of the spot exchange rate:

\[ s(t) = \bar{s} + \left[ \frac{b}{\theta(b + \theta)} \right] \text{NTB}(t). \quad (17) \]

According to this expression, the spot exchange rate will exceed its long-run value by a proportion of the current surplus in the NTB. The factor of proportionality increases with the interest elasticity of relative asset demand \( 1/b \) and decreases with the parameter \( \theta \), which indicates the speed at which the current NTB will tend to approach
zero over time. The smaller the strength of the self-correcting endogenous component of the NTB (i.e., the parameter \( \theta \)), the larger the impact of a current NTB surplus on the level of the spot exchange rate. Intuitively this is so because if \( \theta \) is small, a current surplus in the NTB will be expected to endure over a longer period and therefore to induce important transitional changes in the composition of assets.

We can therefore conclude that a country in NTB surplus should have an exchange rate which exceeds its long-run value (an undervalued currency); as the NTB approaches zero, the exchange rate will accordingly fall toward its long-run value. During the transition period between an initial NTB surplus and its long-run level of zero, we can expect to observe a path of appreciation of the domestic currency.

It should be emphasized that, in spite of having been derived from a full equilibrium portfolio-balance model, expression (17) establishes a direct connection between the spot rate and the current level of the normalized trade balance. In the deterministic setting that we have chosen only the current NTB is required to determine the spot rate, because, given the current NTB, we can derive its entire future time path. It is conceivable, however, that traders might know with certainty that a change in the exogenous component of the NTB will take place at some time in the future. Such a change, which might be of a permanent or temporary nature, will be discounted by rational traders and will thus have an impact on the spot rate. The impact will be proportional to the expected effect on the future trajectory of foreign currency; the precise effect on the spot rate can be computed from (10). It follows clearly from (10) that, other things being equal, the further into the future that this change is expected to take place, the smaller will be its effect, progressively, on the spot rate because, due to the discounting effect of the parameter \( \beta \), expected future levels of foreign-asset holdings carry less weight in the determination of the spot rate.

### III

In this paper I have examined the role of trade flows in the process of exchange rate determination in a full-equilibrium portfolio-balance model under rational expectations. It is concluded that trade flows are a fundamental determinant of both the current level and the future time path of the exchange rate. At any instant in time, the difference between the spot exchange rate and its long-run equilibrium value has

\[ \frac{d\text{NTB}}{dt} = -\theta \text{NTB}(t). \]
been shown to be proportional to the current value of the normalized trade balance. The analysis focused on the "absorption" approach to the trade balance and did not rely on any direct effects of the exchange rate on the trade balance. The main line of causation explored in the text is that from the trade balance to the exchange rate. The exchange rate was assumed to adjust continuously in order to maintain portfolio balance and was shown to respond to those developments in the trade balance which affect the expected time path of foreign-asset holdings. Thus, while it is correct that, in the model presented, the exchange rate must adjust to yield portfolio equilibrium, it is also the case that trade-balance phenomena do play a crucial role in exchange rate determination.

Appendix

We will now analyze the case where the NTB also improves with the level of the nominal exchange rate (if the trade balance deteriorates with a devaluation, a rational expectations solution may not exist). Assume the time path of the NTB is given by

\[ \text{NTB}(t) = \dot{f} = T_0 - \theta f(t) + q s(t), \quad \text{where } q > 0. \]  \hspace{1cm} (A1)

The time path of the exchange rate is described, as in the text, by

\[ \dot{s} = b[a - \ddot{\pi} + s(t) + f(t)]. \]  \hspace{1cm} (A2)

The determinant of the partial derivatives of the linear system (A1), (A2) is negative, implying that the characteristic roots of the system have opposite signs and therefore that the steady-state solution (which is unique) is a saddle point. The rational expectations path is the one described by the negative root of the system and, along that path, the deviations of \( f \) and \( s \) from their steady-state values are negatively related in the following linear form:

\[ (f - \hat{f}) = -k(s - \hat{s}), \]  \hspace{1cm} (A3)

where \( k \) is a positive parameter. (The reader can verify that, along the saddle path describing the rational expectations solution, \( f \) and \( s \) are negatively related by constructing the phase diagram describing the system.)

Equation (A1) can also be written in terms of deviations of the variables from their steady-state values:

\[ f = \text{NTB}(t) = -\theta (f - \hat{f}) + q(s - \hat{s}). \]  \hspace{1cm} (A4)

Substituting (A3) into (A4) and solving for \( s(t) \) we derive

\[ s(t) = \hat{s} + \left( \frac{1}{k\theta + q} \right) \text{NTB}(t), \]  \hspace{1cm} (A5)

which is analogous in all relevant respects to (17) derived in the text.

References

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